

17ABN Tr n°5 Corrigé

(12) Ex1:

- x, y les 2 nombres

- $xy = 18 \Leftrightarrow y = \frac{18}{x}$

- à optimiser: $x^2 + y^2 = x^2 + \left(\frac{18}{x}\right)^2$ (3)

$$f(x) = x^2 + \left(\frac{18}{x}\right)^2$$

$$f'(x) = 2x + 2\left(\frac{18}{x}\right)' \cdot \left(\frac{18}{x}\right)$$

$$= 2x + 2 \cdot 18 \cdot \left(\frac{1}{x}\right)' \cdot \frac{18}{x}$$

$$= 2x + 6 \cdot 18 \cdot \left(-\frac{1}{x^2}\right) \cdot \frac{1}{x}$$

$$= 2 \left(x + \frac{324}{x^3} \right)$$

$$= 2 \left(\frac{x^4 - 324}{x^3} \right)$$

(3)

zéros de $x^4 - 324 = 0$:

$$x^4 - 324 = 0$$

$$x^4 = 324$$

$$x = \pm \sqrt[4]{324}$$

$$= \pm \sqrt[4]{2^2 \cdot 3^4}$$

$$= \pm 3\sqrt{2}$$

$$\approx \pm 4,25$$

x	$-3\sqrt{2}$	0	$3\sqrt{2}$	
$x^4 - 324$	+	0	-	-
x^3	-	-	0	+
$f'(x)$	-	0	1	-
$f(x)$	↗	min	↗	min ↗

(3)

- on veut des nombres négatifs :

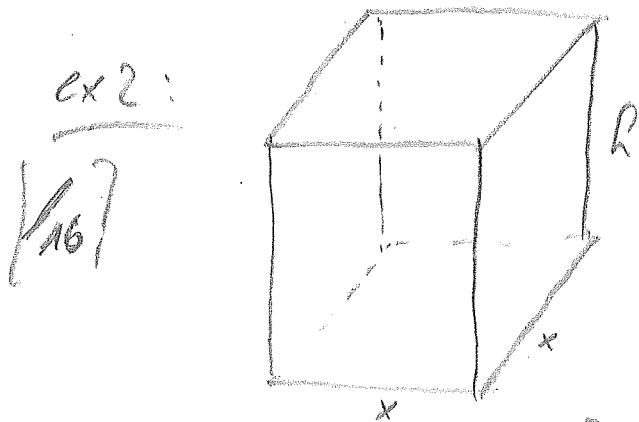
le min atteint pour $x = -3\sqrt{2}$

$$y = \frac{18}{-3\sqrt{2}} = -\frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{6\sqrt{2}}{2} = -3\sqrt{2}$$

(2)

il n'y a pas de max. (1)

Ex 2 :



[16]

x, h les dimensions

à optimiser : aire latérale totale

$$2x^2 + 4xh$$

on sait : $V = 2 \text{ dm}^3 \Leftrightarrow x \cdot h = 2$

$$\Leftrightarrow h = \frac{2}{x}$$

$$\begin{aligned} \text{Donc } f(x) &= 2x^2 + 4x \cdot \frac{2}{x} \\ &= 2x^2 + 8 \end{aligned}$$

$$\begin{aligned} f'(x) &= 4x + 8\left(\frac{1}{x}\right)' \\ &= 4x + 8\left(-\frac{1}{x^2}\right) \\ &= 4x - \frac{8}{x^2} \\ &= \frac{4(x^3 - 2)}{x^2} \end{aligned} \quad (3)$$

x	0	$\sqrt[3]{2}$
$x^3 - 2$	-	0
x^2	+ 0	+ + +
$f'(x)$	- / - 0 + ↗	(3)
$f(x)$	↓ / ↓ min	

La base fait $a = \sqrt[3]{2} \text{ dm}$ et la hauteur $h = \frac{2}{(\sqrt[3]{2})^2} \approx 1,26 \text{ dm}$

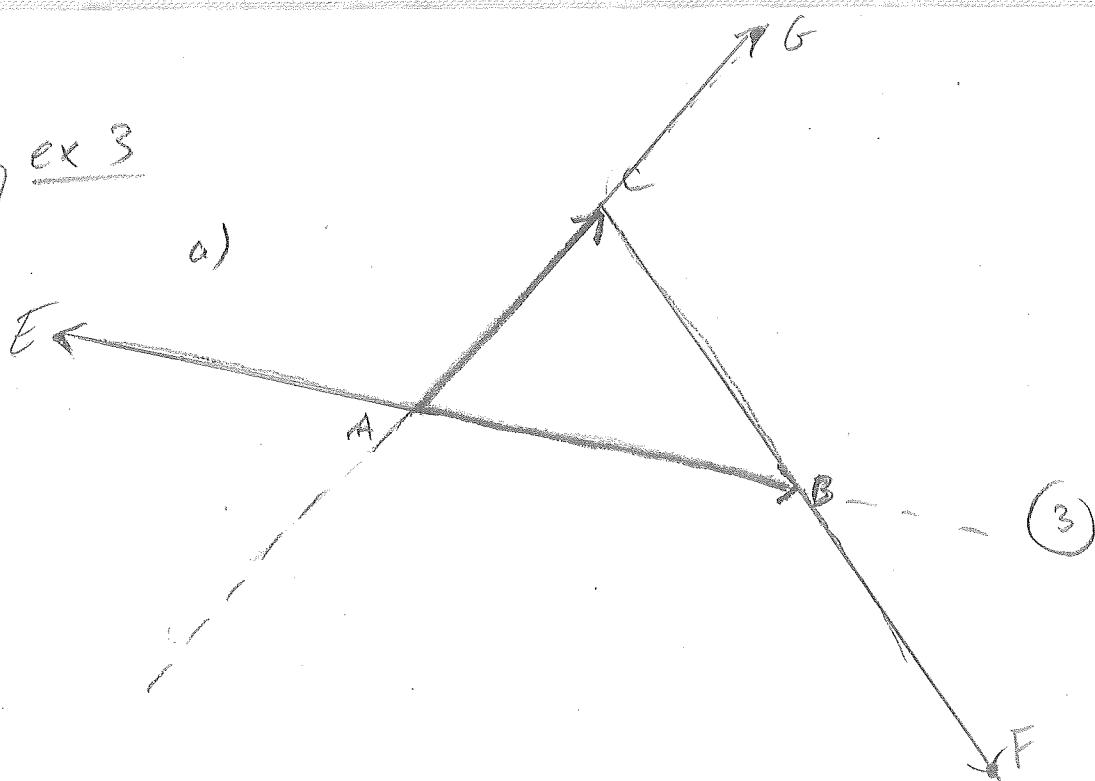
$$x \approx 1,26 \text{ dm}$$

(2)

Il s'agit d'un cube ! (2)

(13) ex 3

a)



$$b) \overrightarrow{AG} = 2\overrightarrow{AC} + 0 \cdot \overrightarrow{AB} \quad (1)$$

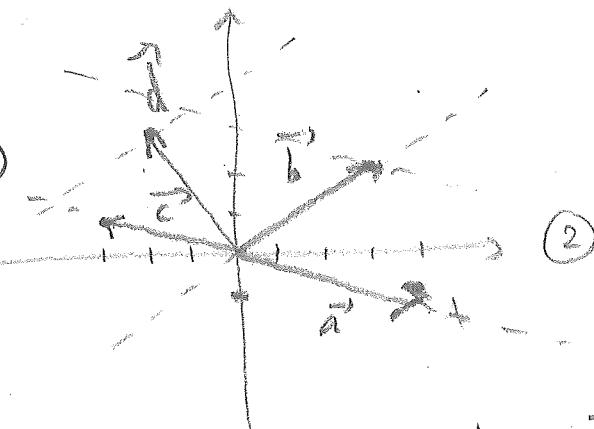
$$\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB} = 1 \cdot \overrightarrow{AC} + (-1) \overrightarrow{AB} \quad (2)$$

$$\begin{aligned}\overrightarrow{FG} &= \overrightarrow{FB} + \overrightarrow{BG} + \overrightarrow{CG} \\ &= 2\overrightarrow{BC} + \overrightarrow{AC} \\ &= 2[\overrightarrow{AC} - \overrightarrow{AB}] + \overrightarrow{AC} \\ &= 3\overrightarrow{AC} + (-2)\overrightarrow{AB} \quad (3)\end{aligned}$$

ex 4

(13)

a)



$$b) \overrightarrow{d} = -\overrightarrow{a} + \overrightarrow{b} \quad (2)$$

$$\begin{aligned}c) \text{ on cherche } \alpha, \beta \in \mathbb{R} \text{ tq } \alpha \overrightarrow{a} + \beta \overrightarrow{b} &= \overrightarrow{d} \\ \Leftrightarrow \alpha(-1) + \beta(2) &= (-3) \\ \Leftrightarrow \begin{cases} 4\alpha + 3\beta + 2 = 0 \\ -\alpha + 2\beta = 3 \end{cases} \quad | \begin{matrix} 4 \\ 4 \end{matrix} \end{aligned}$$

$$\alpha + \beta = 10$$

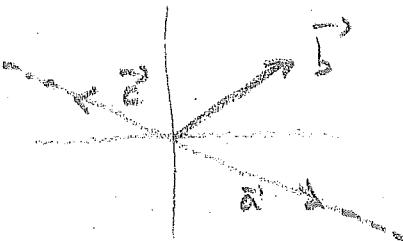
$$\beta = \frac{10}{7}$$

$$d \text{ don } -\alpha + 2 \cdot \frac{10}{7} = 3$$

$$\alpha = \frac{20}{7} - 3 = -\frac{13}{7}$$

$$\left. \begin{aligned} \overrightarrow{d} &= \frac{-13}{7} \overrightarrow{a} + \frac{10}{7} \overrightarrow{b} \\ (4) \end{aligned} \right\}$$

d) géom: on voit que \vec{a} et \vec{c} sont colinéaires !



\vec{b} ne peut être combinaison de \vec{a} et \vec{c}

(2)

alg: il existe α et $\beta \in \mathbb{R}$ tq $\alpha \vec{a} + \beta \vec{c} = \vec{b}$
 $\Leftrightarrow \alpha \begin{pmatrix} 4 \\ -1 \end{pmatrix} + \beta \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$$\begin{cases} 4\alpha + 3\beta = 3 \\ -\alpha + 0,75\beta = 2 \end{cases} \quad (1)$$

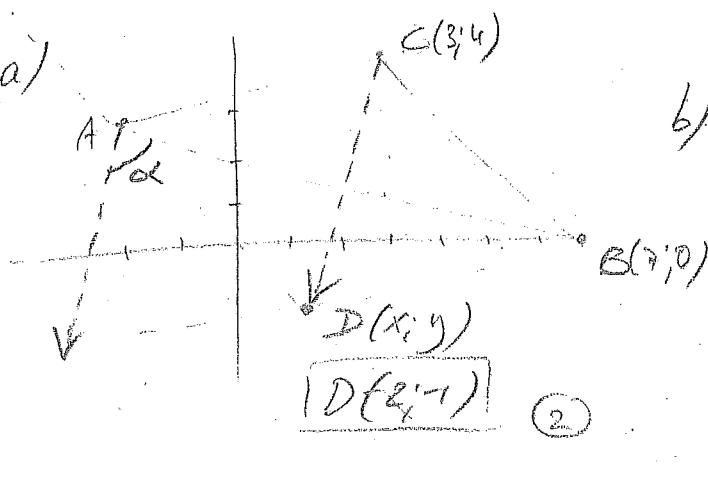
$$\begin{cases} 4\alpha + 3\beta = 3 \\ -4\alpha + 3\beta = 8 \end{cases}$$

$$0 = 11 \quad S = \emptyset$$

(3)

Ex5:

1/3) a)



b) on veut $\vec{AD} = \vec{CB}$

$$\begin{pmatrix} x-(-2) \\ y-3 \end{pmatrix} = \begin{pmatrix} 7-3 \\ 0-4 \end{pmatrix}$$

$$\begin{cases} x+2 = 4 \\ y-3 = -4 \end{cases}$$

$$\begin{cases} x=2 \\ y=-1 \end{cases}$$

$D(2;-1)$ (3)

c) $\vec{CD} = \begin{pmatrix} 2-3 \\ -1-4 \end{pmatrix} = \begin{pmatrix} -1 \\ -5 \end{pmatrix}$

$$\|\vec{CD}\| = \sqrt{(-1)^2 + (-5)^2} = \sqrt{26} \quad (3)$$

d) $\vec{AB} \cdot \vec{CD} = \|\vec{AB}\| \cdot \|\vec{CD}\| \cos(\alpha)$

$$\begin{pmatrix} 7+2 \\ 0+3 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -5 \end{pmatrix} = \sqrt{81+9} \cdot \sqrt{26} \cos(\alpha)$$

$$\therefore -9+15 = \sqrt{90} \sqrt{26} \cos(\alpha)$$

$$\therefore \cos(\alpha) = \frac{6}{\sqrt{90 \cdot 26}} \approx \cos^{-1} 82,9 \quad (5)$$