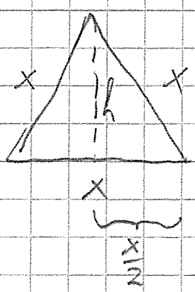
Ma2A : Corrigé du travail de 30' n°4

76 pts
not 2 pts
pr 2 pts [fac] } tot : 78 pts

Ex 1 :

/12 (a) $P_{ABCD} = 2(x+3)+2(x+1) = 4x+8$ et $A_{ABCD} = (x+1)(x+3)$ ②



Pythagore : $x^2 = h^2 + \left(\frac{x}{2}\right)^2 \Leftrightarrow h^2 = x^2 - \frac{x^2}{4} = \frac{3x^2}{4}$

$h = \frac{\sqrt{3}x}{2}$

$\Rightarrow A_{EGF} = \dfrac{x \cdot \frac{\sqrt{3}x}{2}}{2} = \frac{\sqrt{3}}{4}x^2$ ③

$P_{EGF} = 3x$ ①

(b) $4x+8 \leq 3x$

$x \leq -8$

$S = \emptyset$ (car $x > 0$) ②

(c) $(x+1)(x+3) > \frac{\sqrt{3}}{4}x^2$

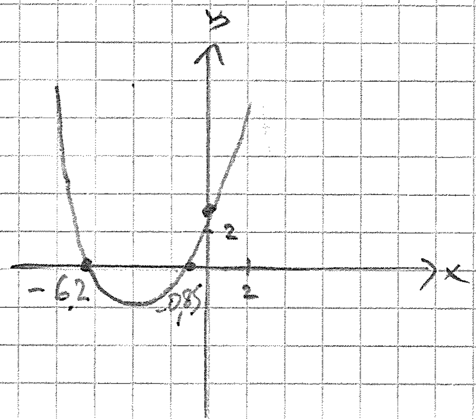
$\Leftrightarrow x^2+4x+3 > \frac{\sqrt{3}}{4}x^2$

$\Leftrightarrow \underbrace{x^2\left(1-\frac{\sqrt{3}}{4}\right)+4x+3}> 0$

$\Delta = 16 - 4\left(1-\frac{\sqrt{3}}{4}\right)\cdot 3 \cong 9{,}2$

$x_{1,2} = \dfrac{-4 \pm \sqrt{\Delta}}{2(1-\sqrt{3}/4)} \nearrow \sim -0{,}85$, $\searrow \sim -6{,}2$

$\hookrightarrow$ OBJ : savoir utiliser la calculatrice...



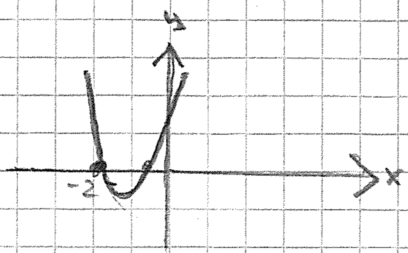
$S = \mathbb{R}_*^+$ (car $x > 0$)

Ex 3

/25 (a) $\underbrace{4x^2+11x+6} > 0$

$\Delta = 121 - 4 \cdot 4 \cdot 6 = 25$

$x_{1,2} = \dfrac{-11 \pm 5}{8} \nearrow -16/8 = -2$, $\searrow -6/8 = -3/4$



$S = \left]-\infty ; -2\right[\cup \left]-\frac{3}{4} ; +\infty\right[$

④

(b) $\dfrac{x}{x-1} > \dfrac{9}{x+3}$ $\qquad D_f = \mathbb{R} \setminus \{-3;1\}$

$\Leftrightarrow \dfrac{x(x+3)}{(x-1)(x+3)} - \dfrac{9(x-1)}{(x+3)(x-1)} > 0$

$\Leftrightarrow \dfrac{x^2+3x-9x+9}{(x-1)(x+3)} > 0$

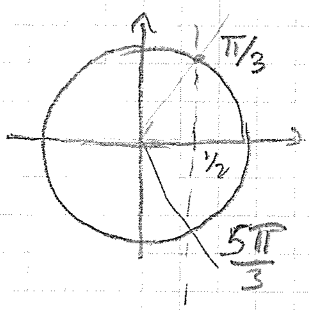
$\Leftrightarrow \dfrac{x^2-6x+9}{(x-1)(x+3)} > 0$

$\Leftrightarrow \dfrac{(x-3)^2}{(x-1)(x+3)} > 0$

x		-3		1		3	
$(x-3)^2$	+	+	+	+	+	0	+
$x-1$	−	−	−	0	+	+	+
$x+3$	−	0	+	+	+	+	+
exp	+	‖	−	‖	+	0	+

$S =]-\infty;-3[\cup]1;3[\cup]3;+\infty[$ (6)

(c) $\cos\left(\dfrac{3x}{2} - \dfrac{\pi}{2}\right) = +\dfrac{1}{2}$



(2)

$\dfrac{3x}{2} - \dfrac{\pi}{2} = \dfrac{\pi}{3} + k2\pi$ ou $\dfrac{3}{2}x - \dfrac{\pi}{2} = \dfrac{5\pi}{3} + k2\pi$

$\Leftrightarrow 3x = \pi + \dfrac{2\pi}{3} + k4\pi$ $\qquad \Leftrightarrow 3x = \pi + \dfrac{10\pi}{3} + k4\pi$

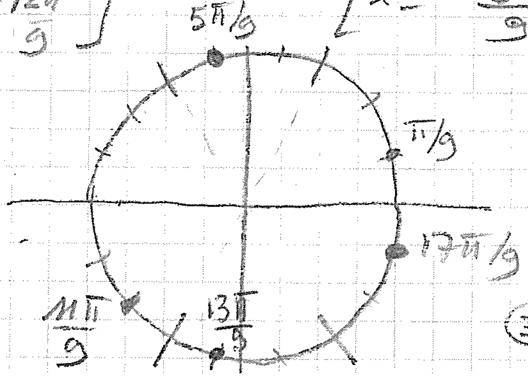
$\Leftrightarrow x = \dfrac{\pi}{3} + \dfrac{2\pi}{9} + k\dfrac{4\pi}{3}$ $\qquad \Leftrightarrow x = \dfrac{\pi}{3} + \dfrac{10\pi}{9} + k\dfrac{4\pi}{3}$

$\Leftrightarrow x = \dfrac{3\pi+2\pi}{9} + k\dfrac{4\pi}{3}$ $\qquad \Leftrightarrow x = \dfrac{3\pi+10\pi}{9} + k\dfrac{4\pi}{3}$

$\Leftrightarrow x = \dfrac{5\pi}{9} + k\dfrac{4\pi}{3}$ $\qquad \Leftrightarrow x = \dfrac{13\pi}{9} + k\dfrac{4\pi}{3}$

$\left[x = \dfrac{5\pi}{9} + k\dfrac{12\pi}{9}\right]$ (3) $\qquad \left[x = \dfrac{13\pi}{9} + k\dfrac{12\pi}{9}\right]$ (3)

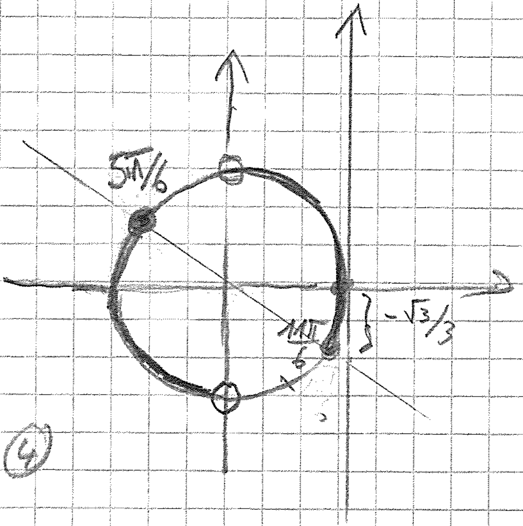
dans $[0;2\pi[$
$k=0$: $5\pi/9$
$k=1$: $17\pi/9$



(3)

dans $[0;2\pi[$
$k=0$: $13\pi/9$
$k=-1$: $\pi/9$

(d) $\tan(x) \geq -\frac{\sqrt{3}}{3}$



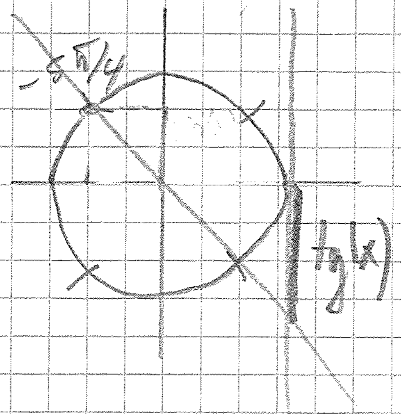
$S: [0; \frac{\pi}{2}[\cup [\frac{5\pi}{6}; \frac{3\pi}{2}[$
$\cup [\frac{11\pi}{6}; 2\pi[$

(4)

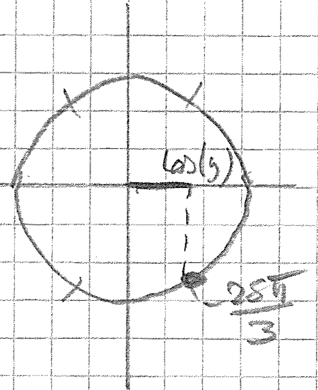
Ex 2

(a)
(b)

[1/8]



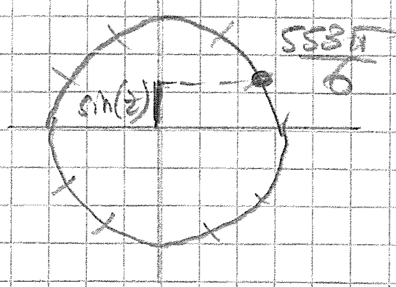
(2)



$-\frac{25\pi}{3} = -\frac{24\pi}{3} - \frac{\pi}{3}$

$= -8\pi - \frac{\pi}{3}$

(3)



$\frac{553\pi}{6} = \frac{546\pi + 7\pi}{6}$

$= 92\pi + \frac{\pi}{6}$

(3)

Ex 4

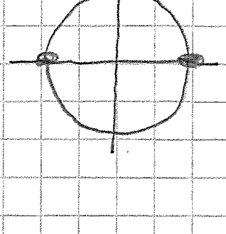
[/11]

$f(x) = 2\sin\left(\frac{3}{2}\left(x + \frac{\pi}{2}\right)\right)$

(a) $D_f = \mathbb{R}$ (1)

Z_f: $2\sin\left(\frac{3}{2}\left(x+\frac{\pi}{2}\right)\right) = 0 \Leftrightarrow \sin\left(\frac{3}{2}\left(x+\frac{\pi}{2}\right)\right) = 0$

$\overset{(\Rightarrow)}{\Leftrightarrow} \frac{3}{2}\left(x+\frac{\pi}{2}\right) = k\pi$



$\Leftrightarrow x + \frac{\pi}{2} = k\frac{2\pi}{3}$

$\Leftrightarrow x = -\frac{\pi}{2} + k\frac{2\pi}{3}$

$Z_f = \left\{-\frac{\pi}{2} + k\frac{2\pi}{3} \mid k \in \mathbb{Z}\right\}$

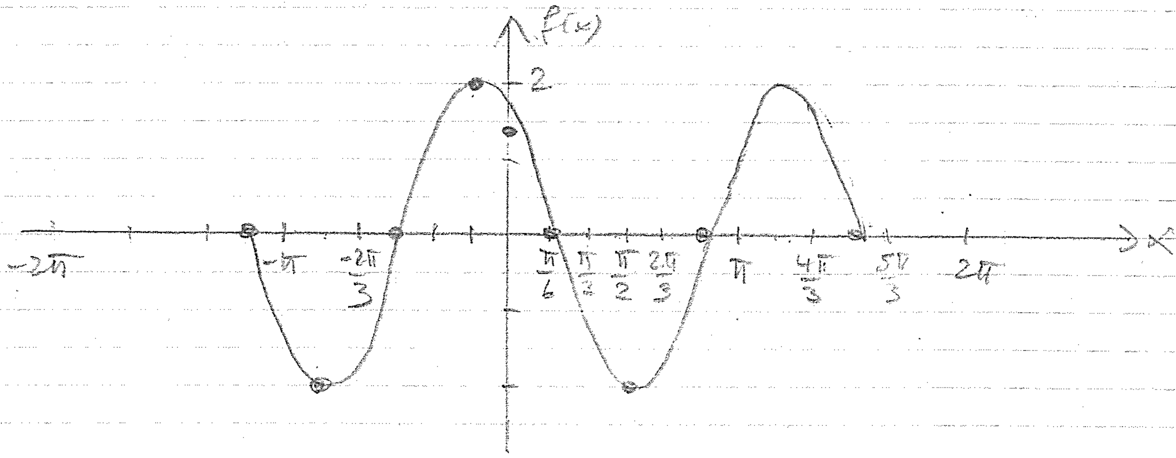
(3)

(b) Comme la période de sin est 2π, celle de f est $\frac{2\pi}{\frac{3}{2}} = \frac{4\pi}{3}$ (1)

(c) $f(0) = 2\sin\left(\frac{3\pi}{4}\right) = 2\cdot\frac{\sqrt{2}}{2} = \sqrt{2}$; $f\left(\frac{\pi}{2}\right) = \dots = -2$ (3)

$f\left(-\frac{\pi}{6}\right) = 2\sin\left(\frac{3}{2}\left(-\frac{\pi}{6}+\frac{\pi}{2}\right)\right) = 2\sin\left(\frac{3}{2}\cdot\frac{2\pi}{6}\right) = 2\sin\left(\frac{\pi}{2}\right) = 2\cdot 1$

(d)



(4)

Ex 5

1) /20 (a) $p :\ x \xrightarrow{e} \frac{1}{x} \xrightarrow{d} \frac{1}{x}+1$, $x \xrightarrow{p} \frac{1}{x}+1$

donc $p(x) = d \circ e(x)$ (2)

⚠ pour r : $r(x) = x^2 + 2x(+1-1)+3$
$= (x+1)^2 - 1 + 3$
$= (x+1)^2 + 2$

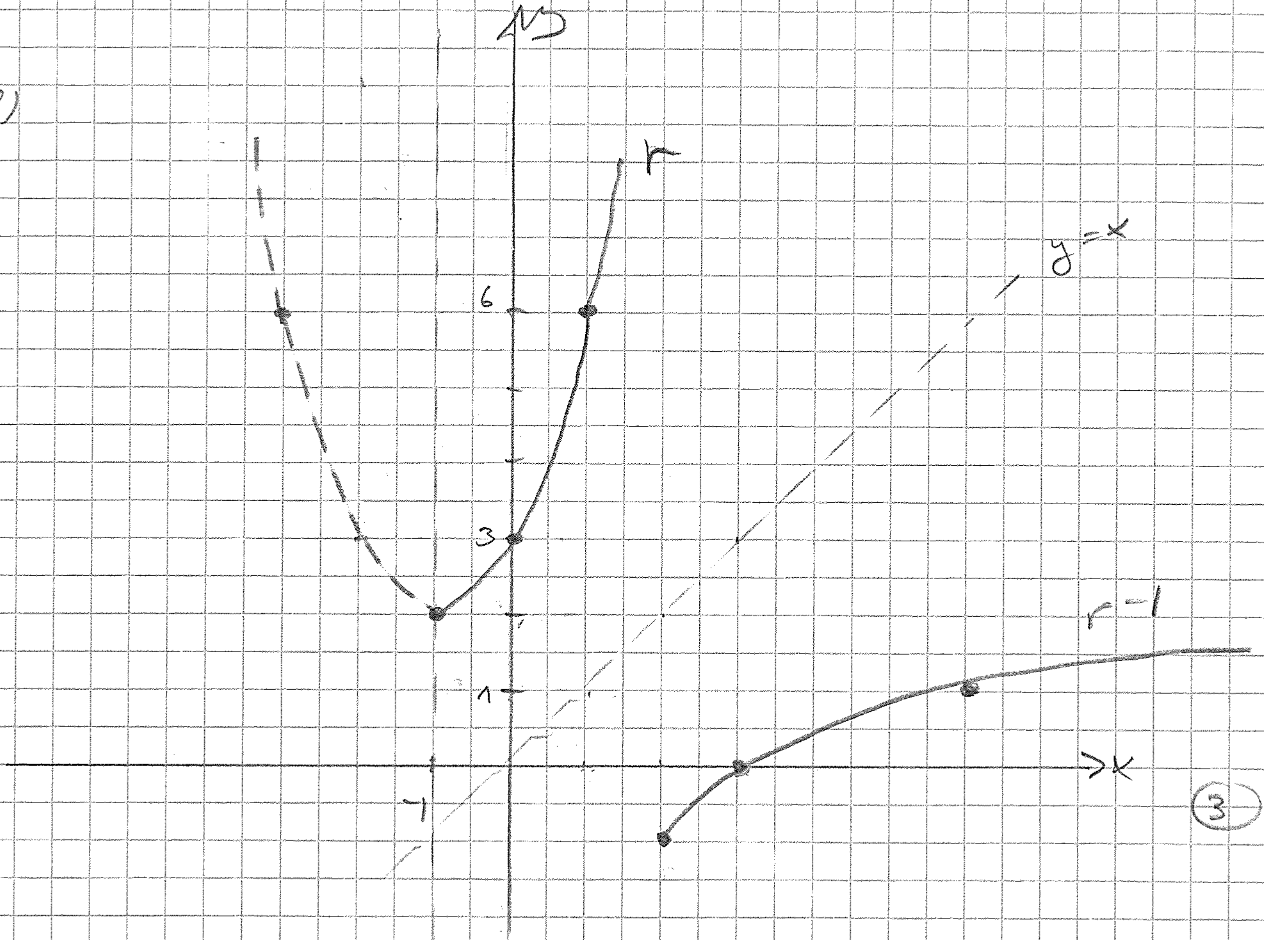
$x \xrightarrow{d} x+1 \xrightarrow{c} (x+1)^2 \xrightarrow{b} (x+1)^2+2$

donc $r(x) = b \circ c \circ d(x)$ (3)

(b) $o \circ p(x) = o(p(x)) = o\left(1+\frac{1}{x}\right) = \frac{1}{1+\frac{1}{x}-1} = \frac{1}{\frac{1}{x}} = x$

$p \circ o(x) = p(o(x)) = p\left(\frac{1}{x-1}\right) = 1 + \frac{1}{\frac{1}{x-1}} = 1 + x - 1 = x$

donc o et p sont réciproques l'une de l'autre (4) (c)(d)



r n'est pas bijective de $\mathbb{R}$ dans $\mathbb{R}$, car, par exemple, 6 a deux préimages (3)

$r : [-1;+\infty[\to [2;+\infty[$ est bijective (3)

r^{-1}: $y = (x+1)^2 + 2 \iff y-2 = (x+1)^2$

$\iff x+1 = \pm\sqrt{y-2}$

$\iff x = -1 \pm \sqrt{y-2}$ ↙ on garde le « + »

$r^{-1}(y) = -1 + \sqrt{y-2}$

ou : $r^{-1} : [2;+\infty[\to [-1;+\infty[$

$x \longmapsto -1+\sqrt{x-2}$ (3)