MA3M Corr. Tr. DO n° 4 — 61 52 53

ex 1 (/3)

$$P_{6;\,3,2} = \frac{6!}{3!\,2!} = 60$$ (3)

ex 2 (/6)

(a)

i) $C^{10}_{6} = \frac{10!}{(10-6)!\,6!} = \frac{10\cdot 9\cdot 8\cdot 7}{1\cdot 2\cdot 3\cdot 4} = 210$ (3)

ii) PRB → (1)

(b) il reste 4 ex à choisir parmi 8 : $C^8_4 = 70$ (2)

ex 3 (/18)

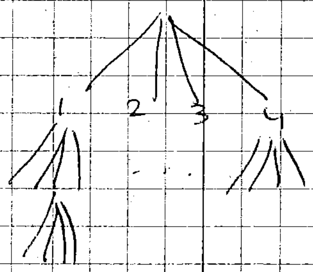
(a) E_1 "obtenir 444"
E_2 "obtenir 111" (2)

(b) $\# \Omega = 4\cdot 4\cdot 4 = 64$ (2)

(c) E_1 et E_2 car $E_1 \cup E_2 = \emptyset$ (2)

(d) C = "obtenir 3 ch. pairs" (2)
D = "obtenir 3 ch. identiques" car $C \cap D = \{$"222", "444"$\}$

(e)



i) $\frac{2}{4}\cdot\frac{2}{4}\cdot\frac{2}{4} = \frac{1}{8}$ (1)

ii) "112" ou "121" ou "211" : $p = \frac{3}{64}$ (1)

iii) E = "somme ≤ 4" = "somme = 4" ∪ "somme = 3"
= "112; 121; 211" ∪ "111"

↳ $p(E) = \frac{4}{64}$ (3)

$\bar{E}$ = "somme > 4" $\Rightarrow p(\bar{E}) = 1 - p(E) = \frac{60}{64}$

iv) 6 possibilités $\Rightarrow p = 6\cdot\frac{1}{64} = \frac{6}{64}$ (2)

v) F = "$\overline{4}\,\overline{4}\,4$" ∪ "$4\,\overline{4}\,\overline{4}$" ∪ "$\overline{4}\,4\,\overline{4}$" (2)

↳ $p(F) = \frac{1}{4}\left(\frac{3}{4}\right)^2 + \frac{3}{4}\cdot\frac{1}{4}\cdot\frac{3}{4} + \left(\frac{3}{4}\right)^2\cdot\frac{1}{4} = 3\left(\frac{3}{4}\right)^2\cdot\frac{1}{4} = \left(\frac{3}{4}\right)^3 = \frac{27}{64}$

iv) $P(A \cap B) = P(\emptyset) = 0 \neq P(A) \cdot P(B) = \frac{1}{8} \cdot \frac{3}{4}$ (3)

donc A et B sont dépendants

Ex 4

[/7] a) Vrai : dém $\binom{n}{p} = C_p^n = \frac{n!}{(n-p)!\,p!}$ (1+2)

$$\binom{n}{n-p} = C_{n-p}^n = \frac{n!}{(n-(n-p))!\,(n-p)!} = \frac{n!}{p!\,(n-p)!}$$

b) faux : c-exemple

on jette une pièce 2 fois (1+3)

$P(PP) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$P(\text{"deux fois la même chose"}) = P(PP \cup FF)$

$\underset{\text{disj}}{=} \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$P(PP \cup \text{"deux fois m. chose"})$

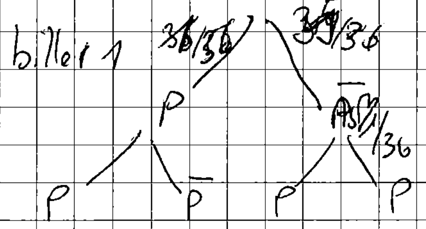
$= P(\text{"deux fois m. chose"}) = \frac{1}{2}$

et $\frac{1}{4} + \frac{1}{2} \neq \frac{1}{2}$!

Ex 5

[/10] A = "avoir au moins un prix"

$\overline{A}$ = "n'avoir aucun prix"



$p(\overline{A}) = \left(\frac{35}{36}\right)^n$

$p(A) = 1 - p(\overline{A}) = 1 - \left(\frac{35}{36}\right)^n$

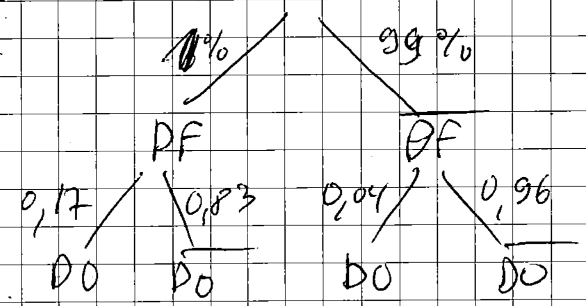
on veut $p(A) > 95\% \iff 1 - \left(\frac{35}{36}\right)^n > 0{,}95$ (6)

$\iff \left(\frac{35}{36}\right)^n < 0{,}05 \iff n \log\left(\frac{35}{36}\right) < \log(0{,}05)$ (3)

$\iff n > \frac{\log(0{,}05)}{\log(35/36)} \simeq 106{,}3$ Il faut au moins 107 lancers ! (1)

ex 6

(18)



(4)

$$P(DF \mid DO) = \frac{P(DF \cap DO)}{P(DO)} = \quad (2)$$

$$= \frac{0{,}0017}{0{,}0413} = \frac{0{,}01 \cdot 0{,}17}{0{,}01 \cdot 0{,}17 + 0{,}99 \cdot 0{,}04} \approx 4{,}1\,\% \quad (2)$$