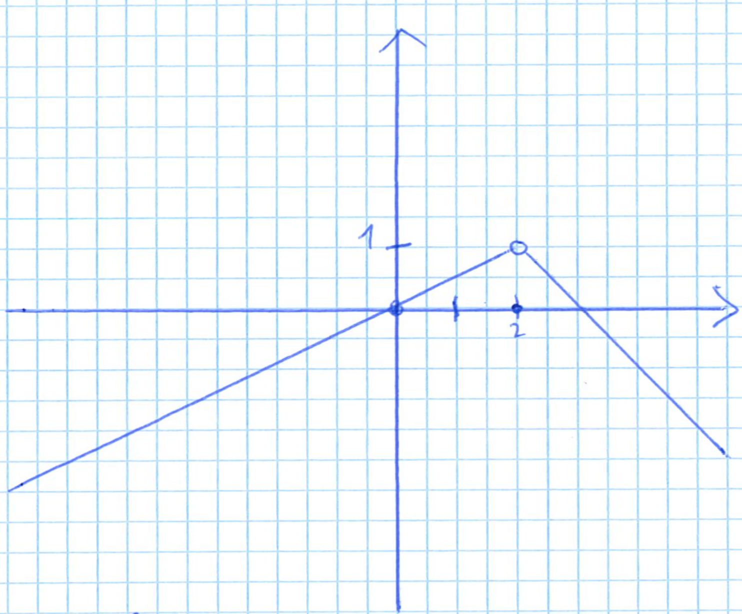
Gr 1 Owen, Thomas, Gabriel, Margaux

Activité 16.4



$\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^-} \frac{1}{2} x = "\frac{2}{2}" = 1$

donc la limite de $x < 2$ est 1

$\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^+} -x + 3 = "-2" + 3 = 1$

donc la limite de $x > 2$ est 1

$\Rightarrow \lim\limits_{x \to 2} f(x) = 1$

+ justifs

$f(2) = 0$ donc $f(2) \neq \lim\limits_{x \to 2} f(x)$

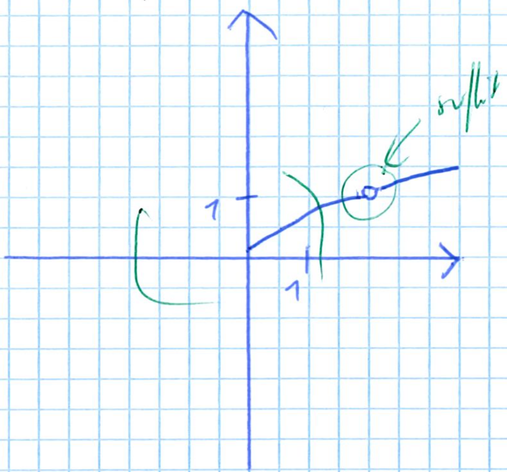
par définition de la continuité, f n'est pas continu en $a = 2$.

et en $a \neq 2$?

+ ...

Owen Thomas Gabriel Morgane (5)

Ex 14 * graphique



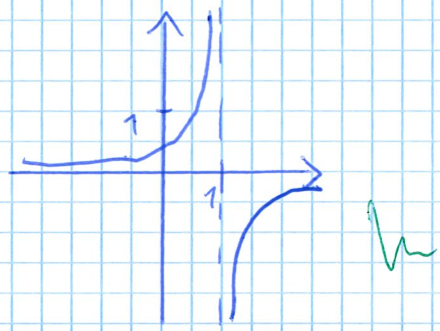
Ex 15

e) $\lim\limits_{x \to -\infty} \sqrt{x^2 - 2x + 4} + 2x$

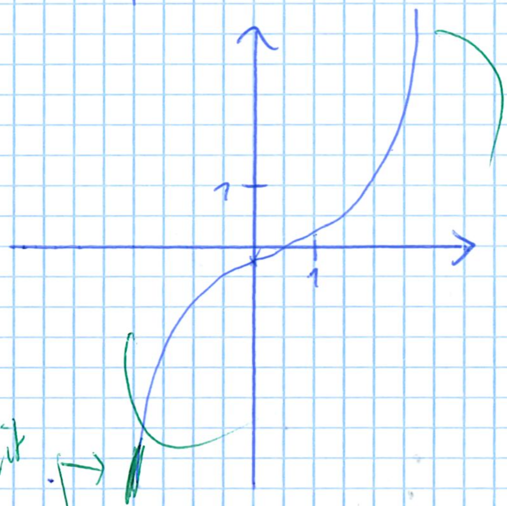
on observe avec la multiplication du conjugué :

$= \lim\limits_{x \to -\infty} \left(\sqrt{x^2 - 2x + 4} + 2x\right) \cdot \dfrac{\sqrt{x^2 - 2x + 4} - 2x}{\sqrt{x^2 - 2x + 4} - 2x}$

Ex 8* graphique :

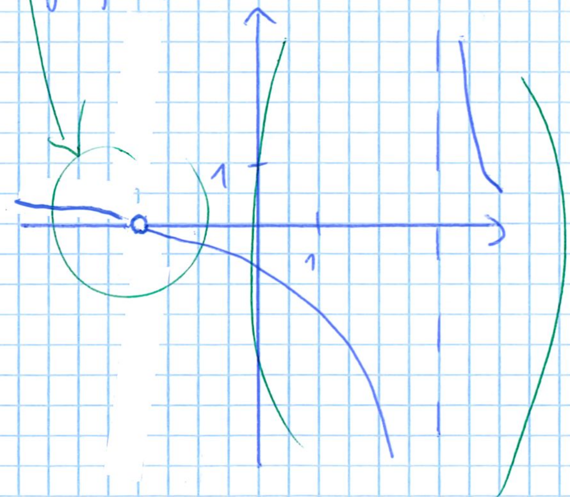


Ex 11* graphique :



suffit

Ex 13* graphique :



③ Thomas Margaux Gabriel Owen

Ex 14

e) $\lim_{x \to 2} \frac{x - \sqrt{x+2}}{\sqrt{4x+1} - 3} = \text{"}\frac{2 - \sqrt{2+2}}{\sqrt{4 \cdot 2+1} - 3}\text{"} = \text{"}\frac{0}{0}\text{"}$ type "$\frac{0}{0}$" avec $\sqrt{\ }$

donc multiplier par conjugué

$\Rightarrow \lim_{x \to 2} \left(\frac{x - \sqrt{x+2}}{\sqrt{4x+1} - 3}\right)\left(\frac{\sqrt{4x+1}+3}{\sqrt{4x+1}+3}\right)$ ✓

$= \lim_{x \to 2} \frac{(x - \sqrt{x+2})(\sqrt{4x+1}+3)}{4x-8}$ ✓ $\cdot \frac{(x+\sqrt{x+2})}{(x+\sqrt{x+2})}$ car $x+\sqrt{x+2}$ pas "dangereux"

$= \lim_{x \to 2} \frac{(x - \sqrt{x+2})(x+\sqrt{x+2})(\sqrt{4x+1}+3)}{(4x-8)(x+\sqrt{x+2})}$

$= \lim_{x \to 2} \frac{(x^2 - (x+2))(\sqrt{4x+1}+3)}{4(x-2)(x+\sqrt{x+2})}$

$= \lim_{x \to 2} \frac{\cancel{(x-2)}(x+1)(\sqrt{4x+1}+3)}{4\cancel{(x-2)}(x+\sqrt{x+2})}$

$= \frac{3 \cdot (3+3)}{4(2+2)} = \frac{18}{16} = \frac{9}{8}$

2

Ex 10

b) $\lim_{x \to 2} f(x) = +\infty$

$f_1(x) = \frac{4}{(x-2)^2}$ car $\lim_{x \to 2^-} = \frac{4}{"(0^-)(0^-)"} = +\infty$

$\lim_{x \to 2^+} = \frac{4}{"(0^+)(0^+)"} = +\infty$

$f_2(x) = \frac{4}{(x-2)^4}$ ✓

//

Ex 11

a) $\lim_{x \to -\infty} x^3 - 2x^2 + 5 = (-\infty)^3 - 2(-\infty)^2 + 5$ [type $-\infty - \infty$ pas indéterminé]

$= -\infty - 2\infty + 5$

$= -\infty$ ✓

int graph? (+...) *

Ex 13

j) $\lim_{x \to -2} \frac{x^2 + 4x + 4}{x^2 - x - 6} = \frac{"(-2)^2" + 4"(-2)" + 4}{"(-2)^2" + "(-2)" - 6} = \frac{"0"}{"0"}$ type $\frac{0}{0}$ indéterminé

donc factoriser

$\Rightarrow \lim_{x \to -2} \frac{(x+2)^{\cancel{2}}}{(x-3)\cancel{(x+2)}} = \lim_{x \to -2} \frac{x+2}{x-3} = "\frac{0}{-5}" = "0"$ ✓

int graph? (+...)

①

Gr 1 Margaux Gabriel Thomas Owen

Ex 8

d) $\lim\limits_{x \to 1} \dfrac{x-2}{x^2+3x-4}$ on essaie

$= \lim\limits_{x \to 1} \dfrac{"1"-2}{"1^2+3\cdot1"-4} = \dfrac{"-1"}{"0"}$

donc limite droite gauche

$\lim\limits_{x \to 1^+} \dfrac{"-1"}{"1^{+}+3^{+}"-4} = \dfrac{"-1"}{"0^+"} = \underline{\underline{-\infty}}$ ✓

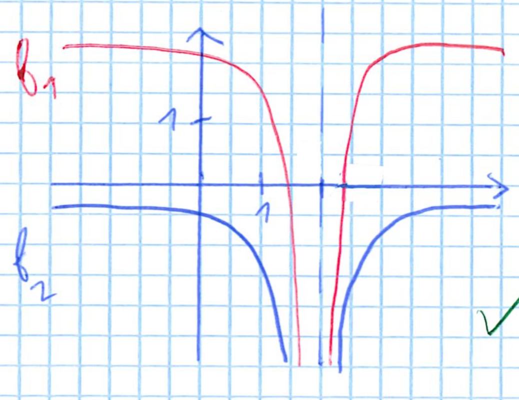
$\lim\limits_{x \to 1^-} \dfrac{"-1"}{"1^{-}+3^{-}"-4} = \dfrac{-1}{0^-} = \underline{\underline{+\infty}}$

Int. graph? +...

*

Ex 9

c) $\lim\limits_{x \to 2} f(x) = -\infty$



✓

Silvia
Camille
Léo
Rebecca

Gr2 Travail de groupe n°2

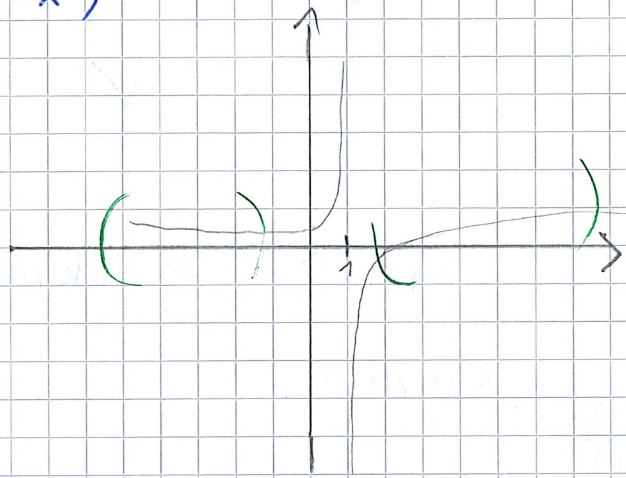
Ex 8G: $\lim\limits_{x \to 1} \frac{3x}{(1-x^2)}$ (limite type " $\frac{1}{0}$ " $\Rightarrow$ donc calcul limite gauche-droite)

$$\lim_{x \to 1^+} \frac{3x}{(1-x^2)} = \frac{3}{(1-1^+)} = \frac{3}{0^-} = -\infty$$

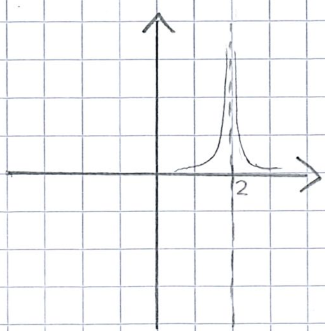
ou f n'est

$$\lim_{x \to 1^-} \frac{3x}{(1-x^2)} = \frac{3}{1-1^-} = \frac{3}{0^+} = +\infty$$

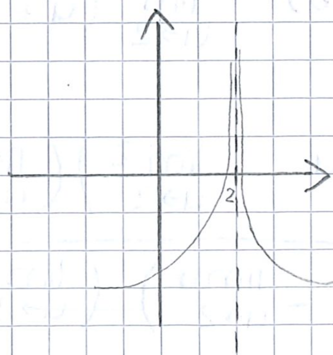
donc $\lim\limits_{x \to 1} \frac{3x}{(1-x^2)}$ n'existe pas



Ex 9B:



la $\lim\limits_{x \to 2} f(x) = +\infty$
pour les deux graphiques



Ex 10A:

$$\lim_{x \to 2} f(x) = \begin{cases} x^2+1 & \text{si } x > 2 \\ x-1 & \text{si } x \leq 2 \end{cases}$$

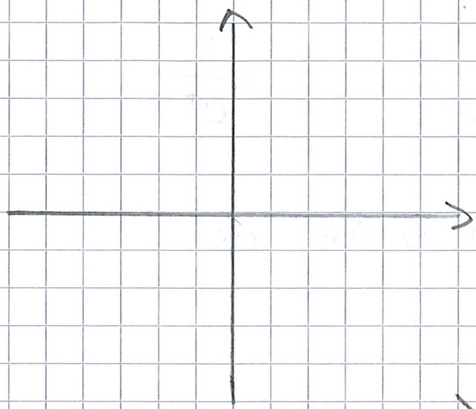
$$\lim_{x \to 2} g(x) = \begin{cases} x^2+3 & \text{si } x > 2 \\ x+1 & \text{si } x \leq 2 \end{cases}$$

Il y a plusieurs possibilités de réponses.

Ex. 110

$$\lim_{x \to +\infty} (x^3 - x^4) \Rightarrow \text{type } [\infty - \infty]$$

$$= \lim_{x \to +\infty} x^4 \left(\frac{1}{x} - 1 \right) = \infty \cdot (-1) = \boxed{-\infty}$$



13e) $\lim_{u \to 2} \frac{u^2 - 4}{u^2 - 5u + 6}$ type "$\frac{0}{0}$" $\Rightarrow$ indétermination

$$= \lim_{u \to 2} \frac{(u-2)(u+2)}{(u-6)(u+1)} \overset{PrL4/PrL3}{=}$$

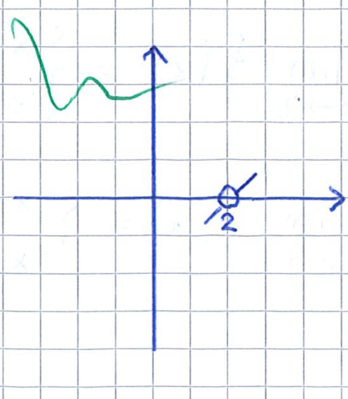
$$\frac{\lim_{u \to 2} (u-2) \cdot \lim_{u \to 2} (u+2)}{\lim_{u \to 2} (u-6) \cdot \lim_{u \to 2} (u+1)}$$

on peut aller plus vite

PrL2/1

$$= \frac{\left(\lim_{u \to 2} u - \lim_{u \to 2} 2 \right)\left(\lim_{u \to 2} u + \lim_{u \to 2} 2 \right)}{\left(\lim_{u \to 2} u - \lim_{u \to 2} 6 \right)\left(\lim_{u \to 2} u + \lim_{u \to 2} 1 \right)}$$

$$= \frac{(2-2)(2+2)}{(2-6)(2+1)} = \frac{0}{-12} = 0$$



Gr2

ex 15d

$$\lim_{x\to 1} \frac{x^4+x^3+x-3}{\sqrt{x}-1} = \lim_{x\to 1} \frac{x^4+x^3+x-3}{\sqrt{x}-1} \cdot \left(\frac{\sqrt{x}+1}{\sqrt{x}+1}\right)$$

$$= \lim_{x\to 1} \frac{(x^4+x^3+x-3)(\sqrt{x}+1)}{x-1}$$

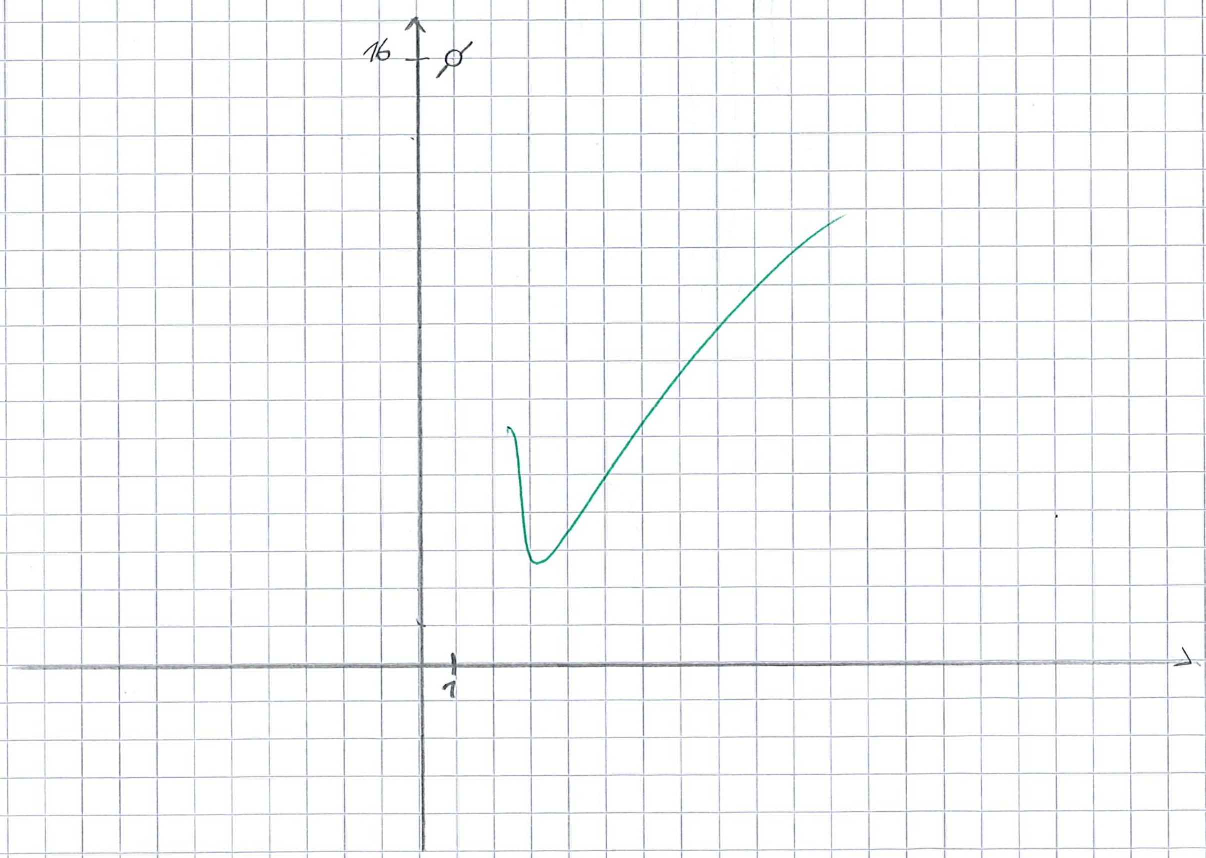
$$= \lim_{x\to 1} \frac{\cancel{(x-1)}(x^3+2x^2+2x+3)(\sqrt{x}+1)}{\cancel{(x-1)}}$$

$$= \lim_{x\to 1} (x^3+2x^2+2x+3)(\sqrt{x}+1)$$

$$= \text{"}(1^3+2\cdot 1^2+2\cdot 1+3)(\sqrt{1}+1)\text{"}$$

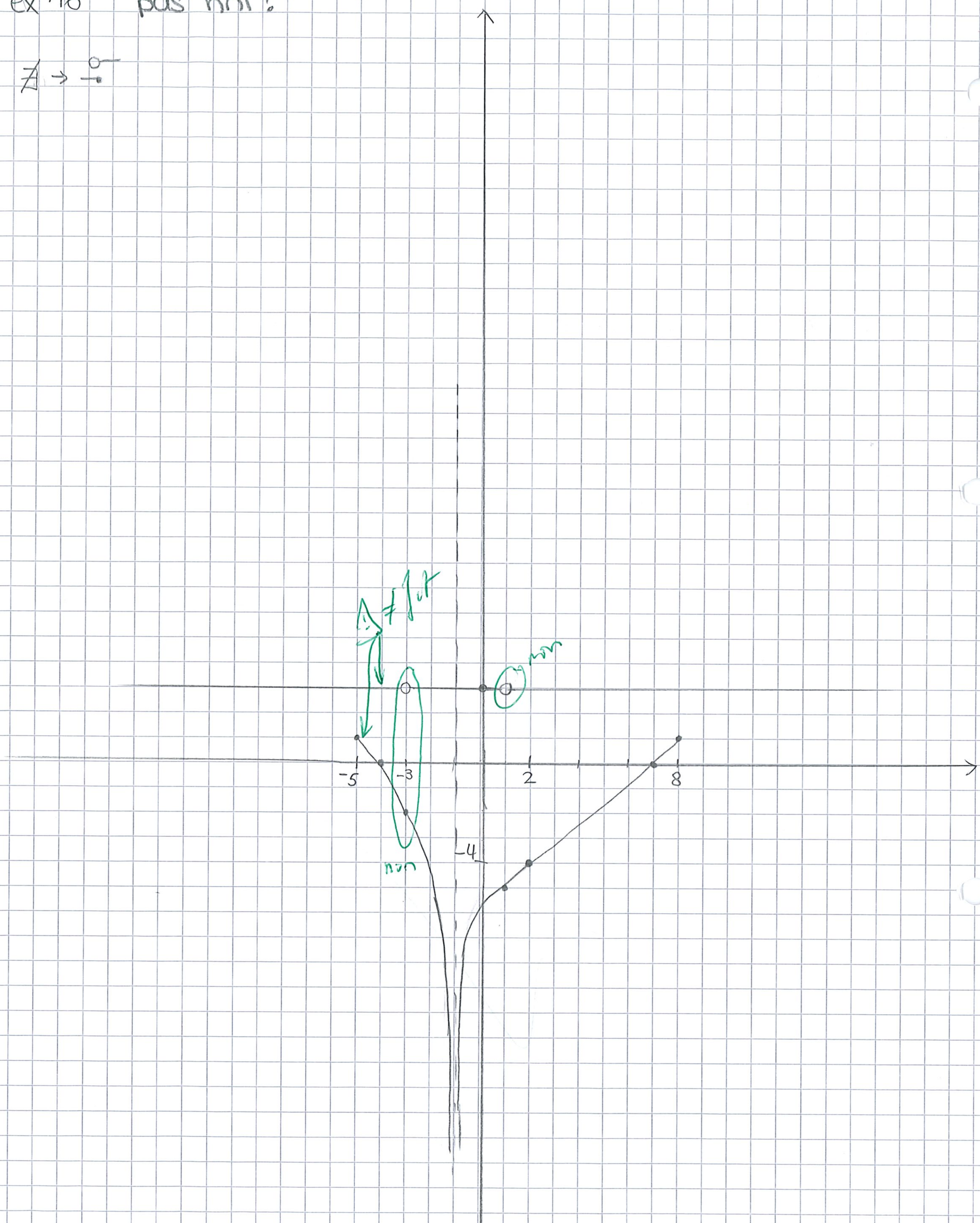
$$= 16$$

$$\boxed{\begin{array}{r|l} & x^3+2x^2+2x+3 \\ x-1 & x^4+x^3+0x^2+x-3 \\ & -(x^4+x^3) \\ & 2x^3+0x^2+x-3 \\ & -(2x^3-2x^2) \\ \hline & 2x^2+x-3 \\ & -(2x^2-2x) \\ \hline & 3x-3 \\ & 3x-3 \\ \hline & 0 \end{array}}$$



ex 16 pas fini !

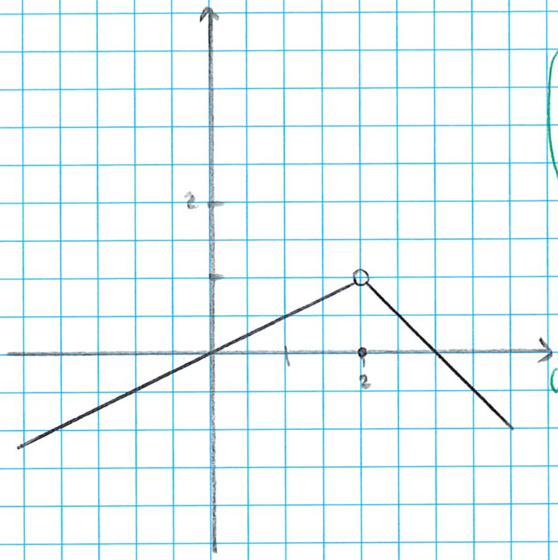
$\nexists \to$



Gr2

activité 16.4

Camille, Rebecca, Leo, Silvia



La fonction f n'est pas continue tout d'abord par la définition intuitive de la continuité en un point car f ne peut pas être représentée sans lever le crayon. ✓

Cela est également justifiable à l'aide du point (3) de la définition mathématique de cont en un point :

f est continue en $a \Leftrightarrow \lim\limits_{x \to a} f(x) = f(a)$

Or, nous avons ~~deux~~ un contre exemples

$\lim\limits_{x \to 2^+} \frac{1}{2}x \neq f(2) = 0$

justifs

+ ...

et

$\lim\limits_{x \to 2^-} (-x+3) \neq f(2) = 0$

+ ...

Gr3 Trevor / Esteban / Joachim / Marten

ex 8

A) $\lim\limits_{x \to -3} \frac{5x}{(x+3)^3} = " \frac{5 \cdot (-3)}{(-3+3)^3} " = " \frac{15}{0} "$ [type $" \frac{1}{0} "$]

$\lim\limits_{x \to -3^-} \frac{5x}{(x+3)^3} = \frac{-15}{0^-} = +\infty$

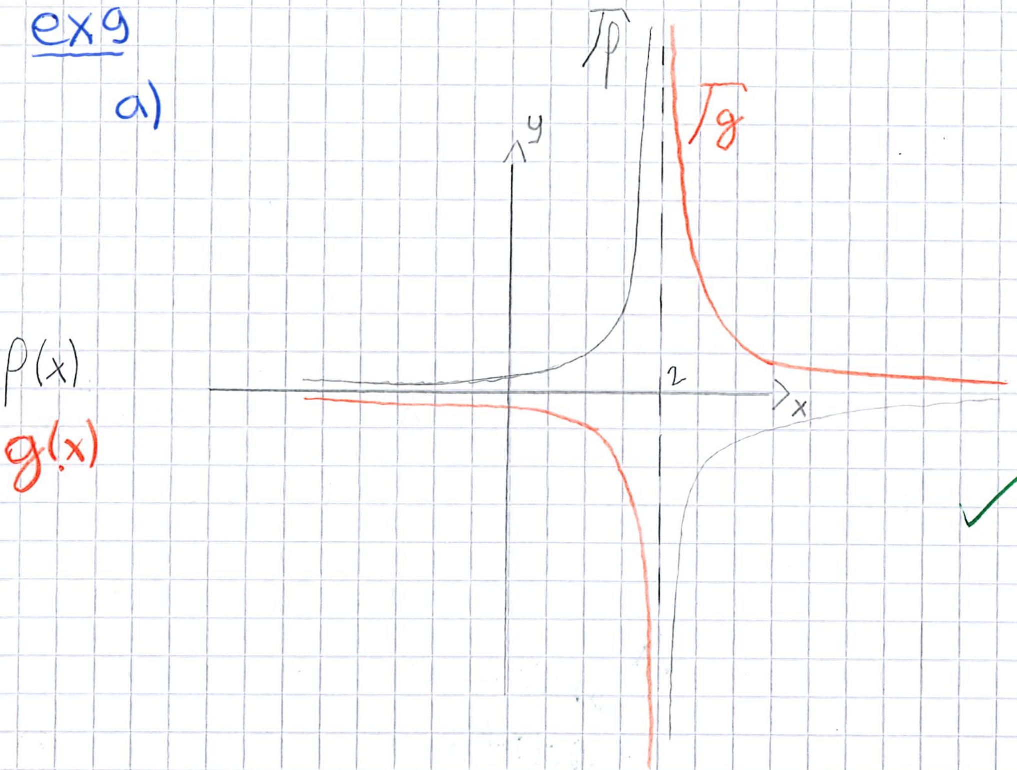
int. graph?

$\lim\limits_{x \to -3^+} \frac{5x}{(x+3)^3} = \frac{-15}{0^+} = -\infty$ ✓

+...

ex 9

a)



✓

ex 10

iii) $\lim\limits_{x \to 2} f(x) = -\infty$, où $f(x) = \frac{-3}{(2-x)^2}$

, où $f(x) = \frac{-5}{(2-x)^2}$ ✓

ex11:

b) $\lim\limits_{x \to +\infty} (x^3 - 2x^2 + 5) = (+\infty)^3 - 2 \cdot (+\infty) + 5 = +\infty - \infty$

[type "$+\infty - \infty$"]

$\lim\limits_{x \to +\infty} x^3 \left(1 - \frac{2}{x} + \frac{5}{x^3}\right)$

$= (+\infty)^3 \left(1 - \frac{2}{+\infty} + \frac{5}{(+\infty)^3}\right)$ ✓

$= +\infty \, (1 - 0 + 0) = +\infty \cdot 1 = +\infty$

int graph?

+...

ex13

g) $\lim\limits_{x \to 1} \frac{x^3 + 3x^2 - 2x - 2}{x^2 - 1} = \frac{1^3 + 3 \cdot 1^2 - 2 \cdot 1 - 2}{1^2 - 1} = \text{"}\frac{0}{0}\text{"}$ type!

$\lim\limits_{x \to 1} \frac{x^3 + 3x^2 - 2x - 2}{(x-1)(x+1)} = \frac{(x-1)(x^2 + 4x + 4) + 2}{(x-1)(x+1)}$ ≠ faut!

division polynomiale

pas ok... résultat pas possible!!!

On est bloqués

$\Leftrightarrow \lim\limits_{x \to 1} \frac{\cancel{(x-1)}(x^2 + 4x + 2)}{\cancel{(x-1)}(x+1)}$ ✓ $= \lim\limits_{x \to 1} \frac{x^2 + 4x + 2}{x+1}$

$\cancel{\Leftrightarrow}$ $=$ not $\frac{1 + 4 + 2}{1 + 1} = \frac{7}{2}$ ✓

+ int graph "

G3

ex14:

c) $\lim\limits_{x \to 1} \dfrac{1-x}{\sqrt{12-3x}-3} = \lim\limits_{x \to 1} \dfrac{(1-x)(\sqrt{12-3x}+3)}{12-3x-3}$

$\Leftrightarrow \lim\limits_{x \to 1} \dfrac{\cancel{(1-x)}(\sqrt{12-3x}+3)}{3\cancel{(1-x)}} = \dfrac{\sqrt{12-3}+3}{3} = 2$

tikt gepl

...

G3 Rebecca, Leo, Camille et Silvia

~~activité 16.4~~

ex 14 b

$$\lim_{x\to 3} \frac{x-3}{\sqrt{x+6}-3} \cdot \left(\frac{\sqrt{x+6}+3}{\sqrt{x+6}+3}\right)$$

$$= \lim_{x\to 3} \frac{(x-3)(\sqrt{x+6}+3)}{(x+6)-9}$$

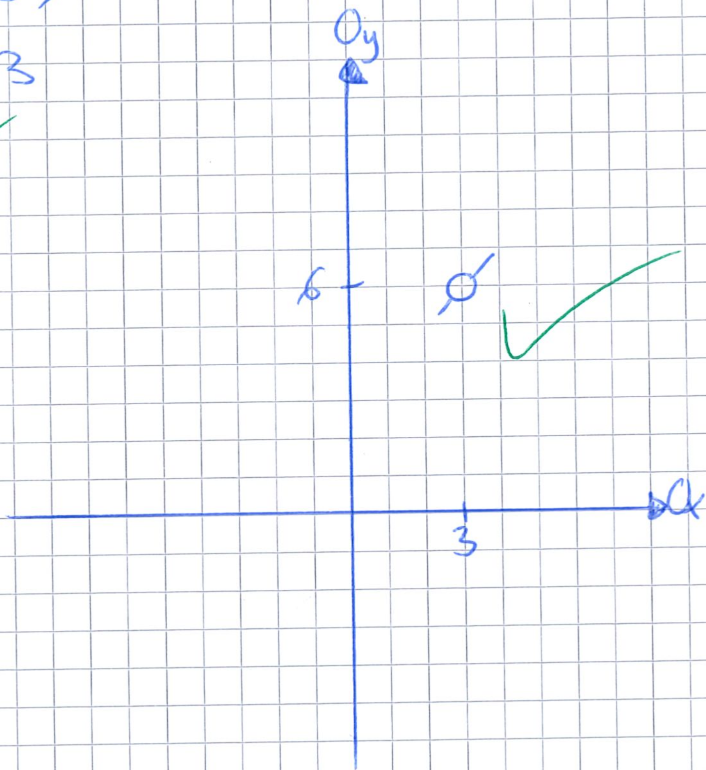
$$= \lim_{x\to 3} \frac{(x-3)(\sqrt{x+6}+3)}{(x-3)}$$

$$= \lim_{x\to 3} \sqrt{x+6}+3$$

$$= \text{"}\sqrt{3+6}+3\text{"}$$

$$= \sqrt{9}+3$$

$$= 6$$



Trevor, Esteban,
Marten & Joachim

Gr3

Travail de groupe

Act 16 :

$\boxed{4}$ $f(2) = 0$ + … justif.

$$\lim_{x\to 2^-} \frac{1}{2}x \overset{L2}{=} \frac{1}{2}\cdot 2 = 1$$

$$\lim_{x\to 2^+} -x+3 \overset{L2}{=} -2+3 = 1$$

$\lim_{x\to 2^-} = \lim_{x\to 2^+} \neq f(2)$ donc la fonction n'est pas continue (par définition).

+ …

Gr4 22/09/2023

Travail de groupe n°2 : Matej, Etienne, Michael, Chloé

8i.

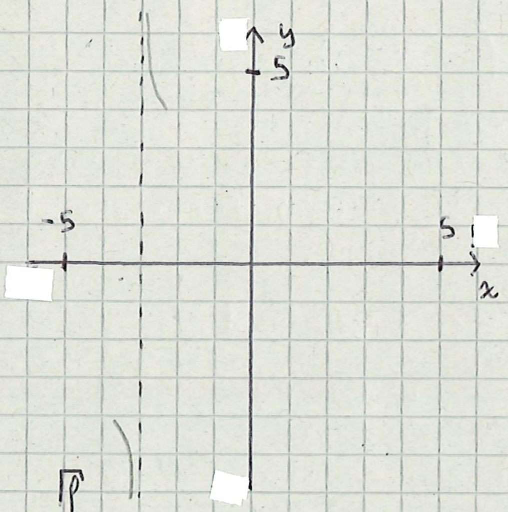
$$\lim_{x \to -3} \frac{5x}{(x+3)^2} \quad \text{type } "\frac{1}{0}"$$

$$\lim_{x \to -3^+} \frac{5x}{(x+3)^2} = \frac{-15}{0^+} = -\infty$$

$$\lim_{x \to -3^-} \frac{5x}{(x+3)^2} = \frac{-15}{0^-} = +\infty$$

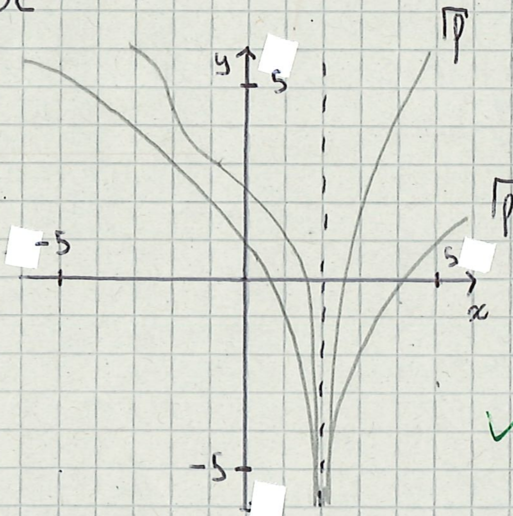
non...

$$\lim_{x \to -3} f(x) \nexists$$



(V)

9c



✓

10 a

$$f(x) = \frac{3x}{(x-2)^2}$$ $\lim\limits_{x \to 2} f(x) = +\infty$ $\exists$ ou $\nexists$?!

$$f(x) = \frac{18x+2}{(x-2)^2}$$

11c. $\lim\limits_{x \to -\infty} (x^4 + 2x)$

$$= (-\infty)^4 + 2(-\infty)$$

$$= +\infty + (-\infty)$$

$$= +\infty - \infty \quad \text{indétermination}$$

$$\lim_{x \to -\infty} x^4 \left(1 + \frac{2}{x^3}\right) = +\infty \cdot 1$$

$$= +\infty \checkmark$$

$\frac{2}{x^3} \to 0$

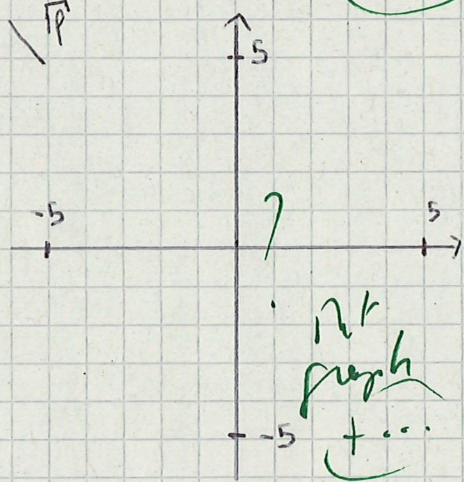
int graph?

+...

13b.

$$\lim_{x \to 1} \frac{x^2 - 1}{x^2 + 3x + 5}$$

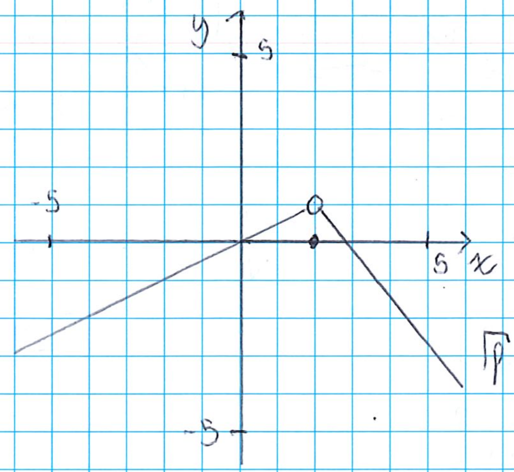
$$= \frac{1-1}{1+3+5} = \frac{0}{9} = 0 \checkmark$$



Gr# Travail de groupe n°2

Chloé, Etienne, Michael, Matej

Act 16.4



$\lim_{x \to 2^-} \frac{1}{2}x = 1$ et $\lim_{x \to 2^+} -x+3 = 1$

donc lorsque $x = 2$, l'image devrait être 1

Cependant, l'énoncé stipule que $f(x) = 0$ si $x = 2$.

$\lim_{x \to 2} 0 = 0$? donc $\lim_{x \to 2^-} \frac{1}{2}x \neq 0$

non, pas de lien

et idem pour $\lim_{x \to 2^+} -x+3 \neq 0$

$$\lim_{x \to a} \frac{1}{2}x \overset{?}{=} f(a) \quad \text{et} \quad \lim_{x \to a} -x+3 \overset{?}{=} f(a)$$

$$\lim_{x \to a} \frac{1}{2} \cdot \lim_{x \to a} x$$

$$= \frac{1}{2} \cdot a$$

[L 1] [L 2]

direct P, L7

$$\lim_{x \to a} -x + \lim_{x \to a} 3$$

$$= -a + 3$$

[L 2] [L 1]

pour quels x ?

$$f(x) = \frac{1}{2}x$$

$$\Leftrightarrow f(a) = \frac{1}{2}a \text{ [def fonction]}$$

$$f(x) = -x+3$$

$$\Leftrightarrow f(a) = -a+3$$

$$\lim_{x \to a} -x+3 = f(a) \text{ [def cont.]}$$

$$\lim_{x \to a} \frac{1}{2}x = \frac{1}{2}a \text{ [def conti]}$$

+ ..