MA1 : exercices de préparation à la semestrielle de juin

Q1 :

(a) i) $x^2 - 3x + 4 = 0$

$\Delta = (-3)^2 - 4\cdot 4 = -7 < 0$

$S = \emptyset$

ii) $3x^2 - 6x - 1 = 0$

$\Delta = (-6)^2 - 4\cdot 3\cdot(-1) = 48$

$x_{1,2} = \frac{6 \pm \sqrt{48}}{6} = \frac{6 \pm \sqrt{16\cdot 3}}{6} = \frac{6 \pm 4\sqrt{3}}{6} = 1 \pm \frac{2\sqrt{3}}{3}$

$S = \left\{1 - \frac{2\sqrt{3}}{3} ; 1 + \frac{2\sqrt{3}}{3}\right\}$

(b) i) $7x^2 + 5x - 2$

$\Delta = 5^2 - 4\cdot 7(-2) = 81$

$x_{1,2} = \frac{-5 \pm \sqrt{81}}{14} = \frac{-5 \pm 9}{14}$

$x_1 = \frac{4}{14} = \frac{2}{7}$ et $x_2 = -\frac{14}{14} = -1$

$7x^2 + 5x - 2 = a(x - x_1)(x - x_2)$

$= 7(x - \frac{2}{7})(x+1)$

$= 7(\frac{7x-2}{7})(x+1)$

$= (7x-2)(x+1)$

ii) $3(x-1)^2 - 9(x-1)$

$= 3(x-1)[(x-1) - 3]$

$= 3(x-1)(x-4)$

iii) $7a^3b^2c - 14a^2b^2c^2 + 28ab^3c$

$= 7ab^2c[a^2 - 2ac + 4b]$

iv) $(5x+4)(9x-5) - (12x+7)(5x+4)$

$= (5x+4)[9x - 5 - (12x+7)]$

$= (5x+4)(-3x-12)$

$= (5x+4)(-3)\cdot(x+4)$

v) $49x^2 + 28x + 4$

$= (7x+2)^2$

vi) $x^2 + 5x - 14 = (x+7)(x-2)$

Q3 : forme canonique : $f(x) = a(x-k)^2 + m$ avec $S = \left(-\frac{b}{2a} ; -\frac{\Delta}{4a}\right) = (-1 ; -3)$

$= a(x+1)^2 - 3$

$(0;-1) \in f \Leftrightarrow -1 = a(0+1)^2 - 3$

$\Leftrightarrow -1 = a - 3$

$\Leftrightarrow a = 2$ $\quad f(x) = 2(x+1)^2 - 3$ forme canonique

$f(x) = 2(x^2 + 2x + 1) - 3 = 2x^2 + 4x - 1$ forme développée

$\Delta = b^2 - 4ac = 4^2 - 4\cdot 2(-1) = 24$

$x_{1,2} = \frac{-4 \pm \sqrt{24}}{4} = \frac{-4 \pm 2\sqrt{6}}{4} = -1 \pm \frac{\sqrt{6}}{2}$ $\Rightarrow$ $f(x) = a(x - x_1)(x - x_2)$

$= 2(x - [-1 - \frac{\sqrt{6}}{2}])(x - [-1 + \frac{\sqrt{6}}{2}])$

$= 2(x + 1 + \sqrt{6}/2)(x + 1 - \sqrt{6}/2)$

forme factorisée

Q2 : $f(x) = -x^2 + 2x + 3 = -(x^2 - 2x - 3) = -(x-3)(x+1)$

$O.O. : f(0) = 3$

$Z_f : f(x) = 0 \Leftrightarrow -(x-3)(x+1) = 0$

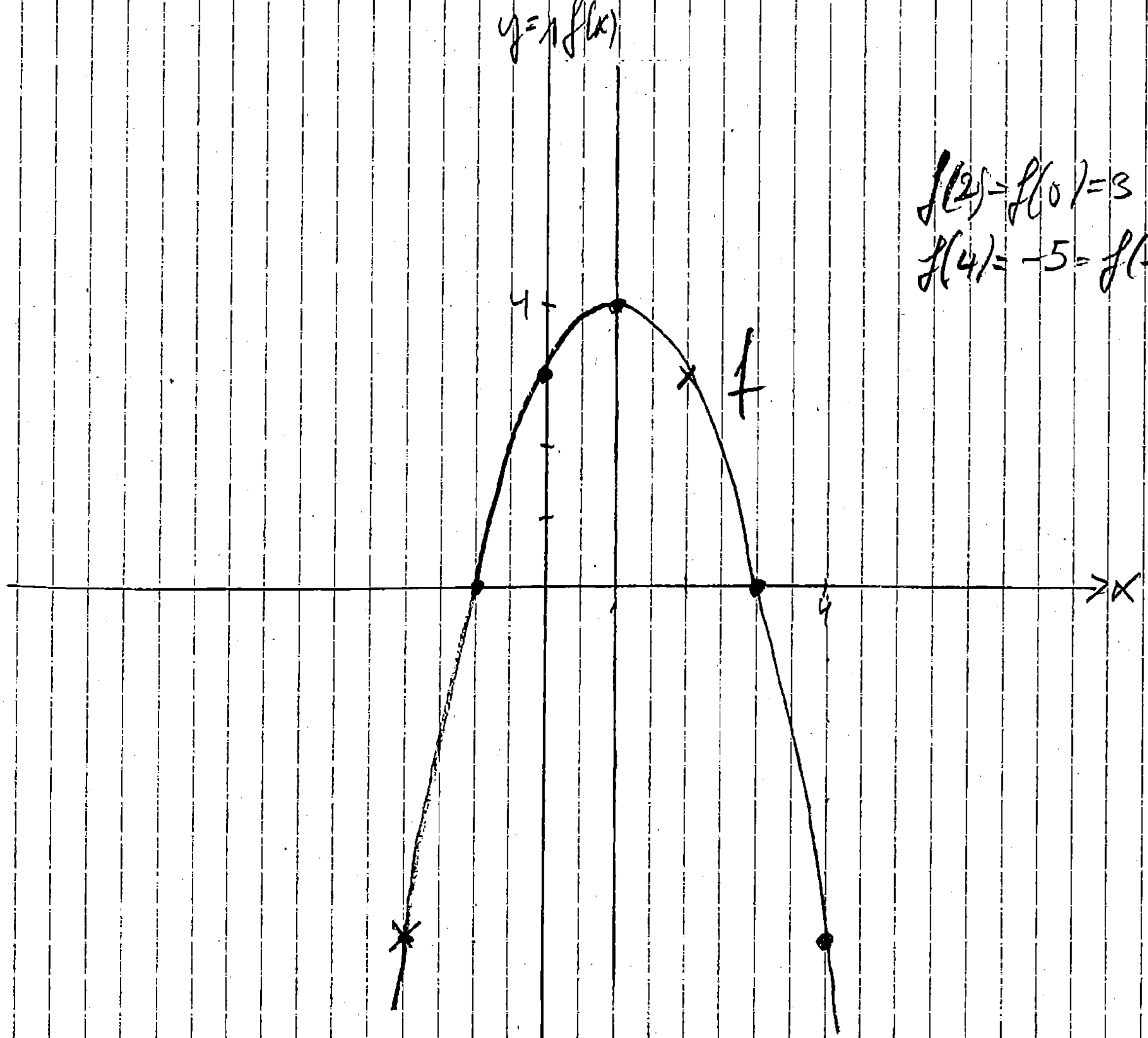
$x-3=0$ ou $x+1=0$

$x=3$ $x=-1$ $Z_f = \{-1; 3\}$

axe : $x = -\frac{b}{2a} = \frac{-2}{-2} = 1$

sommet $S = \left(-\frac{b}{2a}; -\frac{\Delta}{4a}\right)$ ou $S = \left(-\frac{b}{2a}; f\left(-\frac{b}{2a}\right)\right)$

$= (+1; f(1)) = (1; 4)$



$f(2) = f(0) = 3$

$f(4) = -5 = f(-2)$

Q4

(a) $P = 2x + y = 300$

$\Leftrightarrow y = 300 - 2x$

$0 < x < 150 \Rightarrow D_{\text{prob}} =]0 ; 150[$

(b) $A(x) = x \cdot (300 - 2x) = -2x^2 + 300x$

(c) Sommet $S = \left(-\frac{b}{2a} ; f\left(-\frac{b}{2a}\right)\right)$ ou $S = \left(-\frac{b}{2a} ; -\frac{\Delta}{4a}\right)$

$= \left(\frac{-300}{-4} ; \ldots\right)$

$= (75 ; f(75)) = (75 ; -2 \cdot 75^2 + 300 \cdot 75) = (75 ; 11250)$

l'aire est maximale pour $x = 75$ m : elle vaut alors 11250 m²

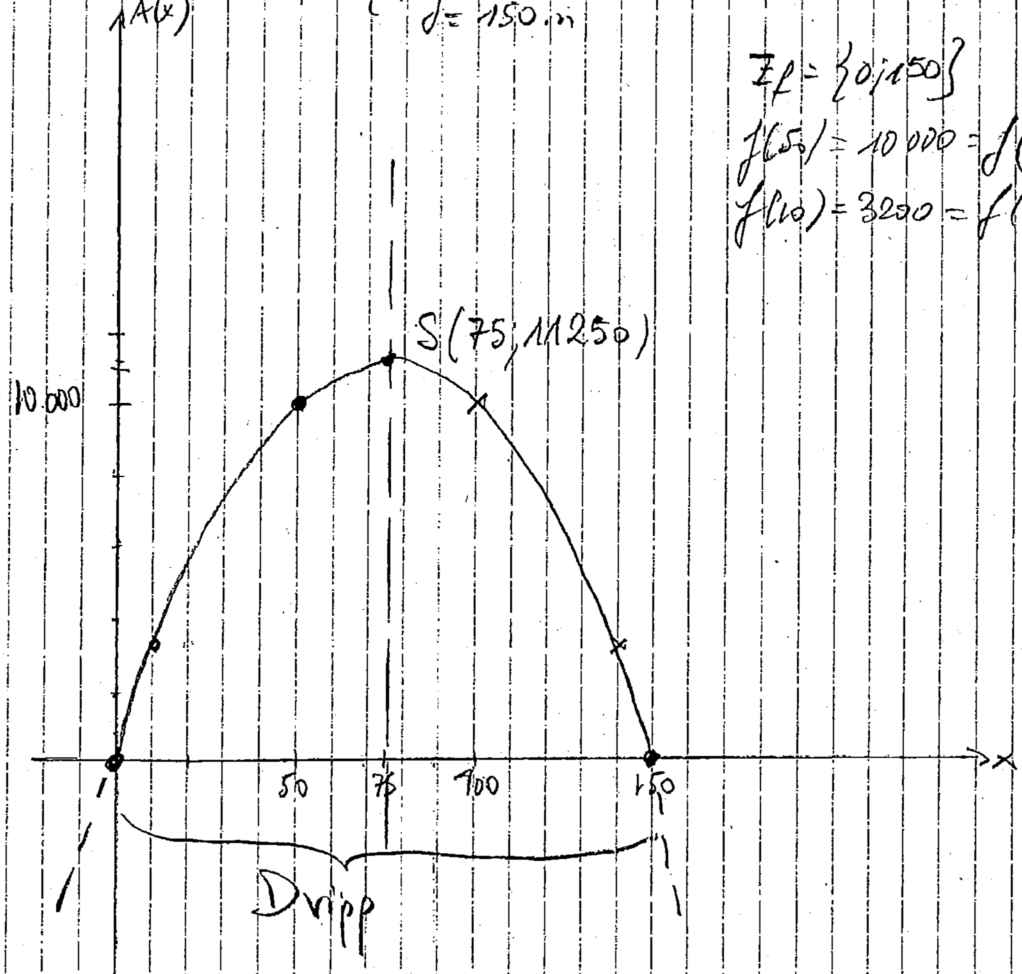
(et $y = 300 - 2 \cdot 75$
$y = 150$ m)

(d)

$Z_f = \{0 ; 150\}$

$f(50) = 10\,000 = f(100)$

$f(10) = 3200 = f(140)$



Q5 :

(a) ΔABD rectangle en A [par thm "cercle de Thalès"]

(b) ΔABD rectangle, 1 côté + 1 angle connus $\Rightarrow$ trigo : $\cos(21) = \dfrac{144}{\overline{BD}}$

$\Leftrightarrow \overline{BD} = \dfrac{144}{\cos(21)} \simeq 154,24$

(c) $\angle AOD = 2\beta$ [par thm "angle au centre / inscrit"]

$= 2 \cdot 21$ [substitution]

$= 42°$

(d) on pose $\angle ACD = \gamma$, $\angle BHA = \varepsilon$, $\angle DHC = \varepsilon'$, $\angle CDH = \delta$ et $\angle HAB = \delta'$

on a : • ε et ε' opposés [def "∠ opp"]

donc $\varepsilon = \varepsilon'$ [thm "∠ opp"]

• $\left.\begin{array}{l}\beta = \gamma \\ \delta = \delta'\end{array}\right\}$ [thm "∠ inscrits"]

d'où $\Delta ABH \sim \Delta CDH$ [def "Δ semblables"]

(e) $\dfrac{\overline{CD}}{\overline{AB}} = \dfrac{\overline{CH}}{\overline{BH}} = \dfrac{\overline{DH}}{\overline{AH}}$ [par "thm Thalès"]

$\Leftrightarrow \dfrac{\overline{CD}}{144} = \dfrac{\overline{CH}}{\overline{BH}} = \dfrac{50}{60}$ [substitution]

$\overline{CD} = \dfrac{144 \cdot 50}{60}$ [$\cdot 144$]

$= 120$ ([simplification])

(f) on pose h la hauteur issue de A dans ΔODA et $H = h \cap [OD]$

on a : $\overline{OA} = \overline{OB}$ [rayons du cercle]

$= \overline{BD}/2$

$= 77,12$

$\sin(42) = \dfrac{\overline{AH}}{\overline{OA}} \Leftrightarrow \overline{AH} = \overline{OA} \sin(42) = 77,12 \sin(42) \simeq 51,6$

$\text{Aire}(\Delta OAD) = \dfrac{\overline{OD} \cdot \overline{AH}}{2} = \dfrac{77 \cdot \overline{AH}}{2} \simeq 1986,7$

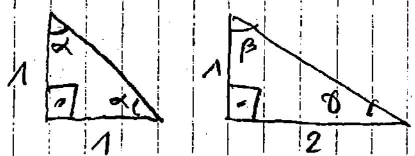
Q6 : $P = \overline{AB} + \overline{BC} + \overline{AC}$

$\cos(30) = \frac{2}{\overline{AB}} \Leftrightarrow \overline{AB} = \frac{2}{\cos(30)} = \frac{2}{\sqrt{3}/2} = 2 \cdot \frac{2}{\sqrt{3}} = \frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$

$\tan(30) = \frac{\overline{BC}}{2} \Leftrightarrow \overline{BC} = 2\tan(30) = \frac{2\sqrt{3}}{3}$

$P = \frac{4\sqrt{3}}{3} + \frac{2\sqrt{3}}{3} + 2 = \frac{6\sqrt{3}}{3} + 2 = 2\sqrt{3} + 2$

Q7 : (a) Faux ; contre exemple



isocèle donc 2 angles égaux

aucun angle égal à un autre

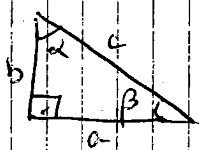
(b) Vrai ; dém : les 2 Δ sont équilatéraux [hyp]

donc ils ont 3 côtés égaux [déf "Δ équil"]

donc 2 côtés égaux

donc ils sont isocèles [déf "Δ isoc"]

(c) (i) vrai ; dém



- $\alpha + \beta + 90 = 180$ [th. $\Sigma \angle \Delta = 180$]
  $\alpha + \beta = 90$ [-90]
  $\beta = 90 - \alpha$ [$-\alpha$]
- $\cos(\beta) = \frac{a}{c}$ [déf "cos"]
  $\sin(\alpha) = \frac{a}{c}$ [déf "sin"]

donc $\sin(\alpha) = \cos(\beta) = \cos(90-\alpha)$ [subst.]

(ii) faux ; c-ex $\alpha = 30 \Rightarrow \cos(2 \cdot 30) \overset{?}{=} 2\cos(30)$
$\cos(60) \overset{?}{=} 2 \cdot \sqrt{3}/2$
$1/2 \overset{?}{=} 2\sqrt{3}/2$ non

(iii) faux ; c-ex $\alpha = 30 \Rightarrow \cos(30-90) \overset{?}{=} \cos(30)$
$\cos(60) \overset{?}{=} \cos(30)$
$\frac{1}{2} \overset{?}{=} \sqrt{3}/2$ non

(iv) faux ; c-ex $\alpha = 5 \rightarrow \sin(5^2) \overset{?}{=} [\sin(5)]^2$
$0{,}42 \overset{?}{=} 0{,}008$ non

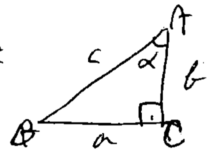
Q7 (suite)

(v) faux : c-ex $\alpha = 45 \Rightarrow \tan(45) \overset{?}{=} \sin(45) - \cos(45)$

$$1 \overset{?}{=} \sqrt{2}/2 - \sqrt{2}/2$$

$$1 \overset{?}{=} 0 \quad \text{non}$$

(vi) vrai : dém



$$\left.\begin{aligned} \sin(\alpha) &= \frac{a}{c} \\ \cos(\alpha) &= \frac{b}{c} \\ \tan(\alpha) &= \frac{a}{b} \end{aligned}\right\} \text{ par définition}$$

donc $\dfrac{\sin(\alpha)}{\cos(\alpha)} = \dfrac{a/c}{b/c} = \dfrac{a}{\cancel{c}} \cdot \dfrac{\cancel{c}}{b} = \dfrac{a}{b} = \tan(\alpha)$

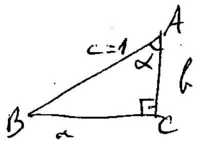
(vii) faux : c-ex $\alpha = 45 \rightarrow \sin^2(45) - \cos^2(45) \overset{?}{=} 1$

$$\left(\frac{\sqrt{2}}{2}\right)^2 - \left(\frac{\sqrt{2}}{2}\right)^2 \overset{?}{=} 1$$

$$\frac{2}{4} - \frac{2}{4} \overset{?}{=} 1$$

$$0 \overset{?}{=} 1 \quad \text{non}$$

(viii) vrai : dém



on choisit $c = 1$

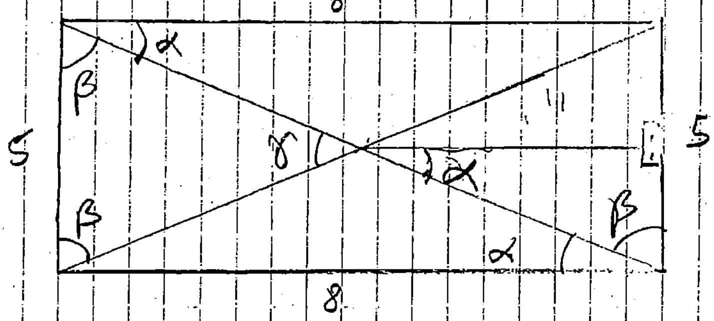
on a : $\left.\begin{aligned} \sin(\alpha) &= \frac{a}{c} = \frac{a}{1} = a \\ \cos(\alpha) &= \frac{b}{c} = b \end{aligned}\right\}$ par définition

donc $\sin^2(\alpha) + \cos^2(\alpha) = a^2 + b^2$

$= 1^2$ [par thm Pythagore]

$= 1$.

Q8 :

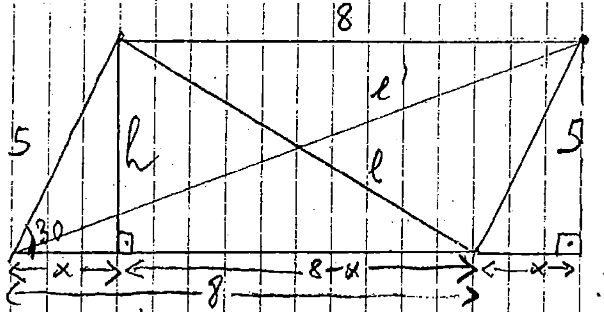


a) $\tan(\alpha) = \frac{5}{8} \Leftrightarrow \alpha = \tan^{-1}(5/8) \simeq 32°$

$\tan(\beta) = \frac{8}{5} \Leftrightarrow \beta = \tan^{-1}(8/5) \simeq 58°$

b) $\gamma = 2\alpha \simeq 64°$

c)



$\sin(30) = \frac{h}{5} \Leftrightarrow h = 5\sin(30) = 5/2$

$\cos(30) = \frac{x}{5} \Leftrightarrow x = 5\cos(30) = 5\sqrt{3}/2$

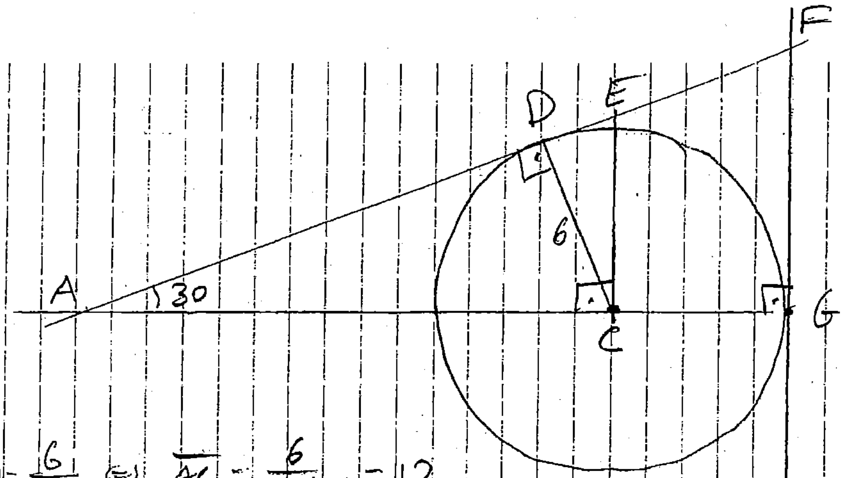
$l^2 = h^2 + (8-x)^2 = \frac{25}{4} + \left(8 - \frac{5\sqrt{3}}{2}\right)^2 = \frac{25}{4} + 64 - 40\sqrt{3} + \frac{25 \cdot 3}{4} = 89 - 40\sqrt{3}$

d'où $l = \sqrt{89 - 40\sqrt{3}} \simeq 4{,}44$

d) $l' = \sqrt{(8+x)^2 + h^2} = \sqrt{\left(8 + \frac{5\sqrt{3}}{2}\right)^2 + \frac{25}{4}} = \sqrt{64 + 40\sqrt{3} + \frac{25 \cdot 3}{4} + \frac{25}{4}}$

$= \sqrt{89 + 40\sqrt{3}} \simeq 12{,}5$

Q9 :



△ACD

$\sin(30) = \frac{6}{\overline{AC}} \Leftrightarrow \overline{AC} = \frac{6}{\sin(30)} = 12$

$\tan(30) = \frac{6}{\overline{AD}} \Leftrightarrow \overline{AD} = \frac{6}{\tan(30)} = 6\sqrt{3}$

△ACE

$\tan(30) = \frac{\overline{EC}}{\overline{AC}} \Leftrightarrow \overline{EC} = \overline{AC}\tan(30) = 4\sqrt{3}$

$\cos(30) = \frac{\overline{AC}}{\overline{AE}} \Leftrightarrow \overline{AE} = \frac{\overline{AC}}{\cos(30)} = \frac{12}{\cos(30)} = 8\sqrt{3}$

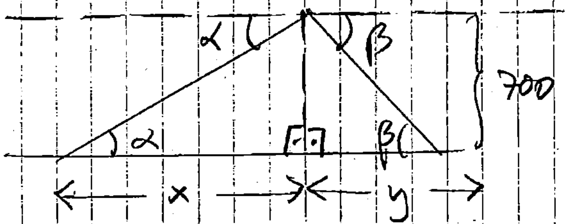
△AGF

$\overline{AG} = \overline{AC} + \overline{CG} = 12 + 6 = 18$

$\tan(30) = \frac{\overline{FG}}{\overline{AG}} \Leftrightarrow \overline{FG} = \overline{AG}\tan(30) = 18 \cdot \tan(30) = 6\sqrt{3}$

$\sin(30) = \frac{\overline{FG}}{\overline{AF}} \Leftrightarrow \overline{AF} = \frac{\overline{FG}}{\sin(30)} = \frac{6\sqrt{3}}{\sin(30)} = 12\sqrt{3}$

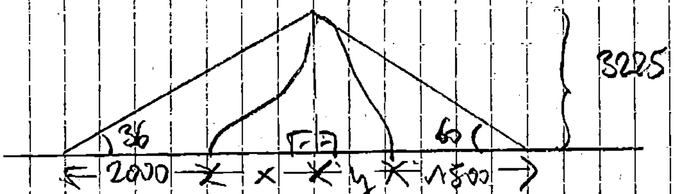
Q10 :



$\tan(\alpha) = \dfrac{700}{x} \Leftrightarrow x = \dfrac{700}{\tan(48)}$ et $\tan(\beta) = \dfrac{700}{y} \Leftrightarrow y = \dfrac{700}{\tan(39)}$

$L = \dfrac{700}{\tan(48)} + \dfrac{700}{\tan(39)} \cong 1494{,}7$ m

Q11 :



$\tan(36) = \dfrac{3225}{2000+x}$ et $\tan(60) = \dfrac{3225}{y+1500}$

$\Leftrightarrow 2000 + x = \dfrac{3225}{\tan(36)}$ | $\Leftrightarrow y + 1500 = \dfrac{3225}{\tan(60)}$

$\Leftrightarrow x = \dfrac{3225}{\tan(36)} - 2000$ | $y = \dfrac{3225}{\tan(60)} - 1500$

L la longueur du tunnel $= x + y \cong 2800{,}8$ m

Q12 : (a) $\angle TAH$ et $\angle TMS$ correspondants [déf "∡ corr"]
(dAH // dMS [hypothèse])
donc $\angle TAH = TMS$ [Ax "∡ corr"]
idem pour $\angle THA = \angle TSM$
$\angle HTA$ commun
d'où $\Delta TAH \sim \Delta TMS$ [déf "Δ semblables"]
donc $\dfrac{\overline{TA}}{\overline{TM}} = \dfrac{\overline{TH}}{\overline{TS}} = \dfrac{\overline{AH}}{\overline{MS}}$ [thm "Thalès"]

(b) $\sin(\angle AHT) = \dfrac{\overline{TA}}{\overline{AH}}$

(c) d'où $\dfrac{\overline{TA}}{\overline{TM}} = \dfrac{\overline{AH}}{\overline{MS}} \Leftrightarrow \dfrac{\overline{TA}}{\overline{AH}} = \dfrac{\overline{TM}}{\overline{MS}}$ [$\cdot\overline{TM}$ et $\div\overline{AH}$]
$= \dfrac{15}{23}$ [substitution] $= \sin(\angle AHT)$