

Test de mathématiques

Date : 29 septembre 2023

Durée : 15'

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Cours : 3Ma2.DF01

Nom :

Points : / 24

Prénom :

Note : / 6

Groupe :

not / 1

Début du travail

Calculer les limites suivantes :

$$(a) \quad \lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{2x^2 - 6x} = \lim_{x \rightarrow 3} \frac{(x-3)(x-2)}{2x(x-3)} = \frac{3-2}{2 \cdot 3} = \frac{1}{6} \quad / 4$$

$$(b) \quad \lim_{x \rightarrow 0} \frac{\sqrt{x^2+1} + \sqrt{2}}{\sqrt{x^2+x} - \sqrt{x+1}} = \frac{\sqrt{1} + \sqrt{2}}{0 - \sqrt{1}} = \frac{1 + \sqrt{2}}{-1} = -\sqrt{2} - 1$$

/2

$$(c) \quad \lim_{x \rightarrow 1} \frac{x-1}{1-x} = \lim_{x \rightarrow 1} \frac{x-1}{-(x-1)} = \lim_{x \rightarrow 1} -1 = -1$$

/2

$$(d) \quad \lim_{x \rightarrow 4} \frac{x-4}{\sqrt{13-x}-3} \cdot \frac{\sqrt{13-x}+3}{\sqrt{13-x}+3} = \lim_{x \rightarrow 4} \frac{(x-4)(\sqrt{13-x}+3)}{13-x-9}$$

$$= \lim_{x \rightarrow 4} \frac{\cancel{x-4}(\sqrt{13-x}+3)}{\cancel{4-x}(-1)} = \frac{\sqrt{9}+3}{-1} = -6$$

/4

$$(e) \quad \lim_{x \rightarrow 1} \frac{1-x}{(1-x)^3} = \lim_{x \rightarrow 1} \frac{1}{(1-x)^2} \quad \text{type "1/0"}$$

$$\Rightarrow \lim_{x \rightarrow 1^+} \frac{1}{(1-x)^2} = \frac{1}{(0^-)^2} = \frac{1}{0^+} = +\infty$$

$$\lim_{x \rightarrow 1^-} \frac{1}{(1-x)^2} = \frac{1}{(0^+)^2} = \frac{1}{0^+} = +\infty$$

$$\Rightarrow \lim_{x \rightarrow 1} f(x) = +\infty$$

/4

$$\begin{aligned}
 \text{(f)} \quad \lim_{x \rightarrow -\infty} \frac{x^2 - 5x + 6}{2x^2 - 6x} &= \lim_{x \rightarrow -\infty} \frac{\cancel{x^2} (1 - 5/x + 6/x^2)}{\cancel{x^2} (2 - 6/x)} \\
 &= \frac{1}{2} \qquad \qquad \qquad /3
 \end{aligned}$$

$$\begin{aligned}
 \text{(g)} \quad \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 - 5x + 6}}{x} &= \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 (1 - 5/x + 6/x^2)}}{x} \\
 &= \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2} \sqrt{1 - 5/x + 6/x^2}}{x} \\
 &= \lim_{x \rightarrow -\infty} \frac{-x \sqrt{1 - 5/x + 6/x^2}}{x} \\
 &= -1 \cdot \sqrt{1} \\
 &= -1 \qquad \qquad \qquad /4
 \end{aligned}$$