

Math 2 : chapitre 5 Corrigé / réponses des exercices

$$\left. \begin{array}{l} (1) \quad (2^2)^3 = 2^6 \\ 2(2^2) = 2^4 \\ (2^{33})^{0.5} = 2^{33/2} = 2^{16.5} \\ 4^{(2^5)} = 4^{25} = (2^2)^{25} = 2^{50} \end{array} \right\} \Rightarrow 2^{(2^2)} < (2^2)^3 < (2^{33})^{0.5} < 4^{(2^5)}$$

(2) (a) 6^4 (b) $-8a^4b^3c$ (c) 2^{15} (d) -3^4

(3) (a) 0 (b) 25 (c) 0,03 (d) 5
(e) 2 (f) 3 (g) -0,3 (h) 0,02

(4) (a) 6 (b) 5 (c) 8 (d) 2

(5) (a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) 3 (d) 2
(e) $\sqrt{2}$ (f) $\sqrt{26}\sqrt{3} (=6\sqrt{26})$

(6) (a) $3\sqrt{2}$ (b) $9\sqrt{3}$ (c) $5\sqrt{2}$ (d) $10\sqrt{10}$ (e) $3\sqrt[5]{2}$ (f) $2\sqrt[4]{5}$

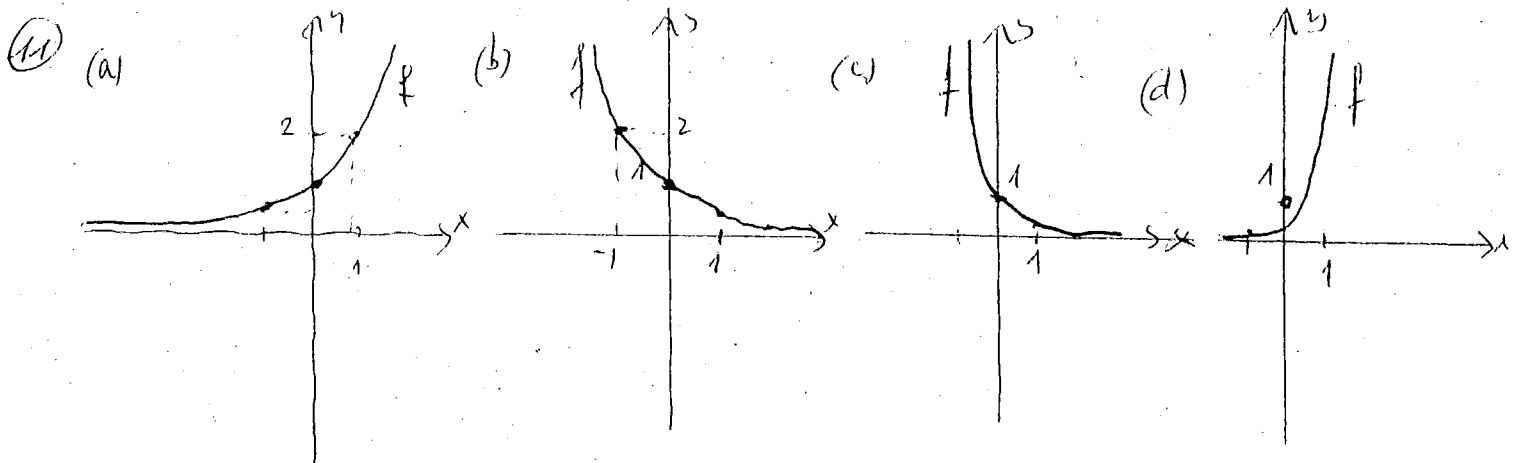
(7) (a) $13\sqrt{10}$ (b) -25

(8) (a) $8\sqrt{a}$ (b) $12\sqrt{a}$ (c) a (d) $\sqrt[4]{a^3}$

(9) (a) $6\sqrt{3^5} = (6\sqrt{3})^5$ (b) $\sqrt[5]{0} (=0)$ (c) $2\sqrt{\frac{1}{9}} (= \frac{2}{3})$

(d) $\sqrt{25} (=5)$ (e) $\sqrt[4]{0,0025}$ (f) $\sqrt[5]{32} (=2)$ (g) $\frac{1}{\sqrt{2}}$

(10) (a) $5^{6/11}$ (b) $4^{1/2}$ (c) a^3 (d) a^2 (e) a^4
(f) a^n (g) a (h) $a^{3/2n}$ (i) $a^{3/2n}$ (j) $a^{-7/6}$ (k) $a^{3/2}$



$$(12) \quad (a) \quad 3^{2x+3} = 3^{x^2}$$

$$\Leftrightarrow 2x+3 = x^2$$

$$\Leftrightarrow x^2 - 2x - 3 = 0$$

$$\Leftrightarrow (x-3)(x+1) = 0$$

$$x = 3 \text{ or } x = -1$$

$$S = \{-1, 3\}$$

$$(c) \quad 2^{-100x} = 0,5^{x-4}$$

$$\Leftrightarrow 2^{-100x} = \left(\frac{1}{2}\right)^{x-4}$$

$$\Leftrightarrow 2^{-100x} = (2^{-1})^{x-4}$$

$$\Leftrightarrow 2^{-100x} = 2^{-x+4}$$

$$\Leftrightarrow -100x = -x+4$$

$$\Leftrightarrow -99x = 4$$

$$\Leftrightarrow x = -\frac{4}{99}$$

$$S = \left\{-\frac{4}{99}\right\}$$

$$(e) \quad 5^{x+2} \cdot 25^{-x} = 625$$

$$\Leftrightarrow 5^{x+2} \cdot (5^2)^{-x} = 5^4$$

$$\Leftrightarrow 5^{x+2} \cdot 5^{-2x} = 5^4$$

$$\Leftrightarrow 5^{x+2-2x} = 5^4$$

$$\Leftrightarrow 5^{-x+2} = 5^4$$

$$\Leftrightarrow -x+2 = 4$$

$$\Leftrightarrow -x = 2$$

$$\Leftrightarrow x = -2$$

$$S = \{-2\}$$

$$(d) \quad 3^{3x+2} = 9^{x^2}$$

$$\Leftrightarrow 3^{3x+2} = (3^2)^{x^2}$$

$$\Leftrightarrow 3^{3x+2} = 3^{2x^2}$$

$$\Leftrightarrow 2x^2 - 3x - 2 = 0$$

$$\Delta = 9 - 4 \cdot 2 \cdot (-2) = 25$$

$$x_{1,2} = \frac{3 \pm 5}{4} \Rightarrow x_1 = \frac{-2}{4} = -\frac{1}{2}$$

$$x_2 = \frac{8}{4} = 2$$

$$S = \left\{-\frac{1}{2}, 2\right\}$$

$$(f) \quad 13^x = 1$$

$$\Leftrightarrow 13^x = 13^0$$

$$\Leftrightarrow x = 0$$

$$S = \{0\}$$

$$(g) \quad 4^{x-3} = 8^{4-x}$$

$$\Leftrightarrow (2^2)^{x-3} = (2^3)^{4-x}$$

$$\Leftrightarrow 2^{2x-6} = 2^{12-3x}$$

$$\Leftrightarrow 2x-6 = 12-3x$$

$$\Leftrightarrow 5x = 18$$

$$\Leftrightarrow x = \frac{18}{5}$$

$$S = \left\{\frac{18}{5}\right\}$$

$$(h) \quad 9^{x+4} = \frac{1}{3^{x^2}}$$

$$\Leftrightarrow (3^2)^{x+4} = 3^{-x^2}$$

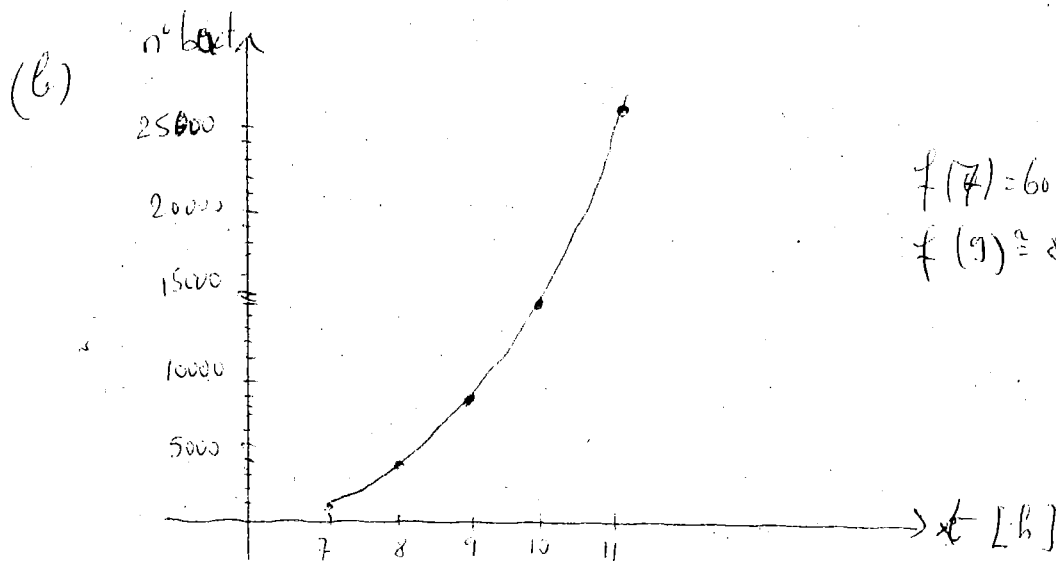
$$\Leftrightarrow 3^{2x+8} = 3^{-x^2}$$

$$\Leftrightarrow 2x+8 = -x^2$$

$$\Leftrightarrow x^2 + 2x + 8 = 0$$

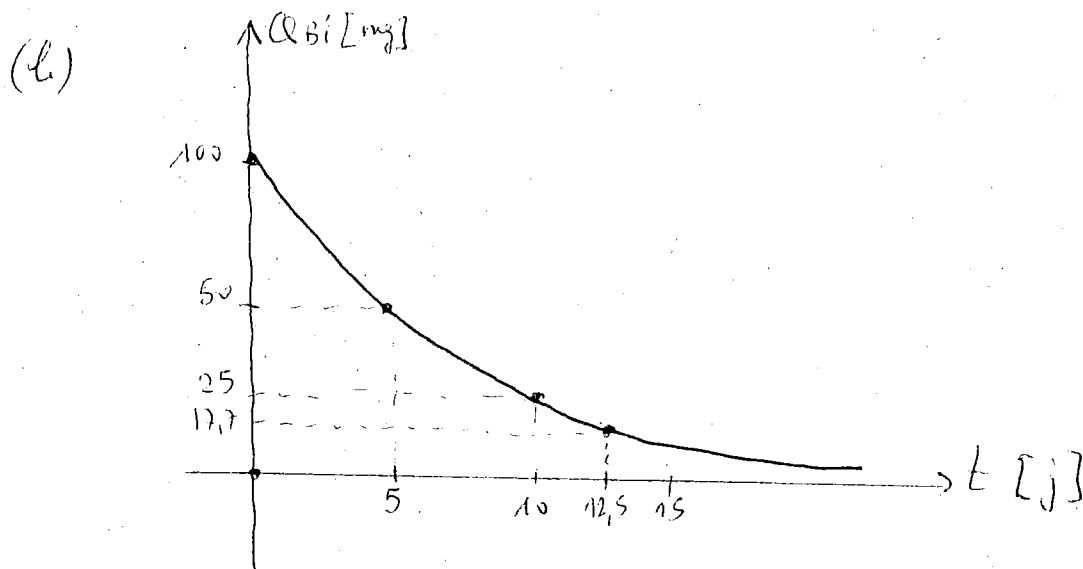
$$\Delta < 0 \quad S = \emptyset$$

ex (13) (a) $f(8) = 600 \cdot 3^4 = 48600$
 $f(10) = 600 \cdot 3^5 = 145800$
 $f(11) = 600 \cdot 3^{5,5} \approx 252533$



$f(7) = 600$
 $f(9) \approx 84178$

ex (14) (a) $f(5) = 100 \cdot 2^{-5/5} = 100 \cdot 2^{-1} = \frac{100}{2} = 50 \text{ mg}$
 $f(10) = 100 \cdot 2^{-2} = \frac{100}{4} = 25 \text{ mg}$
 $f(12,5) = 100 \cdot 2^{-12,5/5} = \frac{100}{2^{2,5}} \approx 17,7 \text{ mg}$



ex (15) $C(t) = C_0 (1+i)^t$
 $\Rightarrow C(t) = 1200 (1+0,02)^{18} \approx 1713,9$

ex 16 $N_0 = 10000$

a) $N(1) = 10000 - 5\% \cdot 10000 = 10000(1 - 0,05) = 9500$ $0,95 = 9500/10000$

b) $N(3) = 10000(0,95)^3 \approx 8573,75$ (h)

c) $N(t) = 10000(0,95)^t$

d) $N(10) = 10000(0,95)^{10} \approx 5987,4$ (h)

ex 17

$C_0 = 24,-$

$i = 6\% = 0,06$

$n = 4$

$t = 1996 - 1626 = 370$

$C_t = C_0 \left(1 + \frac{i}{n}\right)^{nt} = 24 \left(1 + \frac{0,06}{4}\right)^{4 \cdot 370}$

≈ 89115607200

$(89115607200 - 1,03)$

ex 18

13.1.2004 : 12500,-

13.1.2012 : 12500,- + 3027,25

i in annual = 15577,25

$t = 8$ ans

$n = 2$ (capitalisation semestrielle)

$15577,25 = 12500 \left(1 + \frac{i}{2}\right)^{2 \cdot 8}$

$\left(1 + \frac{i}{2}\right)^{16} = \frac{15577,25}{12500}$

$1 + \frac{i}{2} = \sqrt[16]{\frac{15577,25}{12500}}$

$i = \left(\sqrt[16]{\frac{15577,25}{12500}} - 1\right) \cdot 2 \cdot 100 \approx 2,98\%$

ex 19

$i = 11\% = 0,11$

C_0 in annual

$C_n = 100000$

$n = 18$ ans

$100000 = C_0 \cdot e^{0,11 \cdot 18}$

$\Rightarrow C_0 = \frac{100000}{e^{0,11 \cdot 18}} \approx 133092,-$

ex 20

$q = q_0 e^{iT}$

$q_0 = 762$

$i = 2,2\%$

$T = 2010 - 1985 = 25$

$q = 762 \cdot e^{0,022 \cdot 25} \approx 1320,74$ Mio

ex 21

$N(10) = N_0 e^{-0,2 \cdot 10} = N_0 e^{-2}$

$\frac{N(10)}{N_0} = \frac{N_0 e^{-2}}{N_0} = e^{-2} \approx 0,135 \approx 13,5\%$

ex 22 de 1971 à 2014 : 43 ans = t

6,4 : salaire minimum en 1971

aujourd'hui : $6,4 \cdot (1 + 0,05)^{43} = 52,16$

ex 23 $f(x) = 0 \Leftrightarrow x^2(-2e^{-2x}) + 2xe^{-2x} = 0$

$\Leftrightarrow 2xe^{-2x}(-x+1) = 0$

$\Leftrightarrow x=0$ ou $e^{-2x} = 0$ ou $-x+1=0$
 ~~$x=0$~~ $x=1$

done $S = \{0, 1\}$

ex 24 $e^{x^2} = e^{7x-12} \Leftrightarrow x^2 = 7x-12 \Leftrightarrow x^2 - 7x + 12 = 0$

factoriser
et résoudre

$\Leftrightarrow (x-3)(x-4) = 0$

$S = \{3, 4\}$