ex 38 :

a) $\begin{pmatrix}2\\-3\end{pmatrix}\cdot\begin{pmatrix}5\\3\end{pmatrix} = 2\cdot5+(-3)\cdot3 = 10-9 = 1$

b) $\begin{pmatrix}5\\3\end{pmatrix}\cdot\begin{pmatrix}2\\-3\end{pmatrix} = \dots = 1$

c) $\vec{v}\cdot\vec{v} = \begin{pmatrix}2\\-3\end{pmatrix}\cdot\begin{pmatrix}2\\-3\end{pmatrix} = 2\cdot2+(-3)(-3) = 4+9 = 13$

rem : $\vec{v}\cdot\vec{v} = \|\vec{v}\|^2$

d) $\vec{w}\cdot\vec{w} = 5\cdot5+3\cdot3 = 34$

e) $\|\vec{v}\| = \sqrt{2^2+(-3)^2} = \sqrt{13}$

f) $\|\vec{w}\| = \sqrt{5^2+3^2} = \sqrt{34}$

ex 39 :

pour les conj. vraies, démos équivalentes pour $\mathbb{R}^3$

a) $\vec{a}\cdot\vec{b} = \begin{pmatrix}a_1\\a_2\end{pmatrix}\cdot\begin{pmatrix}b_1\\b_2\end{pmatrix} = a_1b_1+a_2b_2 = b_1a_1+b_2a_2 = \begin{pmatrix}b_1\\b_2\end{pmatrix}\cdot\begin{pmatrix}a_1\\a_2\end{pmatrix} = \vec{b}\cdot\vec{a}$

<u>VRAI</u>

b) $\vec{a}\cdot\vec{0} = \begin{pmatrix}a_1\\a_2\end{pmatrix}\cdot\begin{pmatrix}0\\0\end{pmatrix} = a_1\cdot0+a_2\cdot0 = 0$

donc $\vec{a}\cdot\vec{0} = 0$ VRAI

⚠ $\vec{a}\cdot\vec{0} = \vec{0}$ FAUX

$\vec{a}\cdot 0 = 0$ FAUX

c) $\vec{a}\cdot\vec{a} = \begin{pmatrix}a_1\\a_2\end{pmatrix}\cdot\begin{pmatrix}a_1\\a_2\end{pmatrix} = a_1^2+a_2^2 = \left(\sqrt{a_1^2+a_2^2}\right)^2 = \|\vec{a}\|^2$

d) $\alpha(\vec{a}\cdot\vec{b}) = \alpha\left[\begin{pmatrix}a_1\\a_2\end{pmatrix}\cdot\begin{pmatrix}b_1\\b_2\end{pmatrix}\right] = \alpha\left[a_1b_1+a_2b_2\right] = \alpha\left[b_1a_1+b_2a_2\right] = \alpha\left[\begin{pmatrix}b_1\\b_2\end{pmatrix}\cdot\begin{pmatrix}a_1\\a_2\end{pmatrix}\right] = \alpha(\vec{b}\cdot\vec{a})$

donc $\alpha(\vec{a}\cdot\vec{b}) = (\alpha\vec{a})\cdot(\alpha\vec{b})$ est fausse !

contre-ex : $2\cdot\left[\begin{pmatrix}1\\1\end{pmatrix}\cdot\begin{pmatrix}0\\1\end{pmatrix}\right] \overset{?}{=} 2\begin{pmatrix}1\\1\end{pmatrix}\cdot2\begin{pmatrix}0\\1\end{pmatrix}$

$\Leftrightarrow 2\left[0+1\right] \overset{?}{=} \begin{pmatrix}2\\2\end{pmatrix}\cdot\begin{pmatrix}0\\2\end{pmatrix}$

$\Leftrightarrow 2 \overset{?}{=} 2\cdot0+2\cdot2$ non !

e) Faux, contre-exemple : $\begin{pmatrix}1\\1\end{pmatrix}\cdot\left[\begin{pmatrix}1\\1\end{pmatrix}+\begin{pmatrix}1\\1\end{pmatrix}\right] \overset{?}{=} \left[\begin{pmatrix}1\\1\end{pmatrix}+\begin{pmatrix}1\\1\end{pmatrix}\right]\cdot\left[\begin{pmatrix}1\\1\end{pmatrix}+\begin{pmatrix}1\\1\end{pmatrix}\right]$

$\Leftrightarrow \begin{pmatrix}1\\1\end{pmatrix}\cdot\begin{pmatrix}2\\2\end{pmatrix} \overset{?}{=} \begin{pmatrix}2\\2\end{pmatrix}\cdot\begin{pmatrix}2\\2\end{pmatrix}$

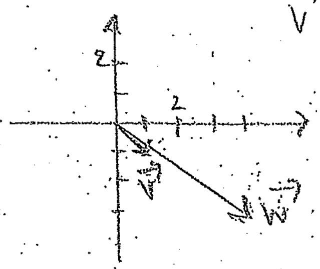
$\Leftrightarrow 2+2 \overset{?}{=} 4+4$ non !

40

a) $\vec{v}\cdot\vec{w} = 1\cdot 4 + (-1)(-3) = 7$

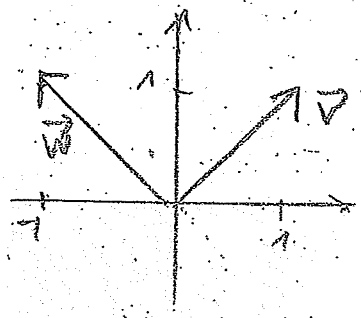
$\vec{v}\cdot\vec{w} = \|\vec{v}\|\cdot\|\vec{w}\|\cos(\alpha) \Leftrightarrow 7 = \sqrt{2}\sqrt{25}\cos(\alpha)$

$\Leftrightarrow \alpha = \cos^{-1}\left(\frac{7}{5\sqrt{2}}\right) \approx 8{,}1°$



b) $\vec{v}\cdot\vec{w} = 0$

$\cos(\alpha) = \frac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\|\vec{w}\|} = 0 \Leftrightarrow \alpha = 90°$



c) $\vec{v}\cdot\vec{w} = 2(-1) + 2\cdot 2 + (-1)\cdot 0 = 2$

$\cos(\alpha) = \frac{\vec{v}\cdot\vec{w}}{\|\vec{v}\|\|\vec{w}\|} = \frac{2}{\sqrt{2^2+2^2+(-1)^2}\cdot\sqrt{(-1)^2+2^2+0^2}} = \frac{2}{\sqrt{9}\cdot\sqrt{5}} = \frac{2}{3\sqrt{5}}$

$\alpha \approx 72{,}7°$

d) $\vec{v}\cdot\vec{w} = 2(-1) + (-3)(-2) + (-1)\cdot 4 = 0$

$\cos(\alpha) = 0 \Leftrightarrow \alpha = 90°$