Ma3N – Corrigé travail 30' n° 1 — 61 pts not /12 fr /12 [/63]

ex2 [/17]

(a) $f(0) = -4$ ①

pb si $x^2 - 1 \neq 0$
$(x-1)(x+1) = 0$
$x - 1 = 0$ ou $x + 1 = 0$
$x = 1$ $x = -1$
$\Rightarrow D_f = \mathbb{R} \setminus \{\pm 1\}$ ②

$f(x) = 0 \Leftrightarrow \frac{x^2+5x+4}{x^2-1} = 0$

$\Leftrightarrow \frac{(x+4)(x+1)}{x^2-1} = 0$

$\Leftrightarrow (x+4)(x+1) = 0$ (et $x^2-1 \neq 0$ ou $x \in D_f$)

$x + 4 = 0$ ou $x + 1 = 0$
$x = -4$ $x = -1$
$\in D_f$ ✓ $\notin D_f$

$\Rightarrow Z_f = \{-4\}$ ③

(b) ②

x		-4		-1		1	
x^2+5x+4	+	0	−	0	+	+	+
x^2-1	+	+	+	0	−	0	+
$f(x)$	+	0	−	//	−	//	+

(c) • $\lim_{x \to -1} f(x)$ type "$\frac{0}{0}$" [cf. tds] ②

$\lim_{x \to -1} \frac{(x+4)\cancel{(x+1)}}{(x-1)\cancel{(x+1)}} = \frac{-1+4}{-1-1} = -\frac{3}{2}$

• $\lim_{x \to 1} f(x)$ type "$\frac{1}{0}$"

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{(x+4)\cancel{(x+1)}}{(x-1)\cancel{(x+1)}} = \frac{5}{0^-} = -\infty$

$\lim_{x \to 1^+} f(x) = \frac{5}{0^+} = +\infty$

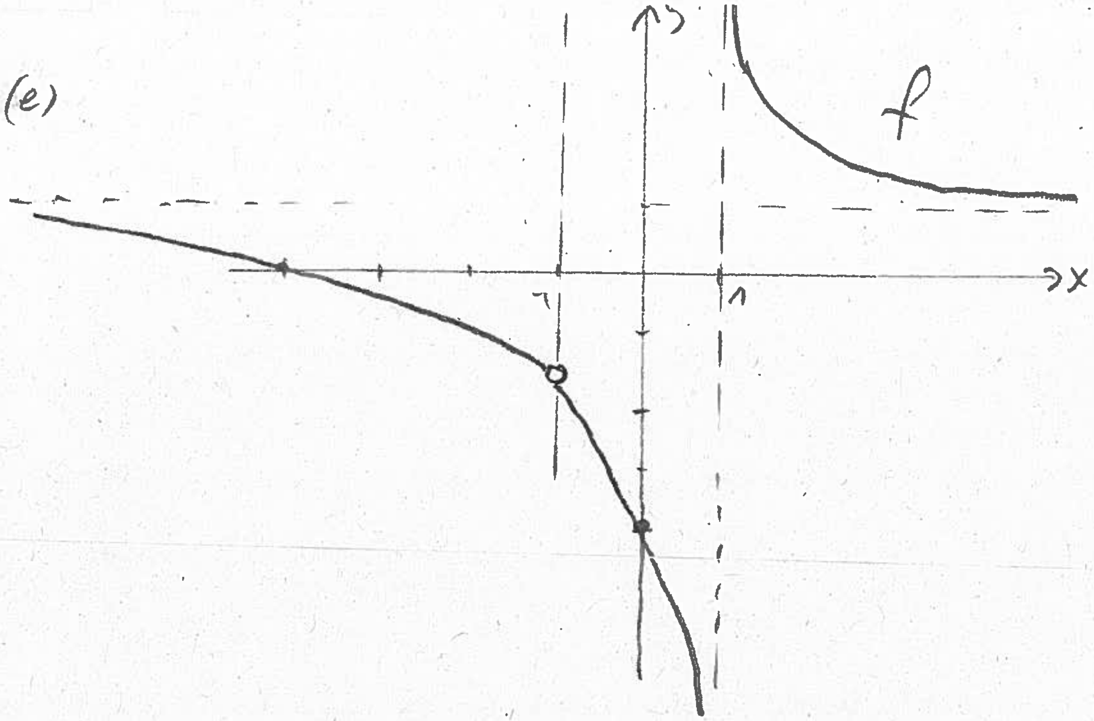
$\Rightarrow \lim_{x \to 1} f(x) \nexists$ ③

(d) $\lim_{x \to -\infty} f(x)$: type $\frac{\infty}{\infty}$ ③

$\lim_{x \to +\infty} \frac{\cancel{x^2}(1 + 5/x + 4/x^2)}{\cancel{x^2}(1 - 1/x^2)} = \frac{1}{1} = 1$

$\lim_{x \to -\infty} \frac{x^2(1 + 5/x + 4/x^2)}{x^2(1 - 1/x^2)} = \frac{1}{1} = 1$

(e)



④

Ex 4 x : nbre passagers en plus des 100 premiers

/14

x+100 : nbre total de passagers

(60 − 0,25·x): nouveau prix

(a) $B(x) = (60-0{,}25\cdot x)\cdot(x+100) - [1000 + 15\cdot x]$

$= 60x - 0{,}25x^2 + 6000 - 25x - 1000 - 15x$ ⑥

$= -0{,}25x^2 + 20x + 5000$

(b) $D_{\text{viable}} = \{1; 2; 3; 4; \dots\}$ (ou $[1; +\infty[$) (ou $[0, +\infty[$) ①

(c) fct de degré 2, concave. Sommet $S = \left(-\frac{b}{2a}; -\frac{\Delta}{4a}\right)$

$\Delta = b^2 - 4ac$

$= 20^2 - 4\cdot\left(-\frac{1}{4}\right)\cdot 5000$

$= 400 + 5000$

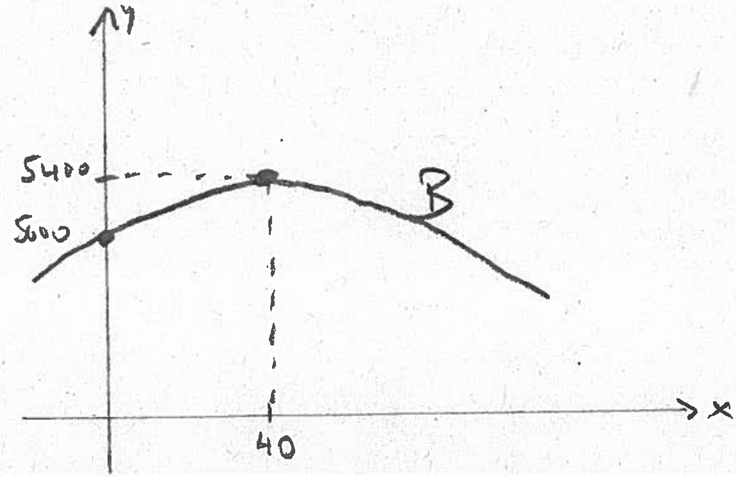
$= 5400$

$= \left(-\frac{20}{-\frac{1}{2}}; -\frac{5400}{4\cdot\left(-\frac{1}{4}\right)}\right)$

$= (40; 5400)$

le bénéfice est maximal pour x = 40 passagers suppl. ④
il vaut alors 5400.-

(d)



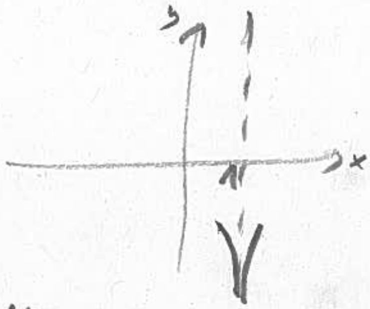
③

ex 1

[/15]

(a) $\lim\limits_{x\to 1} \dfrac{-2}{(1-x)^2}$ type "$\frac{1}{0}$"

$$\left.\begin{aligned} \lim_{x\to 1^-} f(x) &= \frac{-2}{(0^+)^2} = -\infty \\ \lim_{x\to 1^+} f(x) &= \frac{-2}{(0^-)^2} = -\infty \end{aligned}\right\} \Rightarrow \lim_{x\to 1} f(x) \nexists$$



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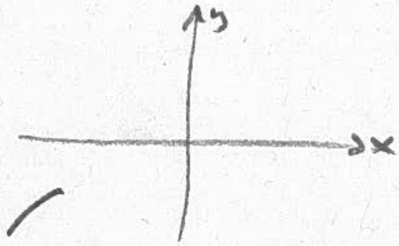
(b) $\lim\limits_{x\to -\infty} x^3 - 4x^2 + \dfrac{2}{x^5} - 1$

$= (-\infty)^3 - 4(-\infty)^2 + \dfrac{2}{(-\infty)^5} - 1$

$= -\infty - 4(\infty) + 0 - 1$

$= -\infty$



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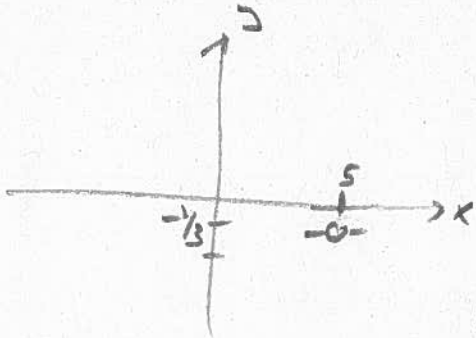
(c) $\lim\limits_{x\to 5} \dfrac{3-\sqrt{2x-1}}{x-5}$ type $\frac{0}{0}$ $\sqrt{}$

$= \lim\limits_{x\to 5} \dfrac{(3-\sqrt{2x-1})}{x-5} \dfrac{(3+\sqrt{2x-1})}{(3+\sqrt{2x-1})}$

$= \lim\limits_{x\to 5} \dfrac{9-(2x-1)}{(x-5)(3+\sqrt{2x-1})} = \lim\limits_{x\to 5} \dfrac{10-2x}{(x-5)(3+\sqrt{2x-1})}$

$= \lim\limits_{x\to 5} \dfrac{-2\cancel{(x-5)}}{\cancel{(x-5)}(3+\sqrt{2x-1})} = \dfrac{-2}{3+3} = -\dfrac{1}{3}$



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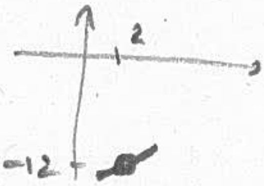
(d) $\lim\limits_{x\to 2} \dfrac{x^2+x-20}{x^2-4x+5} \overset{\text{dir.t}}{=} \dfrac{4+4-20}{4-8+5} = \dfrac{-12}{1} = -12$



3

[4] (a) 6 (e) $-\infty$

/7 (b) 2 (f) -2

(c) $\cancel{\exists}$ (g) 3

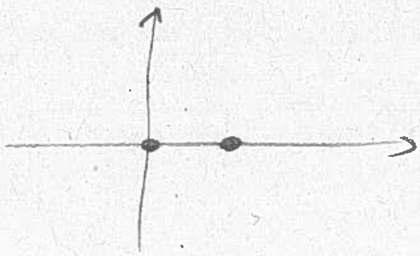
(d) $+\infty$

[5] (a) <u>vrai</u> <u>dem</u> $x = 9{,}\overline{9} = 9 + 0{,}\overline{9} = 9 + 3 \cdot 0{,}\overline{3} = 9 + 3 \cdot \frac{1}{3} = 9 + 1 = 1$

1+3

/8 (b) <u>faux</u> <u>c-ex</u> $f(x) = x^4 \cdot (x-1)$

0 et 1 sont deux zeros successifs



0,5 est-il un extremum local?

$f(0{,}5) = f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^4 \left(-\frac{1}{2}\right) = -\frac{1}{2^5}$

$= -\frac{1}{32}$

$f(0{,}4) \overset{?}{\leq}$

donc 0,5 n'est pas un extr. local

1+3