Théorème « Produit scalaire en composantes » (dans le plan)

Soit $\vec{v}\begin{pmatrix}v_1\\v_2\end{pmatrix}$ et $\vec{w}\begin{pmatrix}w_1\\w_2\end{pmatrix}$ deux vecteurs du plan. Alors on a : $\vec{v}\cdot\vec{w}=v_1 w_1+v_2 w_2$ -

Démonstration

On écrit : $\vec{v}=v_1\begin{pmatrix}1\\0\end{pmatrix}+v_2\begin{pmatrix}0\\1\end{pmatrix}$ et $\vec{w}=w_1\begin{pmatrix}1\\0\end{pmatrix}+w_2\begin{pmatrix}0\\1\end{pmatrix}$,

car [ARG 1: ……………………………………………………………………………………………….]

On a : $\vec{v}\cdot\vec{w}=\left(v_1\begin{pmatrix}1\\0\end{pmatrix}+v_2\begin{pmatrix}0\\1\end{pmatrix}\right)\cdot\left(w_1\begin{pmatrix}1\\0\end{pmatrix}+w_2\begin{pmatrix}0\\1\end{pmatrix}\right)$, car [ARG 2: ……………………………………….]

$$=\left(v_1\begin{pmatrix}1\\0\end{pmatrix}\right)\cdot\left(w_1\begin{pmatrix}1\\0\end{pmatrix}\right)+\left(v_1\begin{pmatrix}1\\0\end{pmatrix}\right)\cdot\left(w_2\begin{pmatrix}0\\1\end{pmatrix}\right)+\left(v_2\begin{pmatrix}0\\1\end{pmatrix}\right)\cdot\left(w_1\begin{pmatrix}1\\0\end{pmatrix}\right)+\left(v_2\begin{pmatrix}0\\1\end{pmatrix}\right)\cdot\left(w_2\begin{pmatrix}0\\1\end{pmatrix}\right),$$

car [ARG 3: ………………………………………………………………………………………….]

$$=v_1\cdot w_1\left(\begin{pmatrix}1\\0\end{pmatrix}\cdot\begin{pmatrix}1\\0\end{pmatrix}\right)+v_1\cdot w_2\left(\begin{pmatrix}1\\0\end{pmatrix}\cdot\begin{pmatrix}0\\1\end{pmatrix}\right)+v_2\cdot w_1\left(\begin{pmatrix}0\\1\end{pmatrix}\cdot\begin{pmatrix}1\\0\end{pmatrix}\right)+v_2\cdot w_2\left(\begin{pmatrix}0\\1\end{pmatrix}\cdot\begin{pmatrix}0\\1\end{pmatrix}\right),$$

car [ARG4 : …………………………………………………………………………………….]

$$=v_1\cdot w_1\left(\begin{pmatrix}1\\0\end{pmatrix}\cdot\begin{pmatrix}1\\0\end{pmatrix}\right)+v_1\cdot w_2(0)+v_2\cdot w_1(0)+v_2\cdot w_2\left(\begin{pmatrix}0\\1\end{pmatrix}\cdot\begin{pmatrix}0\\1\end{pmatrix}\right),$$

car [ARG 5: …………………………………………………………………………………….]

$$=v_1\cdot w_1\left(\begin{pmatrix}1\\0\end{pmatrix}\cdot\begin{pmatrix}1\\0\end{pmatrix}\right)+v_2\cdot w_2\left(\begin{pmatrix}0\\1\end{pmatrix}\cdot\begin{pmatrix}0\\1\end{pmatrix}\right),$$

car [ARG 6: …………………………………………………………………………………….]

$$=v_1\cdot w_1\left(\left\|\begin{matrix}1\\0\end{matrix}\right\|\cdot\left\|\begin{matrix}1\\0\end{matrix}\right\|\cdot\cos(0)\right)+v_2\cdot w_2\left(\left\|\begin{matrix}0\\1\end{matrix}\right\|\cdot\left\|\begin{matrix}0\\1\end{matrix}\right\|\cdot\cos(0)\right),$$

car [ARG 7: …………………………………………………………………………………….]

$$=v_1\cdot w_1(1\cdot 1\cdot\cos(0))+v_2\cdot w_2(1\cdot 1\cdot\cos(0)),$$

car [ARG 8: …………………………………………………………………………………….]

$$=v_1\cdot w_1(1\cdot 1\cdot 1)+v_2\cdot w_2(1\cdot 1\cdot 1),$$

car [ARG 9: …………………………………………………………………………………….]

$$=v_1\cdot w_1+v_2\cdot w_2$$