Limites

1 Soit f - une fonction définie par $f(x)=\frac{x^4+x^3-x^2-x}{1-x}$.

a. $D_{f(x)}=\mathbb{R}\setminus\{1\}$

Zéros de $f(x)$: $x^4+x^3-x^2-x=0$ (et $x\in D_f(x)$)

$\Leftrightarrow$ $x^3(x+1)-x(x+1)=(x^3-x)(x+1)=x(x^2-1)(x+1)=x(x-1)(x+1)^2=0$

$x=0$ ou $x=1$ ou $x=-1$, mais $x=1\notin D_f$; d'où $Z_{f(x)}=\{-1;0\}$

b. $f(1{,}9)\approx-15{,}98$ $f(2{,}1)\approx-20{,}18$

$f(1{,}99)\approx-17{,}8$ $f(2{,}01)\approx-18{,}21$

$f(1{,}999)\approx-17{,}98$ $f(2{,}001)\approx-18{,}02$

(fonction «table» de la calculatrice ou Geogebra)

c. Conjecture : $\lim_{x\to 2} f(x)=-18$

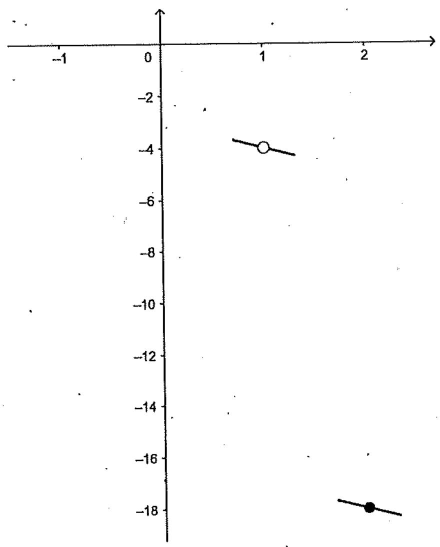
d. $f(0{,}9)\approx-3{,}25$ $f(1{,}1)\approx-4{,}85$

$f(0{,}99)\approx-3{,}92$ $f(1{,}01)\approx-4{,}08$

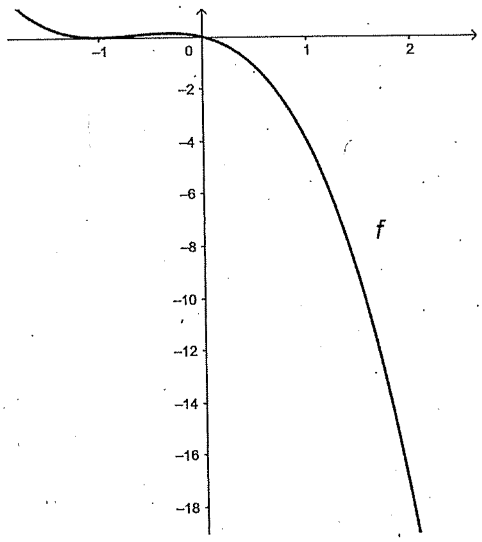
$f(0{,}999)\approx-3{,}99$ $f(1{,}001)\approx-4{,}01$

e. Conjecture : $\lim_{x\to 1} f(x)=-4$

f. « localement »



avec GeoGebra.



Attention !!

GeoGebra ne montre pas le problème à $x=1$

2

a. $\lim\limits_{x \to 1} f(x) = 1$

b. $\lim\limits_{x \to -3^-} f(x) = 4$

c. $\lim\limits_{x \to -3^+} f(x) = 0$

d. $f(-3) = 2$

e. $\lim\limits_{x \to 3^+} f(x) = -\infty$

f. $\lim\limits_{x \to 3^-} f(x) = 1$

g. $\nexists \lim\limits_{x \to 3} f(x)$

h. $f(3) = 1$

i. $\lim\limits_{x \to -1} f(x) = 0$

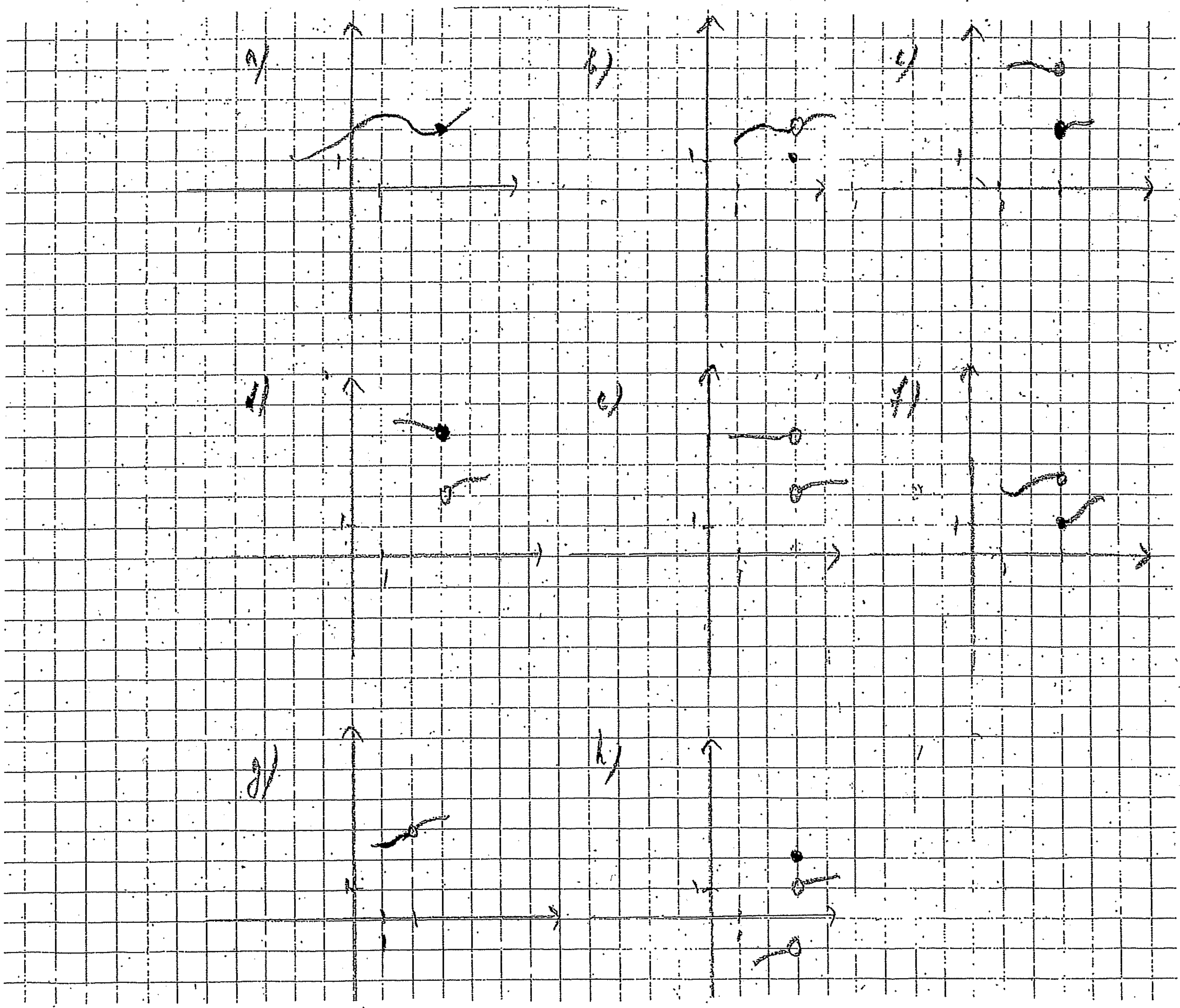
j. $\lim\limits_{x \to 4} f(x) = 4$

k. $\lim\limits_{x \to 2^+} f(x) = 1$

l. $\lim\limits_{x \to 2^-} f(x) = 1$

m. $\lim\limits_{x \to +\infty} f(x) = 9$

3



ex 4

a) Soit $\varepsilon > 0$ qcq :

on veut que $|f(x) - L| < \varepsilon \Leftrightarrow |2x - (-2)| < \varepsilon \Leftrightarrow 2|x+1| < \varepsilon$

$\Leftrightarrow |x+1| < \varepsilon/2$

prenons $\delta = \varepsilon/2$

alors on a : si $|x - (-1)| < \delta$, on a : $|x+1| < \varepsilon/2 \Leftrightarrow |f(x)+2| < \varepsilon$ (vu ci-dessus)

d'où $\lim\limits_{x \to 1} f(x) = 2$

b) Soit $\varepsilon > 0$ qcq :

on veut que $|f(x) - L| < \varepsilon \Leftrightarrow |3x - 1 - 5| < \varepsilon \Leftrightarrow 3|x-2| < \varepsilon$

$\Leftrightarrow |x-2| < \varepsilon/3$

prenons $\delta = \varepsilon/3$

alors on a : si $|x-2| < \delta$, on a : $|x-2| < \frac{\varepsilon}{3} \Leftrightarrow |f(x) - 5| < \varepsilon$ (vu ci-dessus)

d'où $\lim\limits_{x \to 2} f(x) = 5$

c) Soit $\varepsilon > 0$ qcq :

on veut que $|f(x) - a| < \varepsilon \Leftrightarrow |x - a| < \varepsilon$

prenons $\delta = \varepsilon$

alors on a : si $|x - a| < \delta$, on a : $|x - a| < \varepsilon \Leftrightarrow |f(x) - a| < \varepsilon$

d'où $\lim\limits_{x \to a} f(x) = a$

d) Soit $\varepsilon > 0$ qcq :

on veut que $|f(x) - k| < \varepsilon \Leftrightarrow |k - k| < \varepsilon \Leftrightarrow 0 < \varepsilon$

c'est tjs vrai ! on peut prendre δ quelconque

et on aura tjs $|x - k| < \delta \Rightarrow |f(x) - k| < \varepsilon$

donc $\lim\limits_{x \to a} f(x) = k$

5 Bonus

6

a. $\lim\limits_{x\to1}\dfrac{x^2-6x+8}{x^2+5x+4} \overset{PrL4}{=} \dfrac{\lim\limits_{x\to1}[x^2-6x+8]}{\lim\limits_{x\to1}[x^2+5x+4]} \overset{PrL7}{=} \dfrac{1^2-6\cdot1+8}{1^2+5\cdot1+4}=\dfrac{3}{10}$

b. $\lim\limits_{x\to0}\dfrac{\sqrt{x^2-4x+16}}{3} \overset{PrL6}{=} \dfrac{1}{3}\lim\limits_{x\to0}\sqrt{x^2-4x+16} \overset{PrL5}{=} \dfrac{1}{3}\sqrt{\lim\limits_{x\to0}[x^2-4x+16]} \overset{PrL7}{=} \dfrac{1}{3}\sqrt{0^2-4\cdot0+16}=\dfrac{1}{3}\sqrt{16}=\dfrac{4}{3}$

c. $\lim\limits_{x\to-\frac{\pi}{2}}\cos(2x) \overset{PrL5}{=} \cos(\lim\limits_{x\to-\frac{\pi}{2}}2x) \overset{PrL5}{=} \cos[2\cdot(\lim\limits_{x\to-\frac{\pi}{2}}x)] \overset{L2}{=} \cos(2\cdot(-\frac{\pi}{2}))=\cos(-\pi)=-1$

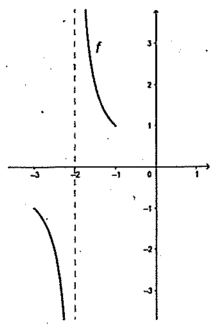
7 Bonus

Limites infinies

8

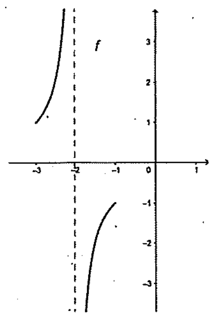
a. $\lim\limits_{x\to-2}\dfrac{2}{2x+4}=$ « $\dfrac{2}{0}$ »: c'est un type $\dfrac{1}{0}$

$$\left.\begin{array}{l}\lim\limits_{x\to-2^-}f(x)=\lim\limits_{x\to-2^-}\dfrac{2}{2\cdot(x+2)}=\lim\limits_{x\to-2^-}\dfrac{1}{x+2}=\dfrac{1}{0^-}=-\infty\\ \lim\limits_{x\to-2^+}f(x)=\lim\limits_{x\to-2^+}\dfrac{2}{2\cdot(x+2)}=\lim\limits_{x\to-2^+}\dfrac{1}{x+2}=\dfrac{1}{0^+}=+\infty\end{array}\right\}\text{ donc }\nexists\lim\limits_{x\to-2}f(x)$$

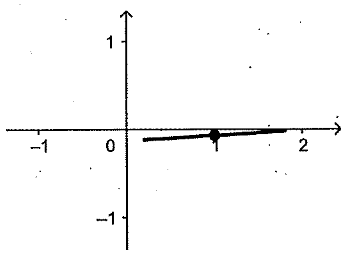


b. $\lim\limits_{x\to-2}\dfrac{-2}{2x+4}=$ « $\dfrac{-2}{0}$ »: c'est un type $\dfrac{1}{0}$

$$\left.\begin{array}{l}\lim\limits_{x\to-2^-}f(x)=\lim\limits_{x\to-2^-}\dfrac{-2}{2\cdot(x+2)}=\lim\limits_{x\to-2^-}\dfrac{-1}{x+2}=\dfrac{-1}{0^-}=+\infty\\ \lim\limits_{x\to-2^+}f(x)=\lim\limits_{x\to-2^+}\dfrac{-2}{2\cdot(x+2)}=\lim\limits_{x\to-2^+}\dfrac{-1}{x+2}=\dfrac{-1}{0^+}=-\infty\end{array}\right\}\text{ donc }\nexists\lim\limits_{x\to-2}f(x)$$

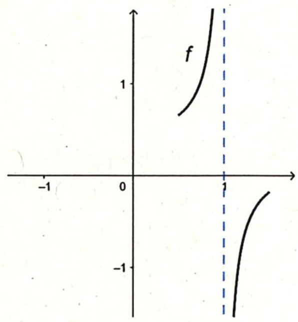


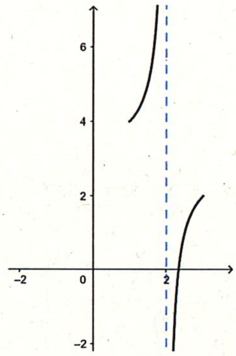
c. $\lim\limits_{x\to1}\dfrac{2-x}{x^2-16}=\dfrac{2-1}{1^2-16}=\dfrac{1}{-15}=-\dfrac{1}{15}$



d. $\lim\limits_{x\to 1}\dfrac{x-2}{x^2+3x-4}=$ « $\dfrac{1}{0}$ »: c'est un type $\dfrac{1}{0}$

$$\left.\begin{array}{l}\lim\limits_{x\to 1^-} f(x)=\lim\limits_{x\to 1^-}\dfrac{x-2}{(x+4)\cdot(x-1)}=\lim\limits_{x\to 1^-}\dfrac{-1}{x+2}=\dfrac{-1}{5\cdot 0^-}=+\infty\\ \lim\limits_{x\to 1^+} f(x)=\lim\limits_{x\to 1^+}\dfrac{x-2}{(x+4)\cdot(x-1)}=\lim\limits_{x\to 1^+}\dfrac{-1}{x+2}=\dfrac{-1}{5\cdot 0^+}=-\infty\end{array}\right\}\text{ donc } \nexists\lim_{x\to 1} f(x)$$



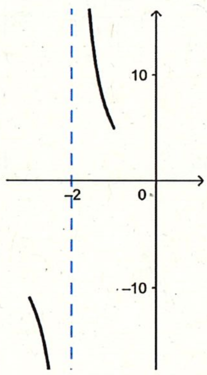


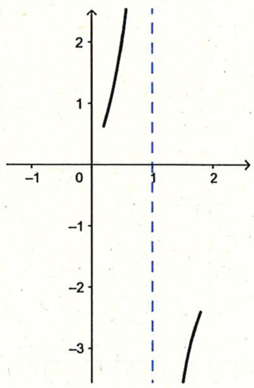
e. $\lim\limits_{x\to 2}\dfrac{3x-7}{x-2}=$ « $\dfrac{-1}{0}$ »: c'est un type $\dfrac{1}{0}$

$$\left.\begin{array}{l}\lim\limits_{x\to 2^-} f(x)=\lim\limits_{x\to 2^-}\dfrac{3\cdot 2-7}{2^--2}=\dfrac{-1}{0^-}=+\infty\\ \lim\limits_{x\to 2^+} f(x)=\lim\limits_{x\to 2^+}\dfrac{3\cdot 2-7}{2^+-2}=\dfrac{-1}{0^+}=-\infty\end{array}\right\}\text{ donc } \nexists\lim_{x\to 2} f(x)$$

f. $\lim\limits_{x\to -2}\dfrac{-3x+2}{x+2}=$ « $\dfrac{8}{0}$ »: c'est un type $\dfrac{1}{0}$

$$\left.\begin{array}{l}\lim\limits_{x\to -2^-} f(x)=\lim\limits_{x\to -2^-}\dfrac{-3\cdot(-2)+2}{-2^-+2}=\dfrac{8}{0^-}=-\infty\\ \lim\limits_{x\to -2^+} f(x)=\lim\limits_{x\to -2^+}\dfrac{-3\cdot(-2)+2}{-2^++2}=\dfrac{8}{0^+}=+\infty\end{array}\right\}\text{ donc } \nexists\lim_{x\to -2} f(x)$$





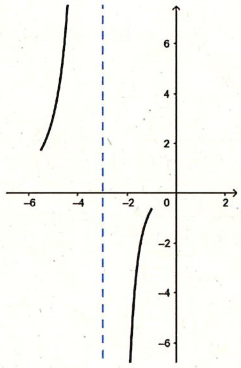
g. $\lim\limits_{x\to 1}\dfrac{3x}{1-x^2}=$ « $\dfrac{3}{0}$ »: c'est un type $\dfrac{1}{0}$

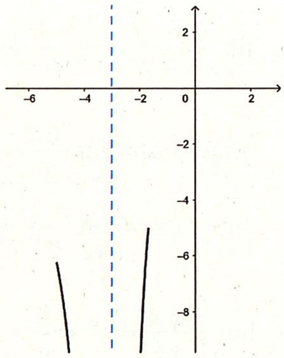
On factorise le plus possible le dénominateur pour contrôler le signe :

$$\left.\begin{array}{l}\lim\limits_{x\to 1^-} f(x)=\lim\limits_{x\to 1^-}\dfrac{3x}{(1+x)(1-x)}=\dfrac{3\cdot 1}{2\cdot 0^+}=+\infty\\ \lim\limits_{x\to 1^+} f(x)=\lim\limits_{x\to 1^+}\dfrac{3x}{(1+x)(1-x)}=\dfrac{3\cdot 1}{2\cdot 0^-}=-\infty\end{array}\right\}\text{ donc } \nexists\lim_{x\to 1} f(x)$$

h. $\lim\limits_{x\to-3}\dfrac{5x}{(x+3)^3}=$ « $\dfrac{-15}{0}$ »: c'est un type $\dfrac{1}{0}$

$$\left.\begin{aligned}\lim_{x\to-3^-} f(x)&=\lim_{x\to-3^-}\frac{5\cdot(-3)}{(0^-)^3}=\frac{-15}{0^-}=+\infty\\ \lim_{x\to-3^+} f(x)&=\lim_{x\to-3^+}\frac{5\cdot(-3)}{(0^+)^3}=\frac{-15}{0^+}=-\infty\end{aligned}\right\}\quad \text{donc } \nexists \lim_{x\to-3} f(x)$$



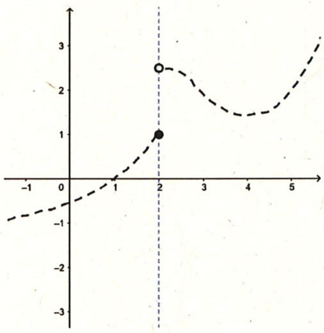


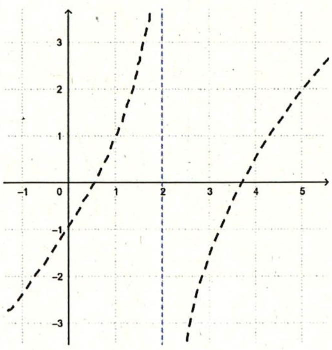
i. $\lim\limits_{x\to-3}\dfrac{5x}{(x+3)^2}=$ « $\dfrac{-15}{0}$ »: c'est un type $\dfrac{1}{0}$

$$\left.\begin{aligned}\lim_{x\to-3^-} f(x)&=\lim_{x\to-3^-}\frac{5\cdot(-3)}{(0^-)^2}=\frac{-15}{0^+}=-\infty\\ \lim_{x\to-3^+} f(x)&=\lim_{x\to-3^+}\frac{5\cdot(-3)}{(0^+)^2}=\frac{-15}{0^+}=-\infty\end{aligned}\right\}\quad \text{donc } \lim_{x\to-3} f(x)=-\infty$$

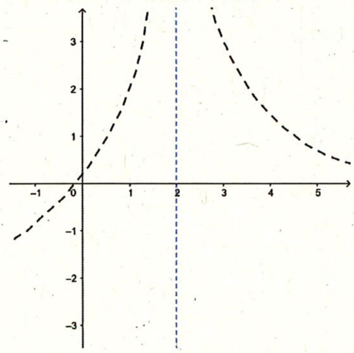
9

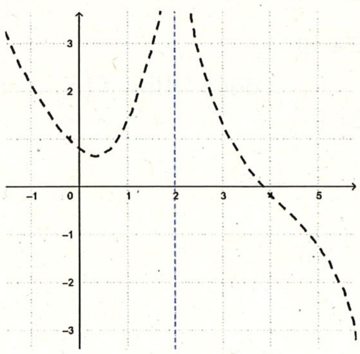
i $\lim\limits_{x\to2} f(x)$ n'existe pas



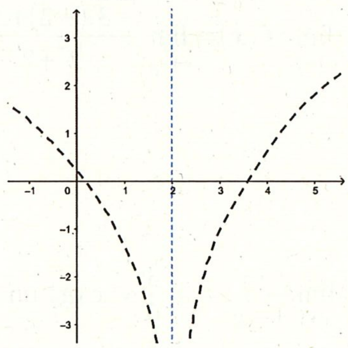


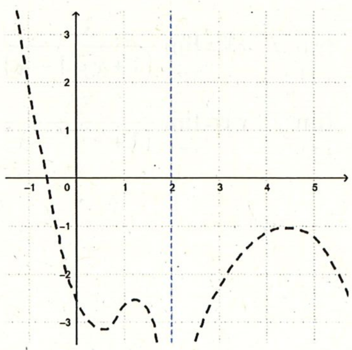
ii $\lim\limits_{x\to2} f(x)=+\infty$





iii $\lim\limits_{x\to2} f(x)=-\infty$





10

i $f_1(x)=\frac{1}{x-2}$, $f_2(x)=\frac{2}{x-2}$.

ii $f_1(x)=\frac{1}{(x-2)^2}$, $f_2(x)=\frac{2}{(x-2)^2}$.

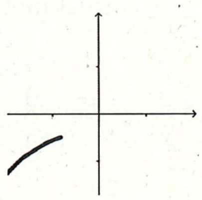
iii $f_1(x)=-\frac{1}{(x-2)^2}$, $f_2(x)=-\frac{2}{(x-2)^2}$.

Limites à l'infini

11 Calculer les limites suivantes et interpréter graphiquement les résultats :

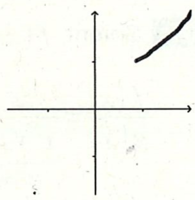
a. $\lim\limits_{x\to-\infty}(x^3-2x^2+5)$: c'est un type $\infty-\infty$

$= \lim\limits_{x\to-\infty} x^3(1-\frac{2}{x}+\frac{5}{x^3})=(-\infty)^3(1-\frac{2}{-\infty}+\frac{5}{(-\infty)^3})=-\infty\cdot(1+0-0)=-\infty\cdot 1=-\infty$



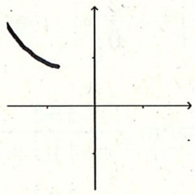
b. $\lim\limits_{x\to+\infty}(x^3-2x^2+5)$: c'est un type $\infty-\infty$

$= \lim\limits_{x\to+\infty} x^3(1-\frac{2}{x}+\frac{5}{x^3})=(+\infty)^3(1-\frac{2}{+\infty}+\frac{5}{(+\infty)^3})=+\infty\cdot(1-0+0)=+\infty\cdot 1=+\infty$



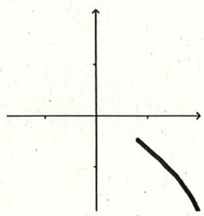
c. $\lim\limits_{x\to-\infty}(x^4+2x)$: c'est un type $\infty-\infty$

$= \lim\limits_{x\to-\infty} x^4(1+\frac{2}{x^3})=(-\infty)^4(1+\frac{2}{(-\infty)^3})=+\infty\cdot(1-0)=+\infty\cdot 1=+\infty$

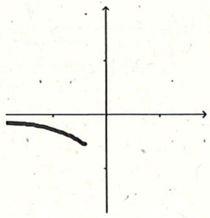


d. $\lim\limits_{x\to+\infty}(x^3-x^4)$: c'est un type $\infty-\infty$

$= \lim\limits_{x\to+\infty} x^4(\frac{1}{x}-1)=(+\infty)^4(\frac{1}{+\infty}-1)=+\infty\cdot(0-1)=+\infty\cdot(-1)=-\infty$

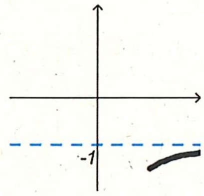


e. $\lim\limits_{x\to-\infty}\frac{-3}{x^2+5}=\frac{-3}{(-\infty)^2+5}=\frac{-3}{+\infty+5}=\frac{-3}{+\infty}=0$

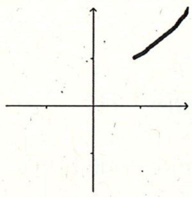


f. $\lim\limits_{x\to+\infty} \dfrac{3x^2+5}{1-3x^2}$: c'est un type $\frac{\infty}{\infty}$

$$= \lim_{x\to+\infty} \frac{x^2(3+\frac{5}{x^2})}{x^2(\frac{1}{x^2}-3)} = \lim_{x\to+\infty} \frac{3+\frac{5}{x^2}}{\frac{1}{x^2}-3} = \frac{3+0}{0-3} = -1$$

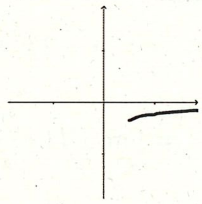


g. $\lim\limits_{x\to+\infty} \dfrac{x^2+x+1}{x}$: c'est un type $\frac{\infty}{\infty}$



$$= \lim_{x\to+\infty} [x+1+\frac{1}{x}] = +\infty+1+\frac{1}{+\infty} = +\infty+1+0 = +\infty$$

h. $\lim\limits_{x\to+\infty} \dfrac{3-x^2}{x^3}$: c'est un type $\frac{\infty}{\infty}$



$$= \lim_{x\to+\infty} [\frac{3}{x^3}-\frac{1}{x}] = \frac{3}{(+\infty)^3}-\frac{1}{+\infty} = 0-0 = 0$$

12 Soient $f(x)=x^2+2x+1$ et $g(x)=x^3+2x^2-x-2$, alors

$$\frac{f(x)}{g(x)} = \frac{x^2+2x+1}{\underbrace{x^3+2x^2-x-2}_{\textit{forme développée}}} = \frac{(x+1)^2}{x^2(x+2)-(x+2)} = \frac{(x+1)^2}{(x^2-1)(x+2)} = \frac{(x+1)^2}{\underbrace{(x-1)(x+1)(x+2)}_{\textit{forme factorisée}}} \underset{x\neq 1}{=} \frac{x+1}{(x-1)(x+2)}$$

a. $\lim\limits_{x\to 1} \left(\dfrac{f(x)}{g(x)}\right) =$ « $\dfrac{2}{0}$ »: : c'est un type $\dfrac{1}{0}$

$$\left.\begin{array}{l} \lim\limits_{x\to 1^-} \dfrac{f(x)}{g(x)} = \lim\limits_{x\to 1^-} \dfrac{x+1}{(x-1)(x+2)} = \dfrac{2}{0^-\cdot 3} = -\infty \\ \lim\limits_{x\to 1^+} \dfrac{f(x)}{g(x)} = \lim\limits_{x\to 1^+} \dfrac{x+1}{(x-1)(x+2)} = \dfrac{2}{0^+\cdot 3} = +\infty \end{array}\right\} \text{donc } \nexists \lim_{x\to 1} \frac{f(x)}{g(x)}$$

b. $\lim\limits_{x\to 1} \left(\dfrac{g(x)}{f(x)}\right) = \dfrac{0}{2} = 0$

c. $\lim\limits_{x\to+\infty} \left(\dfrac{f(x)}{g(x)}\right) = \lim\limits_{x\to+\infty} \dfrac{x^2(1+\frac{2}{x}+\frac{1}{x^2})}{x^3(1+\frac{2}{x}-\frac{1}{x^2}-\frac{2}{x^3})} = \dfrac{1+0+0}{+\infty\cdot(1+0-0-0)} = \dfrac{1}{+\infty\cdot 1} = 0$

d. $\lim_{x\to-\infty}\left(\frac{g(x)}{f(x)}\right)=\lim_{x\to-\infty}\frac{x^3(1+\frac{2}{x}-\frac{1}{x^2}-\frac{2}{x^3})}{x^2(1+\frac{2}{x}+\frac{1}{x^2})}=\frac{-\infty\cdot(1+0-0-0)}{1+0+0}=\frac{-\infty\cdot1}{1}=-\infty$

e. $\lim_{x\to+\infty}(f(x)-g(x))=\lim_{x\to+\infty}(x^2+2x+1-(x^3+2x^2-x-2))=\lim_{x\to+\infty}(x^2+2x+1-x^3-2x^2+x+2)$

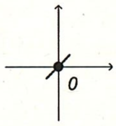
$=\lim_{x\to+\infty}(-x^3-x^2+3x+3)=\lim_{x\to+\infty}x^3(-1-\frac{1}{x}+\frac{3}{x^2}+\frac{3}{x^3})=(+\infty)^3\cdot(-1-0+0+0)=+\infty\cdot(-1)=-\infty$

f. $\lim_{x\to+\infty}\left(\frac{(f(x))^3}{(g(x))^3}\right)=\lim_{x\to+\infty}\left(\frac{f(x)}{g(x)}\right)^3\overset{c.}{=}0^3=0$

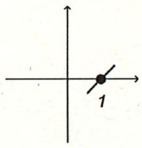
Indéterminations

13 Calculer les limites suivantes et interpréter graphiquement les résultats :

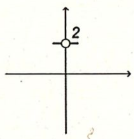
a. $\lim_{x\to0}\frac{x^2}{x+2}\overset{\text{direct}}{=}\frac{0}{2}=0$



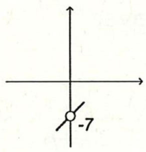
b. $\lim_{x\to1}\frac{x^2-1}{x^2+3x+5}\overset{\text{direct}}{=}\frac{0}{9}=0$



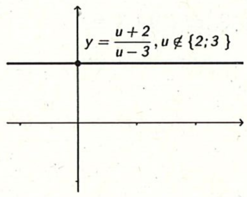
c. $\lim_{x\to0}\frac{2x}{x}\overset{\frac{0}{0}}{=}\lim_{x\to0}\frac{2}{1}=\lim_{x\to0}2\overset{\text{direct}}{=}2$



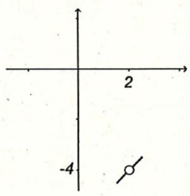
d. $\lim_{x\to0}\frac{5x^2-7x}{x}\overset{\frac{0}{0}}{=}\lim_{x\to0}\frac{x(5x-7)}{x}=\lim_{x\to0}(5x-7)\overset{\text{direct}}{=}-7$



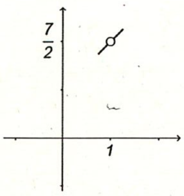
e. $\lim_{x\to2}\frac{u^2-4}{u^2-5u+6}\overset{\text{direct}}{=}\underbrace{\frac{u^2-4}{u^2-5u+6}}_{\text{constante par rapport à }x}=\frac{(u-2)(u+2)}{(u-3)(u-2)}\underset{\substack{u\neq2\\u\neq3}}{=}\frac{u+2}{u-3}$



f. $\lim_{u\to2}\frac{u^2-4}{u^2-5u+6}\overset{\frac{0}{0}}{=}\lim_{x\to2}\frac{(u-2)(u+2)}{(u-3)(u-2)}\underset{\substack{u\neq2\\u\neq3}}{=}\lim_{x\to2}\frac{u+2}{u-3}=\frac{2+2}{2-3}\overset{\text{direct}}{=}-4$

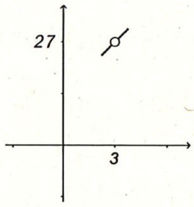


g. $\lim\limits_{x\to 1}\dfrac{x^3+3x^2-2x-2}{x^2-1}\overset{\frac{0}{0}}{=}\lim\limits_{x\to 1}\dfrac{(x-1)(x^2+4x+2)}{(x-1)(x+1)}=\lim\limits_{x\to 1}\dfrac{x^2+4x+2}{x+1}\overset{\text{direct}}{=}\dfrac{7}{2}$

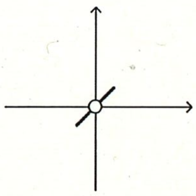


(factorisation intuitive ou division du numérateur par $(x-1)$).

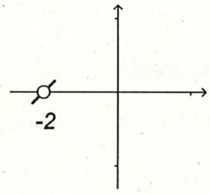
h. $\lim\limits_{x\to 3}\dfrac{x^3-27}{x-3}\overset{\frac{0}{0}}{=}\lim\limits_{x\to 3}\dfrac{(x-3)(x^2+3x+9)}{x-3}=\lim\limits_{x\to 3}(x^2+3x+9)\overset{\text{direct}}{=}27$



i. $\lim\limits_{x\to 0}\dfrac{x^3}{x^2}\overset{\frac{0}{0}}{=}\lim\limits_{x\to 0}x\overset{\text{direct}}{=}0$



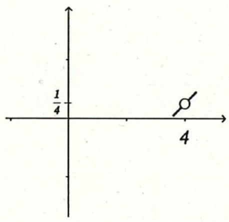
j. $\lim\limits_{x\to -2}\dfrac{x^2+4x+4}{x^2-x-6}\overset{\frac{0}{0}}{=}\lim\limits_{x\to -2}\dfrac{(x+2)^2}{(x+2)(x-3)}=\lim\limits_{x\to -2}\dfrac{x+2}{x-3}\overset{\text{direct}}{=}\dfrac{0}{-5}=0$



14

a. $\lim\limits_{x\to 4}\dfrac{2-\sqrt{x}}{4-x}$: c'est un type $\dfrac{0}{0}$ avec $\sqrt{}$

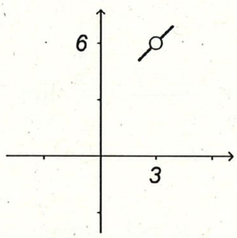
$=\lim\limits_{x\to 4}\dfrac{2-\sqrt{x}}{4-x}\cdot\dfrac{2+\sqrt{x}}{2+\sqrt{x}}=\lim\limits_{x\to 4}\dfrac{4-x}{(4-x)(2+\sqrt{x})}=\lim\limits_{x\to 4}\dfrac{1}{2+\sqrt{x}}=\dfrac{1}{2+\sqrt{4}}=\dfrac{1}{4}$



b. $\lim\limits_{x\to 3}\dfrac{x-3}{\sqrt{x+6}-3}$: c'est un type $\dfrac{0}{0}$ avec $\sqrt{}$

$=\lim\limits_{x\to 3}\dfrac{x-3}{\sqrt{x+6}-3}\cdot\dfrac{\sqrt{x+6}+3}{\sqrt{x+6}+3}=\lim\limits_{x\to 3}\dfrac{(x-3)(\sqrt{x+6}+3)}{(x+6)-9}=\lim\limits_{x\to 3}\dfrac{(x-3)(\sqrt{x+6}+3)}{x-3}$

$=\lim\limits_{x\to 3}(\sqrt{x+6}+3)=\sqrt{9}+3=6$

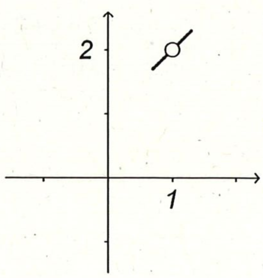


c. $\lim\limits_{x\to 1}\dfrac{1-x}{\sqrt{12-3x}-3}$: c'est un type $\dfrac{0}{0}$ avec $\sqrt{}$

$=\lim\limits_{x\to 1}\dfrac{1-x}{\sqrt{12-3x}-3}\cdot\dfrac{\sqrt{12-3x}+3}{\sqrt{12-3x}+3}=\lim\limits_{x\to 1}\dfrac{(1-x)\cdot(\sqrt{12-3x}+3)}{(12-3x)-9}$

$=\lim\limits_{x\to 1}\dfrac{(1-x)\cdot(\sqrt{12-3x}+3)}{3-3x}=\lim\limits_{x\to 1}\dfrac{(1-x)\cdot(\sqrt{12-3x}+3)}{3(1-x)}$

$=\lim\limits_{x\to 1}\dfrac{\sqrt{12-3x}+3}{3}=\dfrac{\sqrt{9}+3}{3}=\dfrac{6}{3}=2$

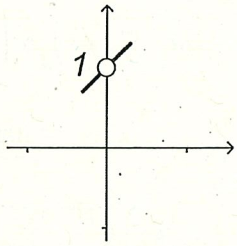


d. $\lim\limits_{x\to 0}\frac{\sqrt{x+1}-\sqrt{x^2-x+1}}{x}$: c'est un type $\frac{0}{0}$ avec $\sqrt{}$

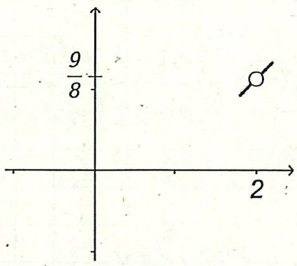
$$=\lim_{x\to 0}\frac{\sqrt{x+1}-\sqrt{x^2-x+1}}{x}\cdot\frac{\sqrt{x+1}+\sqrt{x^2-x+1}}{\sqrt{x+1}+\sqrt{x^2-x+1}}=\lim_{x\to 0}\frac{(x+1)-(x^2-x+1)}{x\cdot(\sqrt{x+1}+\sqrt{x^2-x+1})}$$

$$=\lim_{x\to 0}\frac{-x^2+2x}{x\cdot(\sqrt{x+1}+\sqrt{x^2-x+1})}=\lim_{x\to 0}\frac{x(-x+2)}{x\cdot(\sqrt{x+1}+\sqrt{x^2-x+1})}=\lim_{x\to 0}\frac{-x+2}{\sqrt{x+1}+\sqrt{x^2-x+1}}$$

$$=\frac{0+2}{\sqrt{0+1}+\sqrt{0-0+1}}=\frac{2}{\sqrt{1}+\sqrt{1}}=\frac{2}{1+1}=1$$



e. $\lim\limits_{x\to 2}\frac{x-\sqrt{x+2}}{\sqrt{4x+1}-3}$: c'est un type $\frac{1}{0}$ avec $\sqrt{}$



$$=\lim_{x\to 2}\frac{x-\sqrt{x+2}}{\sqrt{4x+1}-3}\cdot\frac{x+\sqrt{x+2}}{x+\sqrt{x+2}}=\lim_{x\to 2}\frac{x^2-(x+2)}{(\sqrt{4x+1}-3)(x+\sqrt{x+2})}$$

$$=\lim_{x\to 2}\frac{(x-2)(x+1)}{(\sqrt{4x+1}-3)(x+\sqrt{x+2})}=\lim_{x\to 2}\frac{(x-2)(x+1)}{(\sqrt{4x+1}-3)(x+\sqrt{x+2})}\cdot\frac{\sqrt{4x+1}+3}{\sqrt{4x+1}+3}$$

$$=\lim_{x\to 2}\frac{(x-2)(x+1)(\sqrt{4x+1}+3)}{((4x+1)-9)(x+\sqrt{x+2})}=\lim_{x\to 2}\frac{(x-2)(x+1)(\sqrt{4x+1}+3)}{4(x-2)(x+\sqrt{x+2})}$$

$$=\lim_{x\to 2}\frac{(x+1)(\sqrt{4x+1}+3)}{4(x+\sqrt{x+2})}=\frac{(2+1)(\sqrt{4\cdot 2+1}+3)}{4(2+\sqrt{2+2})}=\frac{3\cdot 6}{4\cdot 4}=\frac{9}{8}$$

15

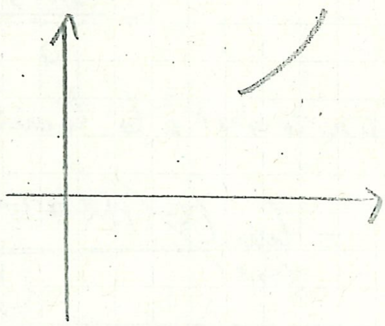
a.Soit $f(x)=3x-\sqrt{x^2-x+1}$

$D_{f(x)}$: pb. Si $x^2-x+1<0$; $\Delta=(-1)^2-4\cdot 1\cdot 1=-3<0$, donc pas de zéros, $a=1>0$, d'où $D_{f(x)}=\mathbb{R}$

$\lim\limits_{x\to+\infty}\left(3x-\sqrt{x^2-x+1}\right)$ indétermination du type $\infty-\infty$

$$\lim_{x\to+\infty}f(x)=\lim_{x\to+\infty}\left(3x-\sqrt{x^2(1-\frac{1}{x}+\frac{1}{x^2})}\right)=\lim_{x\to+\infty}\left(3x-|x|\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}\right)=\lim_{x\to+\infty}\left(3x-x\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}\right)$$

$$=\lim_{x\to+\infty}x\left(3-\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}\right)=(+\infty)\cdot(3-\sqrt{1-0+0})=+\infty\cdot 2=+\infty$$



(Attention ! $\sqrt{x^2}=|x|$ et non $\sqrt{x^2}=x$)