2MA1DF01 Travail n°3 Corrigé tot: 83 pts not: 3 fr: 2 [/86]

1. [/20]

a) $|-2x+6| \geq 2 \Leftrightarrow -2x+6 \geq 2$ ou $-2x+6 \leq -2$

$-2x \geq -4$ $\quad$ $-2x \leq -8$

$x \leq 2$ $\quad$ $x \geq 4$

$S =]-\infty; 2] \cup [4; +\infty[$ (5)

b) $D = \mathbb{R} \setminus \{\pm 1\}$

$$2\frac{x+x^2}{x+1} + \frac{2x^3+4}{(1-x)(1+x)} = \frac{3x}{1-x}$$

$$\Leftrightarrow -\frac{3x}{1-x} + \frac{2x^3+4}{(1-x)(1+x)} = \frac{-2(x+x^2)}{1+x}$$

$$\Leftrightarrow \frac{-3x(1+x)+(2x^3+4)}{(1-x)(1+x)} = \frac{-2(x+x^2)(1+x)}{(1+x)(1+x)}$$

$$\Leftrightarrow -3x-3x^2+\cancel{2x^3}+4 = -2x+\cancel{2x^3}$$

$$\Leftrightarrow -3x^2-x+4=0$$

$$\Leftrightarrow -(3x^2+x-4)=0$$

$\Delta = 1+48 = 49$

$x_{1,2} = \frac{-1\pm 7}{6}$ $\nearrow x_1 = -4/3$ $\searrow x_2 = 1$ (10)

$S = \{-\frac{4}{3}\}$

c) $-2 < 3 - \frac{x}{4} \leq 5$

$\Leftrightarrow -5 < -\frac{x}{4} \leq 2$

$\Leftrightarrow 5 > \frac{x}{4} \geq -2$

$\Leftrightarrow 20 > x \geq -8$ (5)

$S = [-8; 20[$

ou

<u>cas 1</u> $-2x+6 \geq 0$

$-2x \geq -6$

$x \leq 3$

<u>ici</u> : $|-2x+6| \geq 2$

$-2x+6 \geq 2$

$-2x \geq -4$

$x \leq 2$ ok

$S =]-\infty ; 2]$

<u>cas 2</u> : $-2x+6 < 0$

$-2x < -6$

$x > 3$

<u>ici</u> : $|-2x+6| \geq 2$

$-(-2x+6) \geq 2$

$2x-6 \geq 2$

$2x \geq 8$

$x \geq 4$ ok

$S = [4 ; +\infty[$

donc

$S =]-\infty ; 2] \cup [4 ; +\infty[$

$\Leftrightarrow \quad \dfrac{2(x+x^2)(1-x)}{(x+1)(1-x)} + \dfrac{2x^3+4}{(1-x)(1+x)} = \dfrac{3x(1+x)}{(1-x)(1+x)} \quad \Big| \cdot (1-x)(1+x)$ ok si $x \in D$

$\Leftrightarrow \quad 2x + \cancel{2x^2} - \cancel{2x^2} - \cancel{2x^3} + \cancel{2x^3} + 4 = 3x + 3x^2$

$\Leftrightarrow \quad 3x^2 + x - 4 = 0$

$\Delta = 1 - 4 \cdot 3 \cdot (-4)$

$= 49$

$x_{1,2} = \dfrac{-1 \pm \sqrt{49}}{6}$ $\rightarrow x_1 = -4/3 \in D$, $\rightarrow x_2 = 1 \notin D$

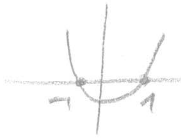
d'où $S = \left\{ -\frac{4}{3} \right\}$

[18]

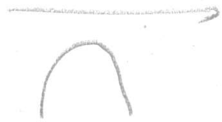
2. a) $x^2-6x+9=0$ $\qquad x^2-1=0$ $\qquad -x^2-x+2=0$

b) $(x-3)^2=0$ $\qquad\qquad -(x^2+x-2)=0$

$-(x+2)(x-1)=0$



$(2x+1)^2=0$ $\qquad -1-3x^2=0$



6

$D_f=\mathbb{R}\setminus\{-2;-\frac{1}{2};1\}$

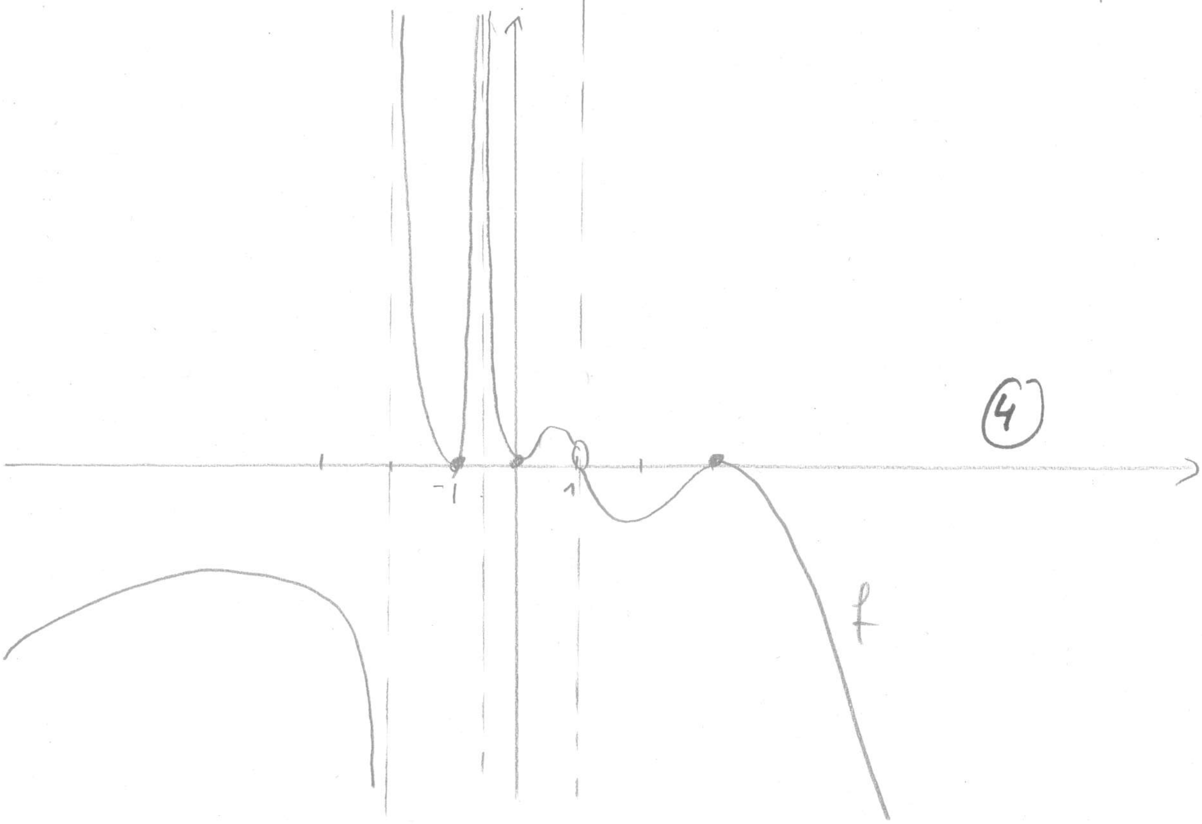
$Z_f=\{-1;0;3\}$

c)

x		-2		-1		$-1/2$		0		1		3	
$-x^4$	−	−	−	−	−	−	−	0	−	−	−	−	−
x^2-6x+9	+	+	+	+	+	+	+	+	+	+	+	0	+
x^2-1	+	+	+	0	−	−	−	−	−	0	+	+	+
$3x+3$	−	−	−	0	+	+	+	+	+	+	+	+	+
-4	−	−	−	−	−	−	−	−	−	−	−	−	−
$1-x$	+	+	+	+	+	+	+	+	+	0	−	−	−
$-x^2-x+2$	−	0	+	+	+	+	+	+	+	0	−	−	−
$(2x+1)^2$	+	+	+	+	+	0	+	+	+	+	+	+	+
$-1-3x^2$	−	−	−	−	−	−	−	−	−	−	−	−	−
$f(x)$	−	‖	+	0	+	‖	+	0	+	‖	−	0	−

8

d)



4

Ex 3 [/31]

a) $f : \mathbb{R} \to \mathbb{R}$, $x \mapsto 9 - x^2$

$D_f = \mathbb{R}$

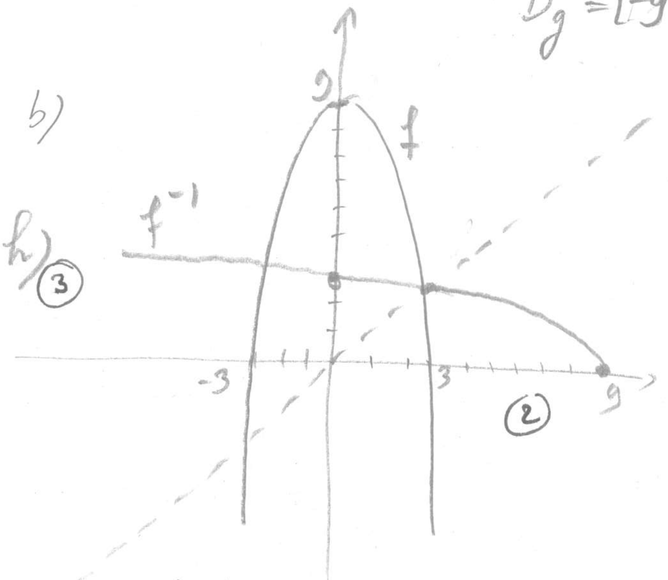
$g : \mathbb{R} \to \mathbb{R}$, $x \mapsto \sqrt{x+9}$

pb si $x + 9 < 0$

$x < 9$

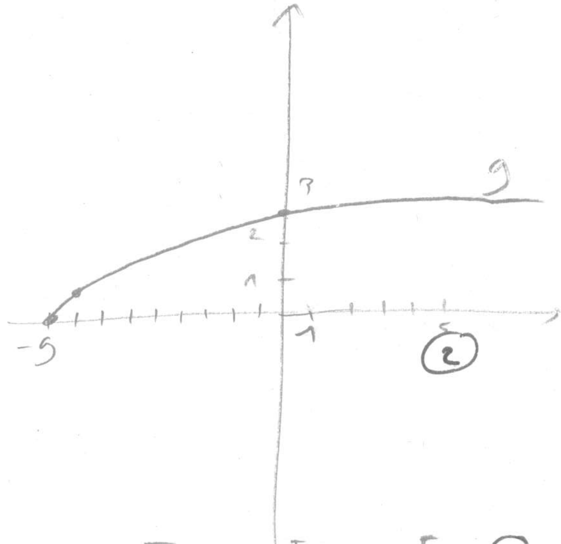
$D_g = [-9 ; +\infty[$ (2)

b)



h) (3)

(2)



(2)

c) $\mathrm{Im}_f =]-\infty ; 9]$ (2) $\mathrm{Im}_g = [0 ; +\infty[$ (2)

d)

$$[-9;+\infty[\xrightarrow{g} [0;+\infty[\subseteq \mathbb{R} \xrightarrow{f}]-\infty;9]$$

$$x \longmapsto \sqrt{x+9} \longmapsto 9 - (\sqrt{x+9})^2 = 9 - (x+9) = -x$$

$f \circ g$

$\mathrm{Im}_g \subseteq D_f$, donc $f \circ g : [-9 ; +\infty[\to]-\infty ; 9]$, $x \longmapsto -x$

$D_{f \circ g} = [-9 ; +\infty[$

$$\mathbb{R} \xrightarrow{f}]-\infty ; 9] \not\subseteq [-9 ; +\infty[\xrightarrow{g} [0 ; +\infty[$$

$$x \longmapsto 9 - x^2 \longmapsto \sqrt{9 - x^2 + 9} = \sqrt{18 - x^2}$$

Il faut restreindre x au départ de f !

$9 - x^2 \geq -9$

$-x^2 \geq -18$

$x^2 \leq 18$

$-\sqrt{18} \leq x \leq \sqrt{18}$

$-3\sqrt{2} \leq x \leq 3\sqrt{2}$

(5)

Conclusion : $g \circ f : [-3\sqrt{2} ; 3\sqrt{2}] \longrightarrow [0 ; +\infty[$

$x \longmapsto \sqrt{18 - x^2}$

e) 10 n'a pas de préimage ! (par exemple) (2)

f) $f : [0;+\infty[\to]-\infty;9]$ est bijective (2)

(ou $f :]-\infty;0] \to]-\infty;9]$)

g) $y = 9 - x^2 \Leftrightarrow x^2 = 9 - y$

$\Leftrightarrow x = \pm\sqrt{9-y}$

on choisit $x = \sqrt{9-y}$ car $x \in [0;+\infty[$

donc $f^{-1}(y) = \sqrt{9-y}$ (4)

ou $f^{-1}(x) = \sqrt{9-x}$

i) $f \circ f^{-1}(x) = f(f^{-1}(x)) = f(\sqrt{9-x}) = 9 - (\sqrt{9-x})^2 = 9-(9-x) = x$

$f^{-1} \circ f(x) = f^{-1}(f(x)) = f^{-1}(9-x^2) = \sqrt{9-(9-x^2)} = \sqrt{x^2} = x$

car $x \geq 0$

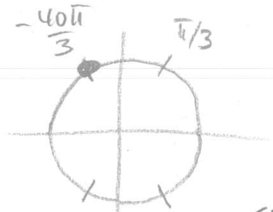
(4)

4. [9]

a) $-\frac{40\pi}{3} = \frac{-39\pi - \pi}{3} = -13\pi - \frac{\pi}{3}$

$\sin\left(-\frac{40\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{3}$

$\cos\left(-\frac{40\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2}$



(3)

b) $\frac{15\pi}{4} = \frac{12\pi + 3\pi}{4} = 3\pi + \frac{3\pi}{4}$

$\sin\left(\frac{15\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = \frac{-\sqrt{2}}{2}$

$\cos\left(\frac{15\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

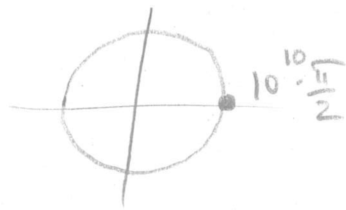
(3)

c) $10^{10} \cdot \frac{\pi}{2} = 100 \cdot 10^{8} \cdot \frac{\pi}{2} = 4 \cdot (25 \cdot 10^{8}) \cdot \frac{\pi}{2}$

$= 2\pi \cdot (25 \cdot 10^{8})$

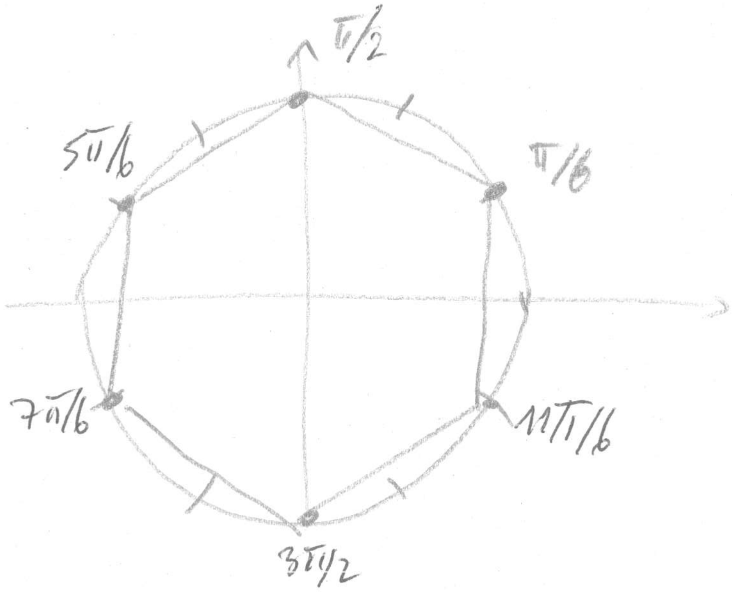
$\sin\left(10^{10} \cdot \frac{\pi}{2}\right) = \sin(0) = 0$

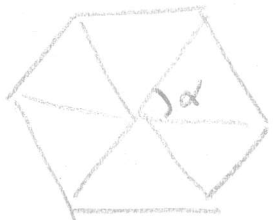
$\cos\left(10^{10} \frac{\pi}{2}\right) = \cos(0) = 1$



(3)

5. [5]





$\alpha = \frac{360}{6} = 60°$

$= \frac{\pi}{3}$

- $\frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi + 2\pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2}$
- $\frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}$

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