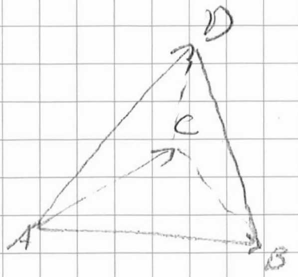
ex78 (a) $V = \frac{1}{6}[\vec{AB}, \vec{AC}, \vec{AD}]$



$\vec{AB}\begin{pmatrix}2\\-2\\1\end{pmatrix}$ $\vec{AC}\begin{pmatrix}4\\0\\6\end{pmatrix}$ $\vec{AD}\begin{pmatrix}-7\\-7\\7\end{pmatrix}$

$\vec{AB} \times \vec{AC} = \begin{pmatrix}2\\-2\\1\end{pmatrix} \times \begin{pmatrix}4\\0\\6\end{pmatrix} = \begin{pmatrix}-12\\-8\\8\end{pmatrix}$

$V = \frac{1}{6}\begin{pmatrix}-12\\-8\\8\end{pmatrix} \cdot \begin{pmatrix}3\\1\\7\end{pmatrix} = \frac{1}{6}\left((-12)(-7)+(-8)(-7)+8\cdot 7\right) = \frac{196}{6} = \frac{98}{3} \simeq 32,7$

(b) Aire base $= \frac{\|\vec{AB} \times \vec{AC}\|}{2} = \frac{\sqrt{144+64+64}}{2} = \frac{\sqrt{272}}{2} = \frac{4\sqrt{17}}{2} = 2\sqrt{17}$

$h = \frac{V}{A} = \frac{98/3}{2\sqrt{17}} = \frac{49}{3\sqrt{17}} = \frac{49\sqrt{17}}{3\cdot 17} = \frac{49}{51}\sqrt{17} \simeq 3,96$

(c) $\cos(\alpha) = \frac{\vec{AB}\cdot\vec{AC}}{\|\vec{AB}\|\|\vec{AC}\|} = \frac{8+0+6}{\sqrt{9}\sqrt{52}} = \frac{14}{3\sqrt{52}} = \frac{14}{3\cdot 2\sqrt{13}} = \frac{7\sqrt{13}}{3\cdot 13} = \frac{7\sqrt{13}}{39}$

$\alpha = \cos^{-1}\left(\frac{7\sqrt{13}}{39}\right) \simeq 49,67°$

(d) Π_1 = plan par A, B, C : $\vec{n_1} = \vec{AB} \times \vec{AC} \perp \Pi_1$ $\vec{n_1}\begin{pmatrix}-12\\-8\\8\end{pmatrix}$

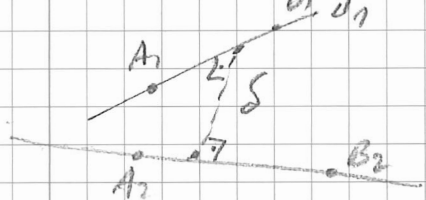
Π_2 = " " A, B, D : $\vec{n_2} = \vec{AB} \times \vec{AD}$

$= \begin{pmatrix}2\\-2\\1\end{pmatrix} \times \begin{pmatrix}-7\\-7\\7\end{pmatrix} = \begin{pmatrix}-7\\-21\\-28\end{pmatrix} \perp \Pi_2$

β = angle entre Π_1 et Π_2 = angle en $\vec{n_1}$ et $\vec{n_2}$

$\beta = \cos^{-1}\left(\frac{\vec{n_1}\cdot\vec{n_2}}{\|\vec{n_1}\|\cdot\|\vec{n_2}\|}\right) = \cos^{-1}\left(\frac{28}{\sqrt{272}\sqrt{1274}}\right) \simeq 87,27°$

ex79



$\delta = \frac{[\vec{A_1B_1}, \vec{A_2B_2}, \vec{A_1A_2}]}{\|\vec{A_1B_1} \times \vec{A_2B_2}\|}$

(a) $\vec{A_1B_1}\begin{pmatrix}9\\6\\3\end{pmatrix}$ et $\vec{A_2B_2}\begin{pmatrix}4\\0\\0\end{pmatrix} \Rightarrow$ non colin.

$\vec{A_1A_2}, \vec{A_1B_1}, \vec{A_2B_2}$ coplanaires $\Leftrightarrow [\vec{A_1A_2}, \vec{A_1B_1}, \vec{A_2B_2}] = 0 \Leftrightarrow \ldots$ oui!

donc $d_1 \cap d_2 = I$:

$d_1 : \begin{pmatrix}x-1\\y-0\\z-1\end{pmatrix} = \lambda\begin{pmatrix}9\\6\\3\end{pmatrix} \Leftrightarrow \begin{cases} ① \; x = 1+9\lambda \\ ② \; y = 6\lambda \\ ③ \; z = 1+3\lambda \end{cases}$

$d_2 : \begin{pmatrix}x-0\\y-2\\z-2\end{pmatrix} = \mu\begin{pmatrix}4\\0\\0\end{pmatrix} \Leftrightarrow \begin{cases} ④ \; x = 4\mu \\ ⑤ \; y = 2 \\ ⑥ \; z = 2 \end{cases} \longrightarrow$ dans ② : $\lambda = 1/3$

↳ dans ① : $x = 4$

dans ③ : $z = 2$

$d_1 \cap d_2 = I(4; 2; 2)$

(b) $\overrightarrow{A_1B_1}\begin{pmatrix}3\\-1\\2\end{pmatrix}$ $\overrightarrow{A_2B_2}\begin{pmatrix}9\\-3\\6\end{pmatrix}$ $\Rightarrow$ colinéaires

$d_1 : \begin{pmatrix}x+4\\y-2\\z-1\end{pmatrix} = \lambda\begin{pmatrix}3\\-1\\2\end{pmatrix}$ $A_2 \overset{?}{\in} d_1 : \begin{pmatrix}0+4\\5-2\\-2-1\end{pmatrix} \overset{?}{=} \lambda\begin{pmatrix}3\\-1\\2\end{pmatrix} \Leftrightarrow \begin{pmatrix}4\\3\\-3\end{pmatrix} \overset{?}{=} \lambda\begin{pmatrix}3\\-1\\2\end{pmatrix}$

non!

$d_1 \parallel d_2$

$d_1 \cap d_2 = \emptyset$

$\delta = 0$

(c) $\overrightarrow{A_1B_1}\begin{pmatrix}-10\\4\\-2\end{pmatrix}$ $\overrightarrow{A_2B_2}\begin{pmatrix}-8\\-3\\-7\end{pmatrix}$ $\Rightarrow$ non colin

$\overrightarrow{A_1A_2}$, $\overrightarrow{A_1B_1}$ et $\overrightarrow{A_2B_2}$ coplanaires $\Leftrightarrow [\overrightarrow{A_1A_2}, \overrightarrow{A_1B_1}, \overrightarrow{A_2B_2}] = 0$

on calcule $[\overrightarrow{A_1A_2}, \overrightarrow{A_1B_1}, \overrightarrow{A_2B_2}] = \ldots \neq 0$

$\Rightarrow$ non coplanaires

$\Rightarrow$ gauches, $d_1 \cap d_2 = \emptyset$

$$\delta = \frac{|[\overrightarrow{A_1B_1}; \overrightarrow{A_2B_2}; \overrightarrow{A_1A_2}]|}{\|\overrightarrow{A_1B_1} \times \overrightarrow{A_2B_2}\|} = \ldots \cong 0{,}48$$

(d) $\overrightarrow{A_1B_1}\begin{pmatrix}1\\1\\2\end{pmatrix}$ $\overrightarrow{A_2B_2}\begin{pmatrix}5\\5\\10\end{pmatrix}$ $\Rightarrow$ colin.

$d_1 : \begin{pmatrix}x-2\\y+3\\z-1\end{pmatrix} = \lambda\begin{pmatrix}1\\1\\2\end{pmatrix}$ $A_2 \overset{?}{\in} d_1 : \begin{pmatrix}0-2\\-5+3\\-3-1\end{pmatrix} \overset{?}{=} \lambda\begin{pmatrix}1\\1\\2\end{pmatrix} \Leftrightarrow \begin{pmatrix}-2\\-2\\-4\end{pmatrix} \overset{?}{=} \lambda\begin{pmatrix}1\\1\\2\end{pmatrix}$

oui!

$\Rightarrow$ confondues

$d_1 \cap d_2 = d_1 = d_2$