Formules de dérivation

19

a. $(x^2-3)' \underset{PrD3}{=} (x^2)'-(3)' \underset{\substack{D1\\D4}}{=} 2x-0=2x$

b. $\left(\frac{2}{x^5}\right)' \underset{PrD7}{=} \frac{-2\cdot(x^5)'}{(x^5)^2} \underset{D6}{=} \frac{-2\cdot 5x^4}{(x^5)^2}=\frac{-10x^4}{x^{10}}=--\frac{10}{x^6}$

ou $\left(\frac{2}{x^5}\right)'=\left(2\cdot\frac{1}{x^5}\right)' \underset{PrD1}{=} 2\cdot\left(\frac{1}{x^5}\right)' \underset{PrD5}{=} 2\cdot\frac{-(x^5)'}{(x^5)^2} \underset{D6}{=} 2\cdot\frac{-5x^4}{x^{10}}=-\frac{10x^2}{x^{10}}=-\frac{10}{x^6}$

c. $(\sqrt{2x^3-3})'=([2x^3-3]^{\frac{1}{2}})' \underset{PrD8}{=} \frac{1}{2}(2x^3-3)^{\frac{1}{2}-1}\cdot(2x^3-3)' \underset{PrD3}{=} \frac{1}{2}(2x^3-3)^{-\frac{1}{2}}\cdot[(2x^3)'-(3)']$

$$\underset{\substack{D1\\PrD1}}{=} \frac{1}{2}\cdot\frac{1}{(2x^3-3)^{\frac{1}{2}}}\cdot(2\cdot 3x^2-0) \underset{D4}{=} \frac{1}{2}\cdot\frac{1}{\sqrt{2x^3-3}}\cdot 6x^2=\frac{6x^2}{2\sqrt{2x^3-3}}=\frac{3x^2}{\sqrt{2x^3-3}}$$

d. $(\sqrt[3]{x+1})'=[(x+1)^{\frac{1}{3}}]' \underset{PrD8}{=} \frac{1}{3}\cdot(x+1)^{\frac{1}{3}-1}\cdot(x+1)' \underset{PrD2}{=} \frac{1}{3}\cdot(x+1)^{-\frac{2}{3}}\cdot(x'+1') \underset{\substack{D1\\D2}}{=} \frac{1}{3}\cdot\frac{1}{(x+1)^{\frac{2}{3}}}\cdot(1+0)=\frac{1}{3\sqrt[3]{(x+1)^2}}$

20

a. $(x^5-10x)'=(x^5)'-(10x)'=5x^4-10$

b. $(x^{100}+100x)'=(x^{100})'+(100x)'=100x^{99}+100$

c. $(x^2+3)'=(x^2)'+(3)'=2x$

d. $(x^2+\pi x^3)'=(x^2)'+(\pi x^3)'=2x+3\pi x^2$

e. $f'(x)=(x^3-3x^2+9)'=(x^3)'-(3x^2)'+(9)'=3x^2-3\cdot 2x+0=3x^2-6x$

f. $f'(x)=\left(\frac{x^5}{5}+\frac{x^3}{3}+\frac{x}{7}\right)'=\left(\frac{x^5}{5}\right)'+\left(\frac{x^3}{3}\right)'+\left(\frac{x}{7}\right)'=\frac{1}{5}\cdot(x^5)'+\frac{1}{3}\cdot(x^3)'+\frac{1}{7}\cdot(x)'=\frac{1}{5}\cdot 5x^4+\frac{1}{3}\cdot 3x^2+\frac{1}{7}\cdot 1=x^4+x^2+\frac{1}{7}$

g. $f'(t)=(t^3+t^2+t+1)'=(t^3)'+(t^2)'+(t)'+(1)'=3t^2+2t+1$

h. $(3\sqrt{x})'=3(\sqrt{x})'=3\cdot\frac{1}{2\sqrt{x}}=\frac{3}{2\sqrt{x}}$ **ou** $(3\sqrt{x})'=(3x^{\frac{1}{2}})'=3(x^{\frac{1}{2}})'=3\cdot\frac{1}{2}\cdot x^{1-\frac{1}{2}}=\frac{3}{2}\cdot x^{-\frac{1}{2}}=\frac{3}{2}\cdot\frac{1}{\sqrt{x}}=\frac{3}{2\sqrt{x}}$

i. $(\sqrt{3x})'=(\sqrt{3}\cdot\sqrt{x})'=(\sqrt{3})'\cdot(\sqrt{x})'=\sqrt{3}\cdot(\sqrt{x})'=\sqrt{3}\cdot\frac{1}{2\sqrt{x}}=\frac{\sqrt{3}}{2\sqrt{x}}$

j. $(\sqrt[3]{x})'=(x^{\frac{1}{3}})'=\frac{1}{3}\cdot x^{\frac{1}{3}-1}=\frac{1}{3}\cdot x^{-\frac{2}{3}}=\frac{1}{3}\cdot\frac{1}{x^{\frac{2}{3}}}=\frac{1}{3}\cdot\frac{1}{(x^2)^{\frac{1}{3}}}=\frac{1}{3\sqrt[3]{x^2}}$

k. $(\sqrt{x^3})'=[(x^3)^{\frac{1}{2}}]'=(x^{\frac{3}{2}})'=\frac{3}{2}\cdot x^{\frac{3}{2}-1}=\frac{3}{2}x^{\frac{1}{2}}=\frac{3}{2}\sqrt{x}$

l. $(\sqrt{2x^3})'=(\sqrt{2}\cdot\sqrt{x^3})'=\sqrt{2}\cdot(\sqrt{x^3})'\overset{k.}{=}\sqrt{2}\cdot\frac{3}{2}\sqrt{x}=\frac{3}{2}\cdot\sqrt{2x}$

m. $(x\sqrt{x})'=(x^1\cdot x^{\frac{1}{2}})'=(x^{1+\frac{1}{2}})'=(x^{\frac{3}{2}})'\overset{k.}{=}\frac{3}{2}\sqrt{x}$

n. $(x^{895})'=895x^{894}$

o. $(x^{-45})'=-45x^{-46}$

p. $(x^{\frac{4}{3}})'=\frac{4}{3}x^{\frac{4}{3}-1}=\frac{4}{3}x^{\frac{1}{3}}$

q. $(x^{\sqrt{2}})'=\sqrt{2}x^{\sqrt{2}-1}$

r. $\left(\frac{4}{x}\right)'=\left(4\cdot\frac{1}{x}\right)'=4\cdot\left(\frac{1}{x}\right)'=4\cdot\left(-\frac{1}{x^2}\right)=-\frac{4}{x^2}$

s. $\left(\frac{-18}{x}\right)'=\left(-18\cdot\frac{1}{x}\right)'=-18\cdot\left(\frac{1}{x}\right)'=-18\cdot\left(\frac{-1}{x^2}\right)=\frac{18}{x^2}$

t. $\left(\frac{1}{x^2}\right)'=(x^{-2})'=-2x^{-2-1}=-2x^{-3}=-2\cdot\frac{1}{x^3}=-\frac{2}{x^3}$

u. $\left(\frac{1}{3x^3}\right)'=\left(\frac{1}{3}\cdot\frac{1}{x^3}\right)'=\frac{1}{3}\cdot\left(\frac{1}{x^3}\right)'=\frac{1}{3}\cdot(x^{-3})'=\frac{1}{3}\cdot(-3x^{-4})=\frac{1}{3}\cdot(-3)\cdot\frac{1}{x^4}=-\frac{1}{x^4}$

v. $\left(\frac{24}{x^2}\right)'=\left(24\cdot\frac{1}{x^2}\right)'=24\cdot\left(\frac{1}{x^2}\right)'\overset{t.}{=}24\cdot\left(-\frac{2}{x^3}\right)=-\frac{48}{x^3}$

w. $\left(\frac{1}{\sqrt{x}}\right)'=[\sqrt{x}^{-1}]'=[(x^{\frac{1}{2}})^{-1}]'=(x^{-\frac{1}{2}})'=-\frac{1}{2}x^{-\frac{1}{2}-1}=-\frac{1}{2}x^{-\frac{3}{2}}=-\frac{1}{2}\cdot\frac{1}{x^{\frac{3}{2}}}=-\frac{1}{2}\cdot\frac{1}{(x^3)^{\frac{1}{2}}}=-\frac{1}{2}\cdot\frac{1}{\sqrt{x^3}}=-\frac{1}{2\sqrt{x^3}}$

x. $\left(\frac{1}{x\sqrt{x}}\right)'=[(x\sqrt{x})^{-1}]'=[(x^1\cdot x^{\frac{1}{2}})^{-1}]'=[(x^{\frac{3}{2}})^{-1}]'=(x^{-\frac{3}{2}})'=-\frac{3}{2}x^{-\frac{3}{2}-1}=-\frac{3}{2}x^{-\frac{5}{2}}=-\frac{3}{2}\cdot\frac{1}{x^{\frac{5}{2}}}=-\frac{3}{2}\cdot\frac{1}{(x^5)^{\frac{1}{2}}}=-\frac{3}{2\sqrt{x^5}}$

y. $\left(x^2-\frac{1}{2x}\right)'=(x^2)'-\left(\frac{1}{2x}\right)'=2x-\left(\frac{1}{2}\cdot\frac{1}{x}\right)'=2x-\frac{1}{2}\cdot\left(\frac{1}{x}\right)'=2x-\frac{1}{2}\cdot\left(-\frac{1}{x^2}\right)=2x+\frac{1}{2x^2}$

z. $\left(\frac{x^2}{x^3+1}\right)'=\frac{(x^2)'\cdot(x^3+1)-(x^2)\cdot(x^3+1)'}{(x^3+1)^2}=\frac{2x\cdot(x^3+1)-x^2\cdot(3x^2+0)'}{(x^3+1)^2}=\frac{2x^4+2x-3x^4}{(x^3+1)^2}=\frac{-x^4+2x}{(x^3+1)^2}$

aa. $\left(\frac{1+2u^3}{2u}\right)'=\frac{(1+2u^3)'\cdot 2u-(1+2u^3)\cdot(2u)'}{(2u)^2}=\frac{(0+6u^2)\cdot 2u-(1+2u^3)\cdot 2}{(2u)^2}=\frac{12u^3-2-4u^3}{(2u)^2}=\frac{8u^3-2}{4u^2}$
$=\frac{2(4u^3-1)}{2\cdot 2u^2}=\frac{4u^3-1}{2u^2}$

ab. $[(2x+3)(3x-7)]' = (2x+3)'(3x-7)+(2x+3)(3x-7)'=2(3x-7)+(2x+3)\cdot 3=6x-14+6x+9$
$= 12x-5$

ac. $(x^2(1+\sqrt{x}))' = (x^2)'(1+\sqrt{x})+(x^2)(1+\sqrt{x})'=2x(1+\sqrt{x})+x^2(0+\frac{1}{2\sqrt{x}})=2x+2x\sqrt{x}+\frac{x^2}{2\sqrt{x}}$
$= 2x+2x\sqrt{x}+\frac{1}{2}x\sqrt{x}=2x+\frac{5}{2}x\sqrt{x}$

ad. $((x^3-x)(x^2-9))' = (x^3-x)'(x^2-9)+(x^3-x)(x^2-9)'=(3x^2-1)(x^2-9)+(x^3-x)(2x-0)$
$= 3x^4-x^2-27x^2+9+2x^4-2x^2=5x^4-30x^2+9$

ae. $\left(\frac{x^2+1}{4x}\right)'=\frac{(x^2+1)'\cdot 4x-(x^2+1)\cdot(4x)'}{(4x)^2}=\frac{2x\cdot 4x-(x^2+1)\cdot 4}{16x^2}=\frac{8x^2-4x^2-4}{16x^2}=\frac{4(x^2-1)}{4\cdot 4x^2}=\frac{x^2-1}{4x^2}$