Exercices de calculs de limites supplémentaires

1]

a) $\lim\limits_{x\to 5} \dfrac{x-5}{\sqrt{2x-1}-3}$: type "$\frac{0}{0}$"

$\lim\limits_{x\to 5} \dfrac{(x-5)\ (\sqrt{2x-1}+3)}{(\sqrt{2x-1}-3)(\sqrt{2x-1}+3)} = \lim\limits_{x\to 5} \dfrac{(x-5)(\sqrt{2x-1}+3)}{(2x-1)-9}$

$= \lim\limits_{x\to 5} \dfrac{\cancel{(x-5)}(\sqrt{2x-1}+3)}{2\cancel{(x-5)}} = \dfrac{\sqrt{2\cdot 5-1}+3}{2} = \dfrac{6}{2} = 3$

b) $\lim\limits_{x\to 0} \dfrac{\sqrt{x^2+3}-\sqrt{x^2+1}}{x^2-1} = \dfrac{\sqrt{0^2+3}-\sqrt{0^2+1}}{0^2-1} = \dfrac{\sqrt{9}-\sqrt{1}}{-1} = -2$

c) $\lim\limits_{x\to 2} \dfrac{-9x}{(4-x^2)^3}$: type "$\frac{1}{0}$"

$\lim\limits_{x\to 2^+} \dfrac{-9x}{[(2-x)(2+x)]^3} = \dfrac{-18}{[0^-\cdot 4]^3} = \dfrac{-18}{(0^-)^3} = \dfrac{-18}{0^-} = +\infty$

$\lim\limits_{x\to 2^-} \dfrac{-9x}{[(2-x)(2+x)]^3} = \dfrac{-18}{[0^+\cdot 4]^3} = \dfrac{-18}{(0^+)^3} = \dfrac{-18}{0^+} = -\infty$

donc $\lim\limits_{x\to 2} f(x) \nexists$

d) $\lim\limits_{x\to 1} \dfrac{x^2-3x+2}{(x^2-1)^2}$: type "$\frac{0}{0}$"

$\lim\limits_{x\to 1} \dfrac{(x-2)\,\cancel{(x-1)}}{(x-1)^{\cancel{2}}(x+1)^2} = \lim\limits_{x\to 1} \dfrac{x-2}{(x-1)(x+1)^2}$: type "$\frac{1}{0}$"

$\lim\limits_{x\to 1^+} \dfrac{x-2}{(x-1)(x+1)^2} = \dfrac{-1}{0^+\cdot 2^2} = -\infty$

$\lim\limits_{x\to 1^-} \dfrac{x-2}{(x-1)(x+1)^2} = \dfrac{-1}{0^-\cdot 2^2} = +\infty$

donc $\lim\limits_{x\to 1} f(x) \nexists$

e) $\lim\limits_{x \to a} \dfrac{x-a}{x-a}$: type "$\dfrac{0}{0}$"

$\lim\limits_{x \to a} \dfrac{\cancel{x-a}}{\cancel{x-a}} = \lim\limits_{x \to a} \dfrac{1}{1} = 1$

f) $\lim\limits_{x \to a} \dfrac{\sqrt{x}-\sqrt{a}}{x-a}$: type "$\dfrac{0}{0}$" avec $\sqrt{\ }$

$\lim\limits_{x \to a} \dfrac{(\sqrt{x}-\sqrt{a})}{(x-a)} \dfrac{(\sqrt{x}+\sqrt{a})}{(\sqrt{x}+\sqrt{a})} = \lim\limits_{x \to a} \dfrac{(\cancel{x-a})}{(\cancel{x-a})(\sqrt{x}+\sqrt{a})} = \dfrac{1}{\sqrt{a}+\sqrt{a}} = \dfrac{1}{2\sqrt{a}}$

(pour $a>0$)

g) $\lim\limits_{x \to a} \dfrac{x^2-a^2}{x-a}$: type "$\dfrac{0}{0}$"

$\lim\limits_{x \to a} \dfrac{x^2-a^2}{x-a} = \lim\limits_{x \to a} \dfrac{\cancel{(x-a)}(x+a)}{\cancel{(x-a)}} = \lim\limits_{x \to a} (x+a) = a+a = 2a$

h) $\lim\limits_{x \to a} \dfrac{x^3-a^3}{x-a}$: type "$\dfrac{0}{0}$"

$\lim\limits_{x \to a} \dfrac{x^3-a^3}{x-a} = \lim\limits_{x \to a} \dfrac{(\cancel{x-a})(x^2+ax+a^2)}{\cancel{x-a}}$, car

$$\begin{array}{rr|l} x^3 & -a^3 & x-a \\ \hline -(x^3-ax^2) & & x^2+ax \\ ax^2 & -a^3 & +a^2 \\ -(ax^2-a^2x) & & \\ a^2x & -a^3 & \\ -(a^2x-a^3) & & \\ & 0 & \end{array}$$

$= \lim\limits_{x \to a} (x^2+ax+a^2)$

$= a^2 + a \cdot a + a^2 = 3a^2$

i) $\lim\limits_{x \to a} \dfrac{\frac{1}{x}-\frac{1}{a}}{x-a}$; type "$\dfrac{0}{0}$"

$\lim\limits_{x \to a} \dfrac{\frac{1}{x}-\frac{1}{a}}{x-a} = \lim\limits_{x \to a} \dfrac{\frac{a-x}{xa}}{x-a} = \lim\limits_{x \to a} \dfrac{a-x}{x \cdot a} \cdot \dfrac{1}{x-a}$

$= \lim\limits_{x \to a} \dfrac{-\cancel{(x-a)}}{x \cdot a} \cdot \dfrac{1}{\cancel{(x-a)}} = \lim\limits_{x \to a} -\dfrac{1}{x \cdot a} = -\dfrac{1}{a \cdot a} = -\dfrac{1}{a^2}$

(pour $a \neq 0$)

j) $\lim\limits_{x\to 2} \dfrac{x^2-3x+2}{x^2-4x+4}$ type "$\frac{0}{0}$"

$= \lim\limits_{x\to 2} \dfrac{\cancel{(x-2)}(x-1)}{(x-2)^{\cancel{2}}} = \lim\limits_{x\to 2} \dfrac{x-1}{x-2}$ type "$\frac{1}{0}$"

$\Rightarrow \begin{cases} \lim\limits_{x\to 2^+} \dfrac{x-1}{x-2} = \dfrac{1}{0^+} = +\infty \\ \lim\limits_{x\to 2^-} \dfrac{x-1}{x-2} = \dfrac{1}{0^-} = -\infty \end{cases}$ donc $\lim\limits_{x\to 2} f(x) \nexists$

k) $\lim\limits_{x\to 3} \dfrac{x^2-5x+6}{2x^2-6x}$ type "$\frac{0}{0}$"

$= \lim\limits_{x\to 3} \dfrac{\cancel{(x-3)}(x-2)}{2x\cancel{(x-3)}} = \dfrac{1}{6}$

l) $\lim\limits_{x\to \infty} \dfrac{(x-1)^2}{x^2+3x}$ type "$\frac{\infty}{\infty}$"

$= \lim\limits_{x\to\infty} \dfrac{x^2-2x+1}{x^2+3x} = \lim\limits_{x\to\infty} \dfrac{\cancel{x^2}(1-2/x+1/x^2)}{\cancel{x^2}(1+3/x)} = \dfrac{1-0+0}{1+0} = 1$

m) $\lim\limits_{x\to 1} \dfrac{\sqrt{x^2+x}-\sqrt{2}}{x-1}$ type "$\frac{0}{0}$" avec $\sqrt{\ }$

$= \lim\limits_{x\to 1} \dfrac{(\sqrt{x^2+x}-\sqrt{2})}{(x-1)} \dfrac{(\sqrt{x^2+x}+\sqrt{2})}{(\sqrt{x^2+x}+\sqrt{2})} = \lim\limits_{x\to 1} \dfrac{(x^2+x)-(2)}{(x-1)(\sqrt{x^2+x}+\sqrt{2})}$

$= \lim\limits_{x\to 1} \dfrac{(x+2)\cancel{(x-1)}}{\cancel{(x-1)}(\sqrt{x^2+x}+\sqrt{2})} = \dfrac{3}{\sqrt{2}+\sqrt{2}} = \dfrac{3}{2\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{3\sqrt{2}}{2\cdot 2} = \dfrac{3\sqrt{2}}{4}$

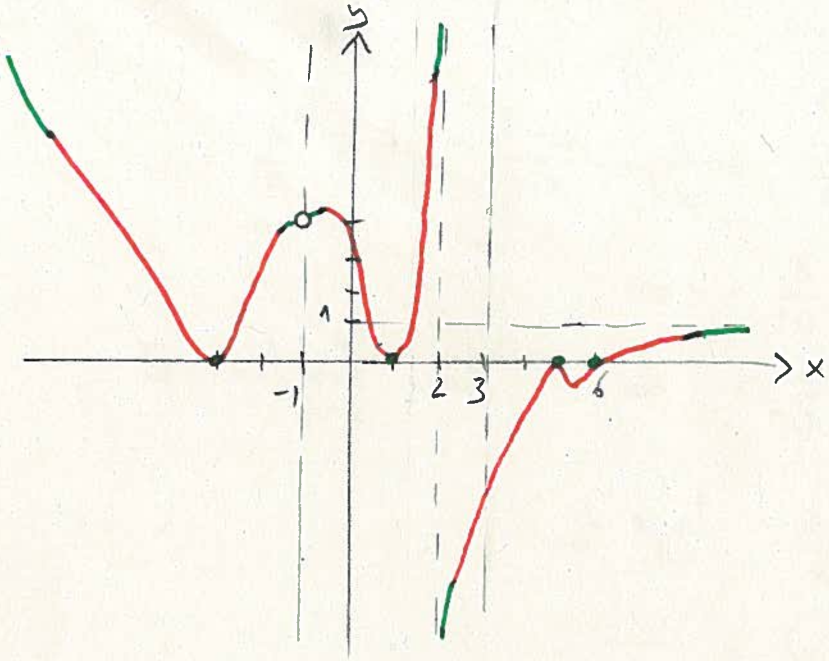
n) $\lim\limits_{x\to 3} \dfrac{x^2+2x-15}{x^2+8x+15} = \dfrac{9+6-15}{9+24+15} = \dfrac{0}{48} = 0$

o) $\lim\limits_{x\to -3} \dfrac{x+1}{(x+3)^2}$ type "$\frac{1}{0}$"

$\Rightarrow \begin{cases} \lim\limits_{x\to -3^+} \dfrac{x+1}{(x+3)^2} = \dfrac{-2}{(0^+)^2} = \dfrac{-2}{0^+} = -\infty \\ \lim\limits_{x\to -3^-} \dfrac{x+1}{(x+3)^2} = \dfrac{-2}{(0^-)^2} = \dfrac{-2}{0^+} = -\infty \end{cases}$ donc $\lim\limits_{x\to -3} f(x) = -\infty$

p)

2]



en vert ce qui est demandé

on complète avec une proposition en rouge (plusieurs possibilités)