Ma2 - Ch7 Réponses exercices

1. a. $\log_3(81)=4$
   b. $\log_3\left(\frac{1}{81}\right)=-4$
   c. $\log_a(c)=b$
   d. $\log_4(2\cdot x)=y$
   e. $\log_7\left(\frac{u+v}{u}\right)=3x$
   f. $\log_{0,3}(2,7)=x$

2. a. $3^4=81$
   b. $3^{-2}=\frac{1}{9}$
   c. $a^b=c$
   d. $4^6=3+y$
   e. $8^3=t-1$
   f. $a^{1/2}=1024$

3. a. $100^{\log(2)}=(10\cdot 10)^{\log(2)}$
   $=10^{\log(2)}\cdot 10^{\log(2)}=2\cdot 2=4$
   b. $\log(10^9)=9$
   c. $\log(0{,}0001)=\log(10^{-4})=-4$
   d. $e^{\ln(5)}=5$
   e. $\ln(e^{-6})=-6$
   f. $e^{3+\ln(2)}=e^3\cdot e^{\ln(2)}=e^3\cdot 2$

4. a. $D=]0;8[$
   $S=\{4\}$
   b. $D=]-\infty;-2/3[$
   $S=\{-1;-2\}$
   c. $D=]4;+\infty[$
   $S=\{13\}$
   d. $D=\mathbb{R}_+^*$
   $S=\{27\}$

5. a. $D=\mathbb{R}_-^*$
   $S=\emptyset$
   b. $D=\mathbb{R}_-^*$
   $S=\{-1\}$
   c. $D=\mathbb{R}_+^*$
   $S=\{2\}$
   d. $S=\left\{\frac{\log(3)}{\log(2)}\right\}\cong\{1,6\}$
   e. $S=\left\{-\frac{\log(180)}{\log\left(\frac{25}{6}\right)}\right\}\cong\{3,6\}$

7. $\log_a\left(\frac{x^3\sqrt{y}}{z^2}\right)$
   $=\log_a(x^3)+\log_a(\sqrt{y})-\log_a(z^2)$
   $=3\log_a(x)+\frac{1}{2}\log_a(y)-2\log_a(z)$

Math 2 Ch 8 Corrigé

ex 6. a. $C(t) = C_0(1+i)^t$

$$25000 = 6000\,(1+0,1)^t$$

$$\Leftrightarrow (1,1)^t = \frac{25}{6}$$

$$\Leftrightarrow \log(1,1^t) = \log\left(\frac{25}{6}\right)$$

$$\Leftrightarrow t \cdot \log(1,1) = \log\left(\frac{25}{6}\right)$$

$$\Leftrightarrow t = \frac{\log(25/6)}{\log(1,1)} \cong 14,97 \text{ ans} \cong 14 \text{ ans } 355 \text{ j } 6 \text{ h } 53' 2''$$

b. $C(t) = C_0\left(1+\frac{i}{12}\right)^{12t}$

$$25000 = 6000\left(1+\frac{0,1}{12}\right)^{12t}$$

$$\Leftrightarrow \frac{25}{6} = \left(\frac{12,1}{12}\right)^{12t}$$

$$\Leftrightarrow \log\left(\frac{12,1}{12}\right)^{12t} = \log\left(\frac{25}{6}\right)$$

$$\Leftrightarrow 12t \cdot \log\left(\frac{12,1}{12}\right) = \log\left(\frac{25}{6}\right)$$

$$\Leftrightarrow t = \frac{\log\left(\frac{25}{6}\right)}{12\log\left(\frac{12,1}{12}\right)} \cong 14,33 \text{ ans}$$

c. $C(t) = C_0 e^{it}$

$$25000 = 6000\, e^{0,1t}$$

Méth 1:

$$\log(25/6) = \log(e^{0,1t})$$

$$\Leftrightarrow 0,1t \cdot \log(e) = \log(25/6)$$

$$\Leftrightarrow t = \frac{\log(25/6)}{0,1 \cdot \log(e)} \cong 14,27 \text{ ans}$$

Méth 2

$$\frac{25}{6} = e^{0,1t} \Leftrightarrow \ln\left(\frac{25}{6}\right) = 0,1t$$

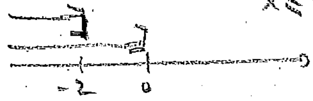
$$\Leftrightarrow t = \frac{\ln\left(\frac{25}{6}\right)}{0,1}$$

$$\cong 14,27 \text{ ans}$$

8. a. $D = \left]-\frac{3}{2} ; +\infty\right[$

$\log_5(2x+3) = \log_5(11 \cdot 3) \Leftrightarrow 2x+3 = 33$
$\Leftrightarrow 2x = 30$
$\Leftrightarrow x = 15$
$S = \{15\}$

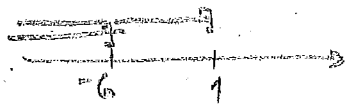
b. D : pb si $x \leq 0$ ou $x+2 \leq 0$
$x \leq -2$



donc $D =]0;+\infty[$

$\log_2(x) + \log_2(x+2) = \log_2(2^3) \Leftrightarrow \log_2(x(x+2)) = \log_2(8)$
$\Leftrightarrow x(x+2) = 8$
$\Leftrightarrow x^2 + 2x - 8 = 0$
$\Leftrightarrow (x+4)(x-2) = 0$
$x = -4$ ou $x = 2$
$S = \{2\}$

c. D : pb si $x+6 \leq 0$ ou $x-1 \leq 0$
$x \leq -6$ $\quad$ $x \leq 1$



donc $D =]1;+\infty[$

$\ln(x+6) - \ln(10) = \ln(x-1) - \ln(2) \Leftrightarrow \ln\left(\frac{x+6}{10}\right) = \ln\left(\frac{x-1}{2}\right)$
$\Leftrightarrow \frac{x+6}{10} = \frac{x-1}{2}$
$\Leftrightarrow 2x+12 = 10x-10$
$\Leftrightarrow 22 = 8x$
$\Leftrightarrow x = \frac{22}{8} = \frac{11}{4} = 2{,}75$
$S = \left\{\frac{11}{4}\right\}$

d. D : pb si $x \leq 0$; $D = \mathbb{R}_+^*$

$2\log_3(x) = 3\log_3(5) \Leftrightarrow \log_3(x^2) = \log_3(5^3)$
$\Leftrightarrow x^2 = 125$
$\Leftrightarrow x = \pm\sqrt{125} = \pm 5\sqrt{5} \simeq \pm 11{,}2$
$S = \{+5\sqrt{5}\}$

e. D : pb si $x+3 \leq 0$ ou $x+5 \leq 0$

$x \leq -3$ $x \leq -5$

donc $D =]-3 ; +\infty[$

$$\log_3(x+3) + \log_3(x+5) = 1 \iff \log_3((x+3)(x+5)) = \log_3(3)$$

$$\iff (x+3)(x+5) = 3$$

$$\iff x^2 + 8x + 12 = 0$$

$$\iff (x+6)(x+2) = 0$$

$x = -6$ ou $x = -2$

$S = \{-2\}$

19. a. $6^x = 7 \iff \log(6^x) = \log(7)$

$$\iff x \cdot \log(6) = \log(7)$$

$$\iff x = \frac{\log(7)}{\log(6)} \simeq 1{,}1$$

b. $5^{x-4} = 2 \iff \log(5^{x-4}) = \log(2)$

$$\iff (x-4)\log(5) = \log(2)$$

$$\iff x\log(5) - 4\log(5) = \log(2)$$

$$\iff x = \frac{4\log(5) + \log(2)}{\log(5)}$$

simplification d'écriture

$$= \frac{\log(5^4) + \log(2)}{\log(5)}$$

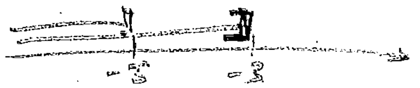
$$= \frac{\log(5^4 \cdot 2)}{\log(5)} = \frac{\log(1250)}{\log(5)} \simeq 4{,}4$$

c. $\left(\frac{1}{2}\right)^x = 100 \iff \log\left(\left(\frac{1}{2}\right)^x\right) = \log(100)$

$$\iff x \log\left(\frac{1}{2}\right) = \log(100)$$

$$\iff x = \frac{\log(10^2)}{\log 1 - \log 2} = \frac{2}{-\log(2)} = -\frac{2}{\log(2)} \simeq -6{,}6$$

e. D : pb si $x+3 \leq 0$ ou $x+5 \leq 0$
$x \leq -3$ $\quad$ $x \leq -5$



donc $D =]-3;+\infty[$

$$\log_3(x+3)+\log_3(x+5)=1 \iff \log_3((x+3)(x+5))=\log_3(3)$$
$$\iff (x+3)(x+5)=3$$
$$\iff x^2+8x+12=0$$
$$\iff (x+6)(x+2)=0$$
$$x=-6 \text{ ou } x=-2$$
$$S=\{-2\}$$

19. a. $6^x=7 \iff \log(6^x)=\log(7)$
$$\iff x\cdot\log(6)=\log(7)$$
$$\iff x=\frac{\log(7)}{\log(6)} \simeq 1{,}1$$

b. $5^{x-4}=2 \iff \log(5^{x-4})=\log(2)$
$$\iff (x-4)\log(5)=\log(2)$$
$$\iff x\log(5)-4\log(5)=\log(2)$$
$$\iff x=\frac{4\log(5)+\log(2)}{\log(5)}$$
simplification d'écriture
$$=\frac{\log(5^4)+\log(2)}{\log(5)}$$
$$=\frac{\log(5^4\cdot 2)}{\log(5)}=\frac{\log(1250)}{\log(5)}\simeq 4{,}4$$

c. $\left(\frac{1}{2}\right)^x=100 \iff \log\left(\frac{1}{2}^x\right)=\log(100)$
$$\iff x\log\left(\frac{1}{2}\right)=\log(100)$$
$$\iff x=\frac{\log(10^2)}{\log 1-\log 2}=\frac{2}{-\log(2)}=-\frac{2}{\log(2)}\simeq-6{,}6$$

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a) $2 \xrightarrow{80\%} 2\cdot 0{,}8 \longrightarrow 2\cdot 0{,}8^2 \longrightarrow 2{,}0{,}8^3 \longrightarrow 2\cdot 0{,}8^4 \quad \dots \quad \longrightarrow 2\cdot 0{,}8^n$

rebond 1 → 1,6 m; r2 → 1,28 m; r3 → 1,024 m; r4 0,8192 ... r_n → $2\cdot 0{,}8^n$ m

b) $2\cdot 0{,}8^n \leq 0{,}1 \Leftrightarrow 0{,}8^n \leq 0{,}05$

$\Leftrightarrow \log 0{,}8^n \leq \log 0{,}05$

$\Leftrightarrow n \log 0{,}8 \leq \log 0{,}05$

$\Leftrightarrow n \leq \dfrac{\log(0{,}05)}{\log(0{,}8)} \cong 13{,}43$

après 14 rebonds !

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a) $C_0(1+i)^{20} = 2\cdot C_0$

$1+i = 2^{1/20}$

$i = 2^{1/20} - 1$

$\cong 0{,}03526$

$\cong 3{,}53\%$

modèle croissance par paliers annuels

ou

$C_n = C_0 e^{it}$

$2C_0 = C_0 e^{i20}$

$20i = \ln(2)$

$i = \dfrac{\ln(2)}{20} \cong 0{,}03466 \cong 3{,}47\%$

modèle croissance continue exponentielle

b) $C_n = 1{,}5\,C_0$

$1{,}5\,C_0 = C_0 e^{it}$

$\Leftrightarrow it = \ln(1{,}5)$

$\Leftrightarrow t = \dfrac{\ln(1{,}5)}{i} \cong \dfrac{\ln(1{,}5)}{0{,}03466} \cong 11{,}5$ ans

c) $C_{100} = C_0 \cdot e^{i \cdot 100}$
$\cong C_0 \cdot e^{0,03466 \cdot 100} \cong C_0 \cdot 34$

d) $C_{200} = C_0 e^{i200}$
$\cong C_0 e^{0,03466 \cdot 200} \cong 1156,3$

<u>14</u> a) $C_{1999} = 6 \cdot 10^9$
$C_{1927} = 2 \cdot 10^9$
$t = 1999 - 1927$
$= 72$ ans

$C_{1999} = C_{1927}\, e^{i \cdot 72}$
$\Leftrightarrow 6 \cdot \cancel{10^9} = 2 \cdot \cancel{10^9}\, e^{i72}$
$\Leftrightarrow 3 = e^{72i}$
$\Leftrightarrow 72i = \ln(3)$
$\Leftrightarrow i = \dfrac{\ln(3)}{72} \cong 0,0153 \cong 1,53\,\%$

b) en 1960 : $C_{1960} = C_{1927}\, e^{i(1960-1927)}$
$= 2 \cdot 10^9 \cdot e^{33i}$ (avec le i calculé en a) et mis en mémoire dans la calculatrice)
$\cong 3,309$ milliards

en 1974 : $C_{1974} = C_{1927}\, e^{i(1974-1927)}$
$= 2 \cdot 10^9 \cdot e^{47i}$
$\cong 4,097$ milliards

en 1987 : $C_{1987} = C_{1927}\, e^{i(1987-1927)}$
$= 2 \cdot 10^9 \cdot e^{60i}$
$\cong 4,996$ milliards

en 1999 : déjà calculé $\rightarrow$ 6 MIA

c) $C(t) = 3 \cdot 10^9$
t inconnu

$C(t) = C_{1927}\, e^{i(t-1927)}$
$\Leftrightarrow 3 \cdot \cancel{10^9} = 2 \cdot \cancel{10^9}\, e^{i(t-1927)}$
$\Leftrightarrow i(t-1927) = \ln(3/2)$
$\Leftrightarrow t = \dfrac{\ln(\frac{3}{2})}{i} + 1927 \cong 1953,57$ ans

<u>en 1954</u>

d) $\left.\begin{array}{l} C(t) = 7\cdot 10^9 \\ t \text{ inconnu} \end{array}\right\}$ $C(t) = C_{1927}\, e^{i(t-1927)}$

$\Leftrightarrow 7\cdot 10^{\cancel{9}} = 2\cdot 10^{\cancel{9}}\, e^{i(t-1927)}$

$\Leftrightarrow t = \dfrac{\ln(7/2)}{i} + 1927 \simeq 2009{,}1$

en 2010 !

e) $130\cdot 10^6\ km^2 = 1{,}3\cdot 10^8\ km^2$
$= 1{,}3\cdot 10^{14}\ m^2$

$\left.\begin{array}{l} C(t) = 1{,}3\cdot 10^{14} \\ t \text{ inconnu} \end{array}\right\}$ $C(t) = C_{1927}\, e^{i(t-1927)}$

$\Leftrightarrow 1{,}3\cdot 10^{14} = 2\cdot 10^9\, e^{i(t-1927)}$

$\Leftrightarrow t = \dfrac{\ln\left(\frac{1{,}3\cdot 10^{14}}{2\cdot 10^9}\right)}{i} + 1927 \simeq 2653{,}3$

en 2654 !

15 / $N(t) = N_0\, a^t$

a) $\begin{cases} N(3) = N_0\, a^3 = 200\,000 & \Leftrightarrow N_0 = 200\,000/a^3 \\ N(4{,}5) = N_0\, a^{4,5} = 1\,600\,000 & \Leftrightarrow N_0 = 1\,600\,000/a^{4,5} \end{cases}$

$\Rightarrow \dfrac{200\,000}{a^3} = \dfrac{1\,600\,000}{a^{4,5}} \Leftrightarrow 2a^{4,5} = 16a^3$

$\Leftrightarrow a^{4,5} = 8a^3$

$\Leftrightarrow a^3(a^{1,5} - 8) = 0$

$a = 0$ ou $a^{1,5} = 8$

$a = 8^{\frac{1}{1,5}} = 8^{\frac{1}{\frac{3}{2}}}$

$= 8^{2/3} = 4$

$N_0 = \dfrac{200\,000}{4^3} = 3125$

b) $N(5) = 3125\cdot 4^5 = 3\,200\,000$

c) $\cancel{8}\,800\,000 = 3125\cdot 4^t$

$\Leftrightarrow 256 = 4^t$

$\Leftrightarrow \dfrac{\log(256)}{\log(4)} = t$

$\Leftrightarrow t = 4$

d) $10 N_0 = N_0\cdot 4^t \Leftrightarrow t = \dfrac{\log(10)}{\log(4)} \simeq 1{,}66\ j \simeq 1j\ 15h\ 51'\ 47''$