MATH 2A Travail du 7 mai 2009 <u>Corrigé</u> 73 pts
not 3 pts
tot 76 pts
fac : fr 2 pts

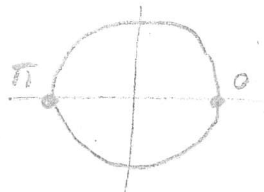
Ex 1 /15

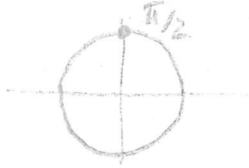
a) $\sin^2(x) = \sin(x)$

$\Leftrightarrow \sin(x)[\sin(x) - 1] = 0$

$\Leftrightarrow \sin(x) = 0$ ou $\sin(x) = 1$

(4)

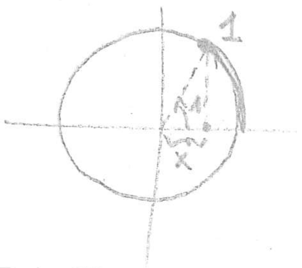




$S_{\mathbb{R}} = \{k\pi ; \frac{\pi}{2} + k2\pi \mid k \in \mathbb{Z}\}$ et $S_{[0;2\pi[} = \{0 ; \frac{\pi}{2} ; \pi\}$

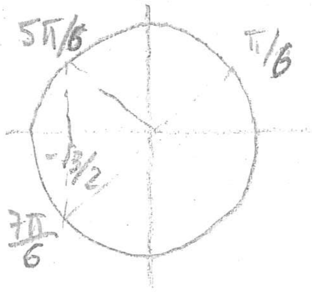
b) $\cos^{-1}(x) = 1$

(3)



$x \cong$

c) $\cos\left(\frac{\pi}{4} - 3x\right) = -\frac{\sqrt{3}}{2}$



$\frac{\pi}{4} - 3x = \frac{5\pi}{6} + k2\pi$ ou $\frac{\pi}{4} - 3x = \frac{7\pi}{6} + k2\pi$

$\Leftrightarrow -3x = \frac{5\pi}{6} - \frac{\pi}{4} + k2\pi$ | $\Leftrightarrow -3x = \frac{7\pi}{6} - \frac{\pi}{4} + k2\pi$

$\Leftrightarrow -3x = \frac{7\pi}{12} + k2\pi$ | $\Leftrightarrow -3x = \frac{11\pi}{12} + k2\pi$

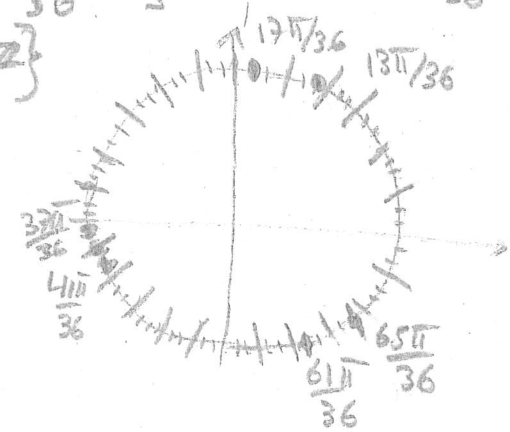
$\Leftrightarrow x = -\frac{7\pi}{36} - \frac{k2\pi}{3}$ | $\Leftrightarrow x = -\frac{11\pi}{36} - \frac{k2\pi}{3}$

$S_{\mathbb{R}} = \left\{-\frac{7\pi}{36} + k\frac{2\pi}{3} ; -\frac{11\pi}{36} + k\frac{2\pi}{3} \mid k \in \mathbb{Z}\right\}$

(8)

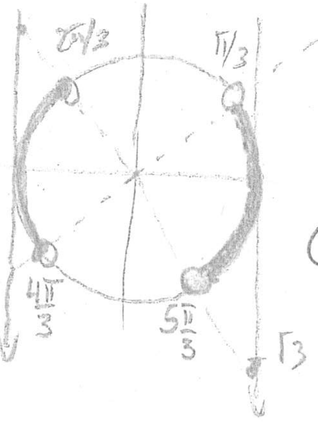
$x = -\frac{7\pi}{36} + k\frac{24\pi}{36}$: $\frac{17\pi}{36} ; \frac{41\pi}{36} ; \frac{65\pi}{36}$

$x = -\frac{11\pi}{36} + k\frac{24\pi}{36}$: $\frac{13\pi}{36} ; \frac{37\pi}{36} ; \frac{61\pi}{36}$



Ex2 : $\tan^2(x) < 3$

[18] $\Leftrightarrow -\sqrt{3} < \tan(x) < \sqrt{3}$ (2)



(3)

$S = \left]0; \frac{\pi}{3}\right[\cup \left]\frac{2\pi}{3}; \frac{4\pi}{3}\right[\cup \left]\frac{5\pi}{3}; 2\pi\right[$ (3)

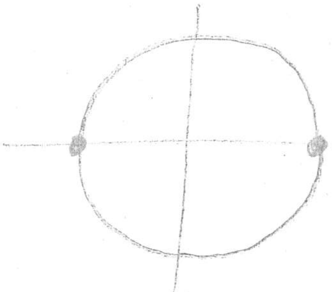
Ex 3

/18

a) $D_f = \mathbb{R}$ (1)

b) $4\sin\left(\frac{x}{2} - \frac{\pi}{3}\right) = 0$

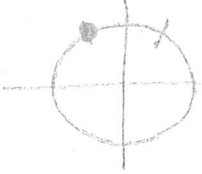
$\Leftrightarrow \sin\left(\frac{x}{2} - \frac{\pi}{3}\right) = 0$



$\frac{x}{2} - \frac{\pi}{3} = k\pi$

$\Leftrightarrow \frac{x}{2} = \frac{\pi}{3} + k\pi$

$\Leftrightarrow x = \frac{2\pi}{3} + k2\pi$ (5)



$Z_f = \left\{\frac{2\pi}{3} + k2\pi\right\}$

c) $f\left(-\frac{\pi}{3}\right) = 4\sin\left(-\frac{\pi}{2}\right) = -4$

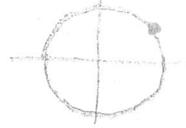
$f\left(\frac{5\pi}{3}\right) = 4\sin\left(\frac{5\pi}{6} - \frac{\pi}{3}\right) = 4\sin\left(\frac{3\pi}{6}\right) = 4\cdot 1 = 4$

(4) $f(0) = 4\sin\left(-\frac{\pi}{3}\right) = 4\cdot\left(-\frac{\sqrt{3}}{2}\right) = -2\sqrt{3}$

$f(\pi) = 4\sin\left(\frac{\pi}{2} - \frac{\pi}{3}\right) = 4\sin\left(\frac{\pi}{6}\right) = 4\cdot\frac{1}{2} = 2$

d) $f(x+P) = f(P)$

$\Leftrightarrow \cancel{4}\sin\left(\frac{x+P}{2} - \frac{\pi}{3}\right) = \cancel{4}\sin\left(\frac{x}{2} - \frac{\pi}{3}\right)$



ou

$\frac{x+P}{2} - \cancel{\frac{\pi}{3}} = \frac{x}{2} - \cancel{\frac{\pi}{3}} + k2\pi$

$\Leftrightarrow \frac{P}{2} = k2\pi$

$\Leftrightarrow P = k4\pi$

Plus petit $P > 0$:

$P = 4\pi$

$\frac{x+P}{2} - \frac{\pi}{3} = \pi - \left(\frac{x}{2} - \frac{\pi}{3}\right)$

$\Leftrightarrow \frac{x}{2} + \frac{P}{2} - \frac{\pi}{3} = -\frac{x}{2} + \frac{\pi}{3} + \pi$

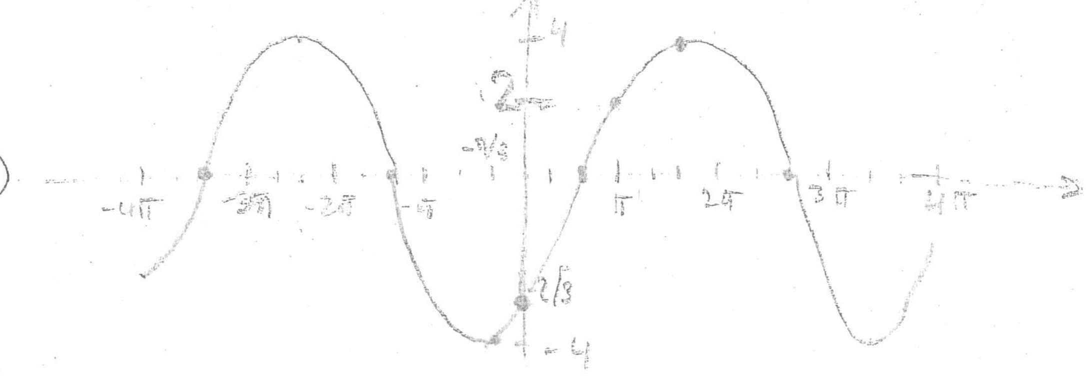
$\Leftrightarrow x + \frac{P}{2} = \frac{5\pi}{3}$

P dépend de x

$P = 4\pi$ (4)

e)

(4)



Ex 4

[9] a) vrai

(5) demo : $1+\tan^2(x) = 1+\frac{\sin^2(x)}{\cos^2(x)} = \frac{\cos^2(x)+\sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$

b) faux

(4) C-Ex : $x=\frac{\pi}{2}$: $\sin(2\frac{\pi}{2}) \overset{?}{=} 2\sin(\frac{\pi}{2})$

$\sin(\pi) \overset{?}{=} 2\sin(\frac{\pi}{2})$

$0 \overset{?}{=} 2\cdot 1$

non

Exercice 5 *(environ 5 points)*

[/15]

On considère le :

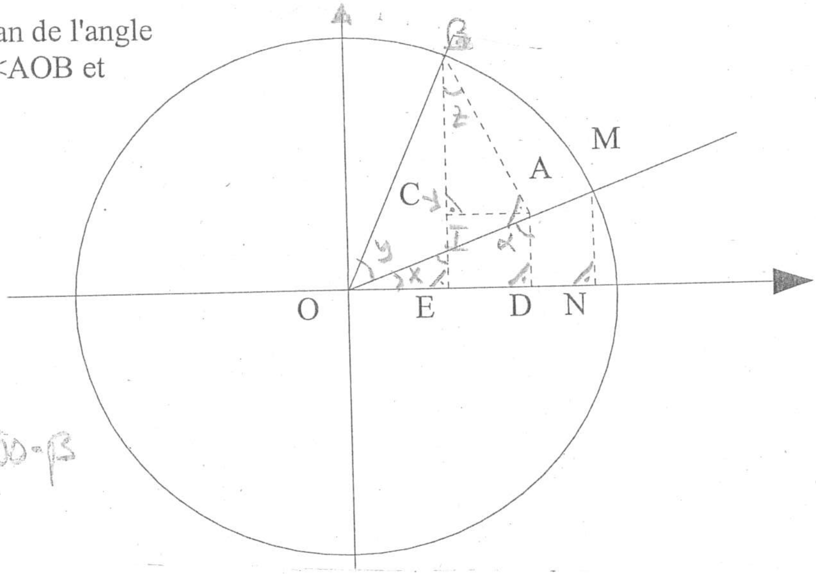
Théorème: Si $x \in \mathbb{R}$, alors on a: $\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$

On donne la démonstration ci-dessous:

(a) Les angles <OAB, <ODA, <ONM et < BCA sont droits.

Posons x la mesure en radian de l'angle <NOM, y celle de l'angle <AOB et z celle de l'angle <CBA.

Montrer que x est égal à z.



ΔODA : $x = 180 - 90 - \alpha = 90 - \alpha$

$\Delta OEI \sim \Delta ODA$: $\beta = \alpha$

β et γ opposés : $\gamma = \beta$

ΔABI : $z = 180 - 90 - \gamma = 90 - \gamma = 90 - \beta$
$= 90 - \alpha$

(6) donc $x = z$

(b) On a :

$\cos(x+y) = \overline{OE}$, car [ARG 1 ..... déf cos .....] 2

$= [\overline{OD}] - \overline{DE}$

$= \overline{OD} - [\overline{A}]\overline{C}$

[6] $= \dfrac{\overline{OD}}{\overline{OB}} - \dfrac{\overline{AC}}{\overline{OB}}$, car [ARG 2 ..... $\overline{OB} = 1$ car rayon cercle trig. .....] 2

$= (\dfrac{\overline{OD}}{\overline{OB}}) \cdot (\dfrac{[\overline{OA}]}{\overline{OA}}) - (\dfrac{\overline{AC}}{\overline{OB}}) \cdot (\dfrac{\overline{BA}}{\overline{BA}})$, car [ARG 3 ..... on mult. par 1 .....] 2

$= (\dfrac{\overline{OD}}{\overline{OA}}) \cdot (\dfrac{\overline{OA}}{[\overline{OB}]}) - (\dfrac{\overline{AC}}{\overline{BA}}) \cdot (\dfrac{\overline{BA}}{\overline{OB}})$

$= (\dfrac{\overline{ON}}{\overline{OM}}) \cdot (\dfrac{\overline{OA}}{\overline{OB}}) - (\dfrac{\overline{MN}}{\overline{OM}}) \cdot (\dfrac{\overline{BA}}{\overline{OB}})$, car [ARG 4 ..... $\Delta ONM \sim \Delta ODA$; $\Delta ONM \sim \Delta BAC$ } donc, par Thalès .....] 4

$= \overline{ON} \cdot \overline{OA} - \overline{MN} \cdot \overline{BA}$, car [ARG 5 ..... $\overline{OM} = \overline{OB} = 1$ car rayon cercle trig. .....] 4

$= \cos(x)\cos[\ y\ ] - \sin[\ x\ ]\sin(y)$

Remplir chacun des [............] directement sur l'énoncé et donner pour chaque [ARG] le ou les arguments nécessaires.

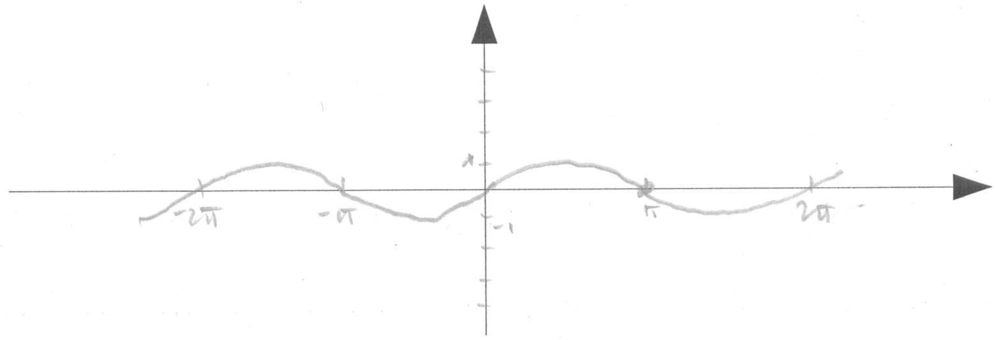
tot 18/
→ 9,5

Exercice 6 *(environ 3 points)*

/8

Tracer dans le même repère ci-dessous, en utilisant des couleurs différentes - une représentation graphique des fonctions réelles suivantes. On ne demande pas de calcul, mais seulement l'allure générale des fonctions ainsi qu'un choix pertinent de l'échelle.

(a) $f(x)=\sin(x)$



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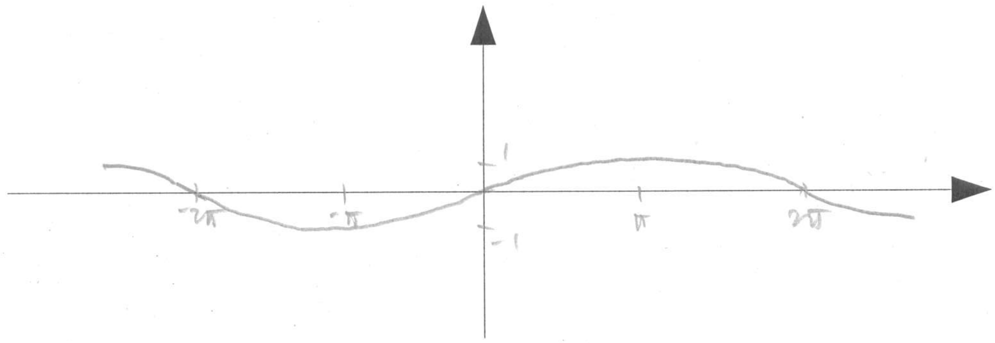
(b) $f(x)=\sin(\frac{x}{2})$



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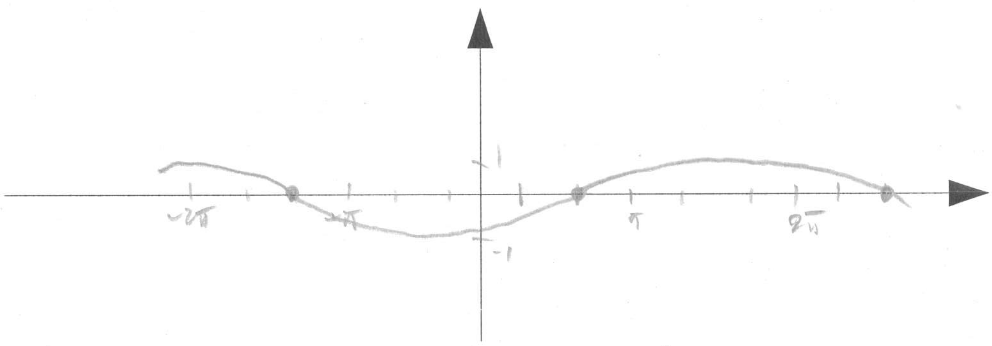
(c) $f(x)=\sin(\frac{x}{2}-\frac{\pi}{3})$



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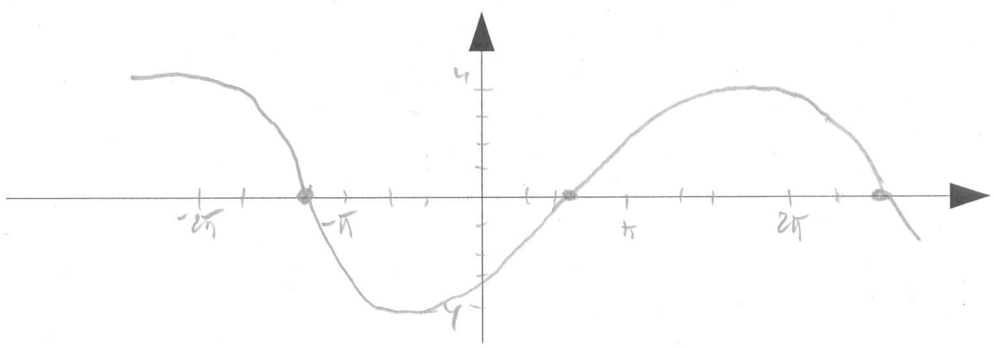
(d) $f(x)=4\sin(\frac{x}{2}-\frac{\pi}{3})$



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