Ma2 2008-09 Corrigé semestrielle juin 2009

[total 66 pts]

[/12]

Question 1 :

D : pb si $x=0$

pb si $x+1=0$, $x=-1$ } donc $D=\mathbb{R}\setminus\{-1;0\}$ (2)

$$\frac{1}{x}\leq\frac{4x+1}{x+1}\iff\frac{1}{x}-\frac{4x+1}{x+1}\leq 0$$ (1)

$$\iff\frac{(x+1)-x(4x+1)}{x(x+1)}\leq 0$$

$$\iff\frac{\cancel{x}+1-4x^2-\cancel{x}}{x(x+1)}\leq 0$$

$$\iff\frac{-4x^2+1}{x(x+1)}\leq 0$$

$$\iff\frac{1-4x^2}{x(x+1)}\leq 0$$

$$\iff\frac{(1-2x)(1+2x)}{x(x+1)}\leq 0$$ (3)

tds :

x		-1		$-1/2$		0		$1/2$	
$1-2x$	+	+	+	+	+	+	+	0	−
$1+2x$	−	−	−	0	+	+	+	+	+
x	−	−	−	−	−	0	+	+	+
$x+1$	−	0	+	+	+	+	+	+	+
$\frac{(1-2x)(1+2x)}{x(x+1)}$	−	||	+	0	−	||	+	0	−

(4)

$$S=\left]-\infty;-1\right[\cup\left[-\frac{1}{2};0\right[\cup\left[\frac{1}{2};+\infty\right[$$ (2)

[1/14]

Question 2 :

a) $180 = 45 \cdot 4^{-\frac{x}{5}}$

$\Leftrightarrow \frac{180}{45} = 4^{-x/5}$

$\Leftrightarrow 4 = 4^{-x/5}$

$\Leftrightarrow 4^1 = 4^{-x/5}$

$\Leftrightarrow 1 = -\frac{x}{5}$

$\Leftrightarrow 5 = -x$

$\Leftrightarrow x = -5$

$S = \{-5\}$ 5

b) D : pb si $x-2 \leq 0$
$x \leq 2$

pb si $x+1 \leq 0$
$x \leq -1$

donc $D =]2 ; +\infty[$ 2

$\log(x-2) + \log(x+1) = 1$

$\Leftrightarrow \log((x-2)(x+1)) = \log(10)$ 4

$\Leftrightarrow (x-2)(x+1) = 10$

$\Leftrightarrow x^2 - 2x + x - 2 - 10 = 0$

$\Leftrightarrow x^2 - x - 12 = 0$

$\Leftrightarrow (x-4)(x+3) = 0$

$x = 4$ ou $x = -3$
$\in D$ $\notin D$ 2

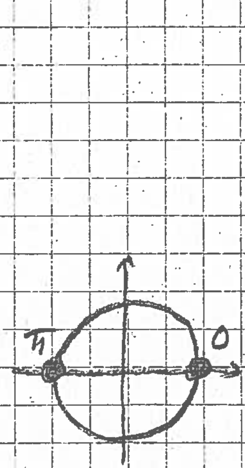
$S = \{4\}$ 1

[/20]

Question 3 :

a) $f(x)=0 \iff -3\sin\left(\frac{\pi}{2}-2x\right)=0$

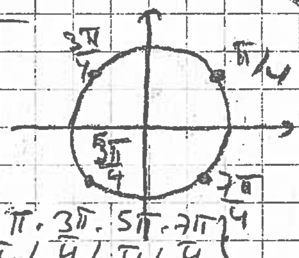
$\iff \sin\left(\frac{\pi}{2}-2x\right)=0$



$\frac{\pi}{2}-2x=0+k\pi$

$\iff -2x=-\frac{\pi}{2}+k\pi$

$\iff x=\frac{\pi}{4}-\frac{k\pi}{2}$



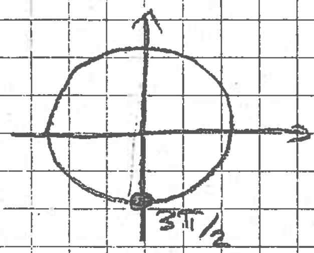
(4)

$S_{[-\pi,2\pi]}=\left\{-\frac{3\pi}{4};-\frac{\pi}{4};\frac{\pi}{4};\frac{3\pi}{4};\frac{5\pi}{4};\frac{7\pi}{4}\right\}$ (2)

b) $f(x)=3 \iff -3\sin\left(\frac{\pi}{2}-2x\right)=3$

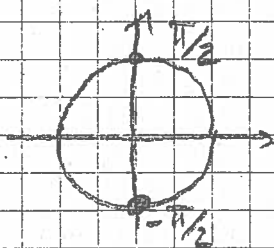
$\iff \sin\left(\frac{\pi}{2}-2x\right)=-1$

$\iff \frac{\pi}{2}-2x=\frac{3\pi}{2}+k2\pi$



$\iff -2x=\pi+k2\pi$

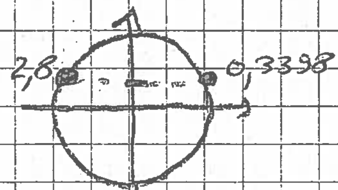
(4) $\iff x=-\frac{\pi}{2}-k\pi$



(2) $S_{[-\pi,2\pi]}=\left\{-\frac{\pi}{2};\frac{\pi}{2};\frac{3\pi}{2}\right\}$

c) $f(x)=1 \iff -3\sin\left(\frac{\pi}{2}-2x\right)=1$

$\iff \sin\left(\frac{\pi}{2}-2x\right)=-\frac{1}{3}$



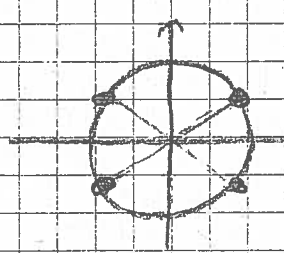
$\frac{\pi}{2}-2x\cong 0,3398+k2\pi$

(2) $\iff x\cong 0,6155-k\pi$ ou $\frac{\pi}{2}-2x\cong 2,8018+k2\pi$

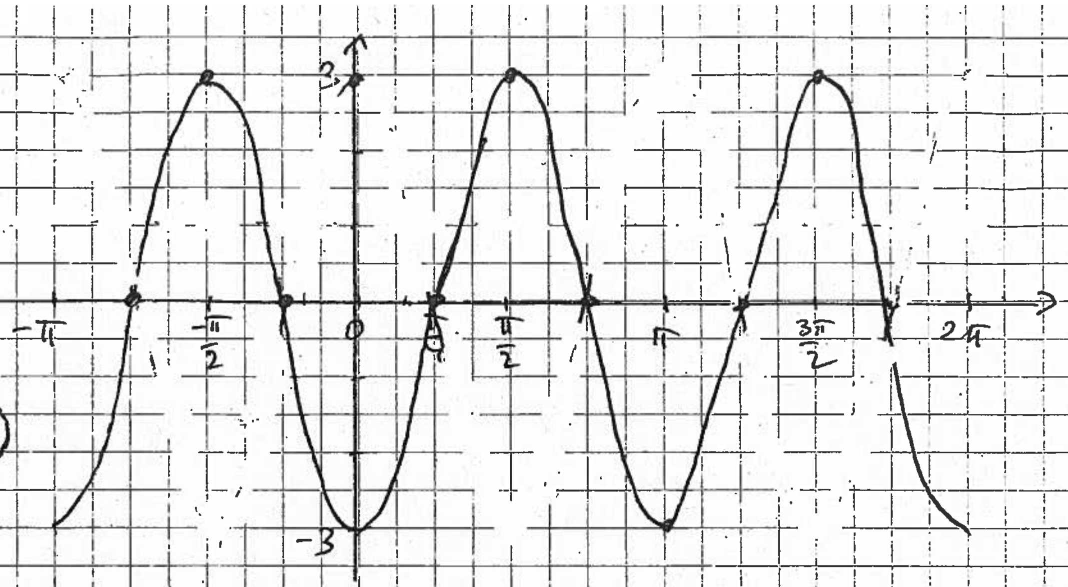
$\iff x\cong -0,6155-k\pi$

(2) $S_{\mathbb{R}}\cong\{\pm 0,62+k\pi\}$

$(\cong\{\pm 35,3°+k\pi\})$



d)



2+2

1/20

Question 4

Modèle continu :

a) $C(t) = C_0 \cdot e^{it}$

$\Leftrightarrow 3\cancel{C_0} = \cancel{C_0} e^{0,06 \cdot t}$

$\Leftrightarrow 0,06t = \ln(3)$

$\Leftrightarrow t = \frac{\ln(3)}{0,06} \cong 18,31 \text{ ans}$

b) 2 ans $\Rightarrow$ 24 mois

$C(t) = C_0 e^{it}$

dimin. de 10% $\Rightarrow$ reste 90%

$0,9\cancel{C_0} = \cancel{C_0} e^{i 24}$

$\Leftrightarrow i = \frac{\ln(0,9)}{24} \cong -0,00439$

$\cong -0,44\%$

Modèle exp "par année" :

2 a) $C(t) = C_0 (1+i)^t$

2 $\Leftrightarrow 3\cancel{C_0} = \cancel{C_0} (1,06)^t$

$\Leftrightarrow \log(3) = \log[(1,06)^t]$

2 $\Leftrightarrow \log(3) = t \log(1,06)$

$\Leftrightarrow t = \frac{\log(3)}{\log(1,06)} \cong 18,85 \text{ ans}$

b) $0,9\cancel{C_0} = \cancel{C_0}(1+i)^{24}$

$1+i = \sqrt[24]{0,9}$

$i = \sqrt[24]{0,9} - 1 \cong -0,00438$

$\cong -0,438\%$

c) $C(t) = C_0 e^{0,06 \cdot t} \cdot e^{-0,03(20-t)}$

$\Leftrightarrow 2\cancel{C_0} = \cancel{C_0} e^{0,06t - 0,03(20-t)}$

$\Leftrightarrow 0,06t - 0,03(20-t) = \ln 2$

$\Leftrightarrow t = \frac{\ln(2) + 0,03 \cdot 20}{0,09} \cong 14,37 \text{ ans}$

3

5

$C(t) = C_0 (1+0,06)^t \cdot (1-0,03)^{20-t}$

$\Leftrightarrow 2\cancel{C_0} = \cancel{C_0} (1,06)^t (0,97)^{20-t}$

$\Leftrightarrow 2 = \left(\frac{1,06}{0,97}\right)^t (0,97)^{20}$

$\Leftrightarrow t = \frac{\log(2/0,97^{20})}{\log(1,06/0,97)} \cong 14,66 \text{ ans}$