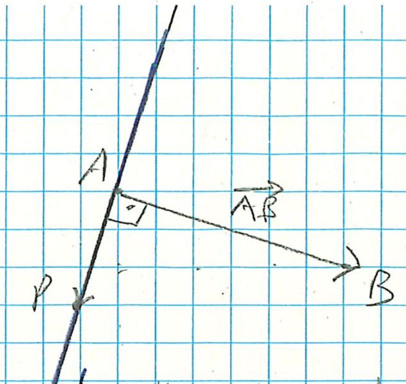
ex 41

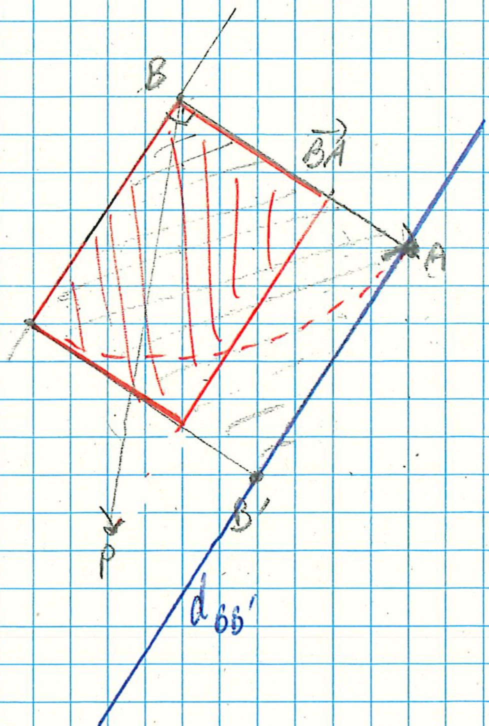
a)



d est l'ensemble des pts P tels que $\vec{AB}\cdot\vec{AP}=0$

(càd $\vec{AB}\perp\vec{AP}$)

b) $\vec{BA}\cdot\vec{BP} = \|\vec{AB}\|^2$



Géométriquement:

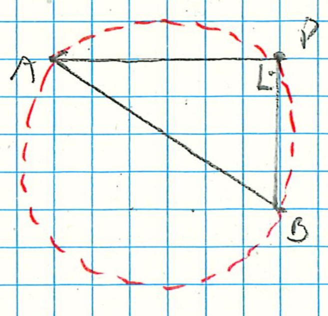
$\|\vec{AB}\|^2$ = aire du carré de côté $\|\vec{AB}\|$ ou $\|\vec{BA}\|$

$\vec{BA}\cdot\vec{BP}$: aire du rectangle rouge

Pour que les 2 aires soient égales:

$P \in d_{BB'}$

c) $\vec{PA}\cdot\vec{PB}=0 \Leftrightarrow \vec{PA}\perp\vec{PB}$

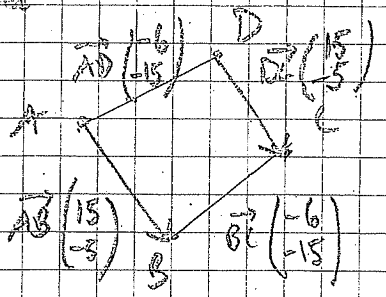


$P \in$ cercle de centre M et de rayon $\frac{\|\vec{AB}\|}{2}$

où M : milieu de [AB]

(Cercle de Thalès)

ex42 schéma



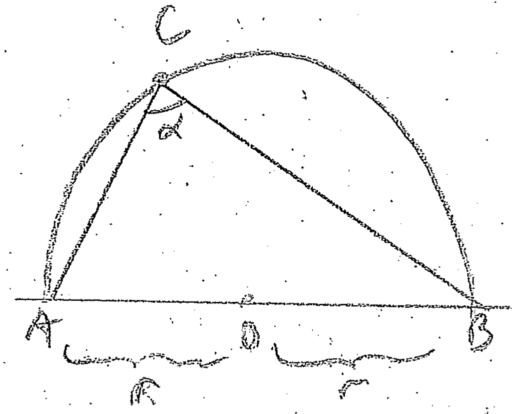
paralélisme ? $\vec{AD} = \vec{BC}$, $\vec{AB} = \vec{DC}$ $\Rightarrow$ ABCD parallélogramme

longueurs ? $\|\vec{AD}\| = \|\vec{BC}\| = \sqrt{261}$, $\|\vec{AB}\| = \|\vec{DC}\| = \sqrt{250}$ $\Rightarrow$ ABCD pas un losange

angles ? $\vec{AB} \cdot \vec{AD} = \begin{pmatrix}15\\-5\end{pmatrix} \cdot \begin{pmatrix}-6\\-15\end{pmatrix} = 15 \cdot (-6) + (-5)(-15) \neq 0$

donc $\vec{AB} \not\perp \vec{AD}$ $\Rightarrow$ ABCD n'est pas un rectangle (donc pas un carré)

ex43



à démontrer : $\alpha = 90°$

$\Leftrightarrow \vec{CA} \cdot \vec{CB} = 0$

on a : • $\|\vec{OA}\| = \|\vec{OB}\| = r$

• $\vec{OA} = -\vec{OB} = \vec{BO} = -\vec{AO}$

on a : $\vec{CA} \cdot \vec{CB} = (\vec{CO} + \vec{OA}) \cdot (\vec{CO} + \vec{OB})$

$= \vec{CO} \cdot \vec{CO} + \vec{OA} \cdot \vec{CO} + \vec{CO} \cdot \vec{OB} + \vec{OA} \cdot \vec{OB}$

$= \|\vec{CO}\|^2 + (-\vec{OB} \cdot \vec{CO}) + \vec{CO} \cdot \vec{OB} + \vec{OA}(-\vec{OA})$

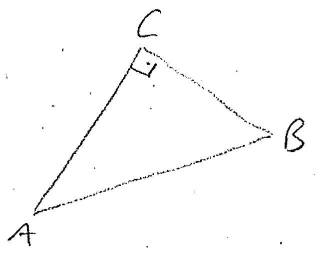
$= r^2 - \vec{OB} \cdot \vec{CO} + \vec{CO} \cdot \vec{OB} - \vec{OA} \cdot \vec{OA}$

$= r^2 - \cancel{\vec{CO} \cdot \vec{OB}} + \cancel{\vec{CO} \cdot \vec{OB}} - \|\vec{OA}\|^2$

$= r^2 - r^2$

$= 0$ cqfd

ex 44



on veut $\overrightarrow{CA} \cdot \overrightarrow{CB} = 0$

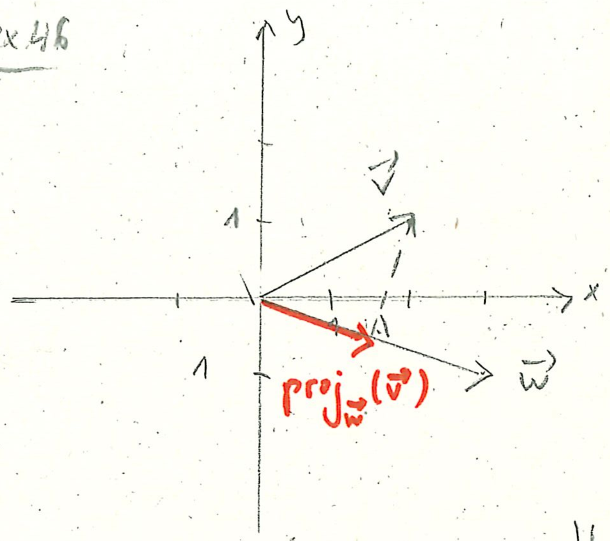
$\Leftrightarrow \begin{pmatrix} 5-3 \\ 2-4 \end{pmatrix} \cdot \begin{pmatrix} x-3 \\ 1-4 \end{pmatrix} = 0$

$\Leftrightarrow 2(x-3) + (-2)(-3) = 0$

$\Leftrightarrow 2x - 6 + 6 = 0$

$\Leftrightarrow x = 0$

ex 46



$$proj_{\vec{w}}(\vec{v}) = \frac{\vec{v}\cdot\vec{w}}{\|\vec{w}\|^2}\cdot\vec{w}$$

$$= \frac{\begin{pmatrix}2\\1\end{pmatrix}\begin{pmatrix}3\\-1\end{pmatrix}}{\left(\sqrt{3^2+(-1)^2}\right)^2}\begin{pmatrix}3\\-1\end{pmatrix}$$

$$= \frac{2\cdot3+1\cdot(-1)}{(\sqrt{10})^2}\begin{pmatrix}3\\-1\end{pmatrix}$$

$$= \frac{5}{10}\begin{pmatrix}3\\-1\end{pmatrix} = \begin{pmatrix}3/2\\-1/2\end{pmatrix}$$

$$\|proj_{\vec{w}}(\vec{w})\| = \sqrt{\left(\frac{3}{2}\right)^2+\left(-\frac{1}{2}\right)^2}$$

$$= \sqrt{\frac{9}{4}+\frac{1}{4}} = \sqrt{\frac{10}{4}} = \frac{\sqrt{10}}{2} \cong 1{,}58$$

ex 47

a) vecteur normal à d_1 : $\vec{n}_1\begin{pmatrix}1\\-4\end{pmatrix}$

" " " d_2 : $\vec{n}_2\begin{pmatrix}4\\1\end{pmatrix}$

$\vec{n}_1\cdot\vec{n}_2 = \begin{pmatrix}1\\-4\end{pmatrix}\begin{pmatrix}4\\1\end{pmatrix} = 4\cdot1+(-4)\cdot1 = 0$

donc $d_1 \perp d_2$

b) vecteur normal à d_1 : $\vec{n}_1\begin{pmatrix}1/9\\-1/6\end{pmatrix}$

" " " d_2 : $\vec{n}_2\begin{pmatrix}2\\-3\end{pmatrix}$

$\vec{n}_1\cdot\vec{n}_2 = \begin{pmatrix}1/9\\1/6\end{pmatrix}\begin{pmatrix}2\\-3\end{pmatrix} \neq 0$

donc $d_1 \not\perp d_2$

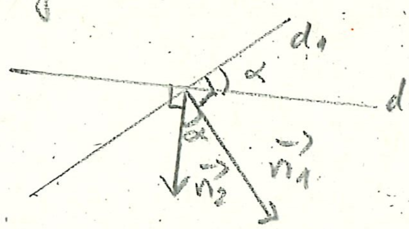
$\vec{n}_1 \overset{?}{=} \lambda\vec{n}_2 \Leftrightarrow \begin{pmatrix}1/9\\-1/6\end{pmatrix} \overset{?}{=} \lambda\begin{pmatrix}2\\-3\end{pmatrix}$

$\Leftrightarrow \begin{cases}1/9 = 2\lambda\\ -1/6 = -3\lambda\end{cases} \Leftrightarrow \begin{cases}\lambda = 1/18\\ \lambda = 1/18\end{cases}$ ok $\Rightarrow \vec{n}_1, \vec{n}_2$ colinéaires

$\Rightarrow d_1 \parallel d_2$

ex 48

l'angle entre d_1 et d_2 est celui entre 2 vecteurs normaux de d_1 et d_2



$\vec{n}_1\begin{pmatrix}3\\2\end{pmatrix}$

$\vec{n}_2\begin{pmatrix}4\\-1\end{pmatrix}$

$$\cos(\alpha) = \frac{\vec{n}_1\cdot\vec{n}_2}{\|\vec{n}_1\|\cdot\|\vec{n}_2\|}$$

$$= \frac{12-2}{\sqrt{13}\,\sqrt{17}} = \frac{10}{\sqrt{17\cdot13}}$$

$$\alpha = \cos^{-1}\left(\frac{10}{\sqrt{17\cdot13}}\right) \cong 47{,}7^\circ$$

(et l'autre angle est $180-\alpha$)

ex49 $d_1 : 2x - y - 5 = 0$ $\vec{n_1}\begin{pmatrix}2\\-1\end{pmatrix} \perp d_1$

$d_2 : m_1x + m_2y + 3 = 0$ $\vec{n_2}\begin{pmatrix}m_1\\m_2\end{pmatrix} \perp d_2$

a) $d_1 \perp d_2 \Leftrightarrow \vec{n_1} \perp \vec{n_2} \Leftrightarrow \begin{pmatrix}m_1\\m_2\end{pmatrix} \cdot \begin{pmatrix}2\\-1\end{pmatrix} = 0 \Leftrightarrow 2m_1 - m_2 = 0$

il suffit de choisir $m_2 = 2m_1$; par ex : $m_1 = 1$, $m_2 = 2$

b) $d_1 \parallel d_2 \Leftrightarrow \vec{n_1} = \lambda \vec{n_2}$

$\Leftrightarrow \begin{pmatrix}m_1\\m_2\end{pmatrix} = \lambda \begin{pmatrix}2\\-1\end{pmatrix} \Leftrightarrow \begin{cases} m_1 = 2\lambda \\ m_2 = -\lambda \end{cases} \begin{vmatrix} \\ 2 \end{vmatrix}$

$m_1 + 2m_2 = 0$

$\Leftrightarrow m_1 = -2m_2$

il suffit de choisir $m_1 = -2m_2$; par ex : $m_2 = 1$, $m_1 = -2$

ex45 a) $\vec{AB} = \begin{pmatrix}-4-3\\2-(-5)\end{pmatrix} = \begin{pmatrix}-7\\7\end{pmatrix}$. Soit $M = (x;y) \in d_1$.

On a : $\vec{AM} = \lambda \vec{AB} \Leftrightarrow \begin{pmatrix}x-3\\y+5\end{pmatrix} = \lambda \begin{pmatrix}-7\\7\end{pmatrix}$

$\Leftrightarrow \begin{cases} x-3 = -7\lambda \\ y+5 = 7\lambda \end{cases} \begin{vmatrix} 1 \\ 1 \end{vmatrix}$

$+$ $[x + y + 2 = 0]$

b) vect normal à d_1 : $\vec{n}\begin{pmatrix}1\\1\end{pmatrix}$

$\Rightarrow$ vect directeur de d_1 : $\vec{v}\begin{pmatrix}-1\\1\end{pmatrix}$, car $\begin{pmatrix}1\\1\end{pmatrix} \cdot \begin{pmatrix}-1\\1\end{pmatrix} = 0$

$\Rightarrow$ vect directeur de d_2 : $\vec{v}\begin{pmatrix}-1\\1\end{pmatrix}$

$M(x;y) \in d_2 \Leftrightarrow \vec{CM} = \lambda \vec{v} \Leftrightarrow \begin{pmatrix}x-3\\y+1\end{pmatrix} = \lambda \begin{pmatrix}-1\\1\end{pmatrix} \Leftrightarrow \begin{cases} x-3 = -\lambda \\ y+1 = \lambda \end{cases}$

$[x + y - 2 = 0]$

ou

$M(x;y) \in d_2 \Leftrightarrow \vec{CM} \cdot \vec{n} = 0 \Leftrightarrow \begin{pmatrix}x-3\\y+1\end{pmatrix} \cdot \begin{pmatrix}1\\1\end{pmatrix} = 0 \Leftrightarrow (x-3)\cdot 1 + (y+1)\cdot 1 = 0$

$\Leftrightarrow [x + y - 2 = 0]$

c) $\vec{n}\begin{pmatrix}1\\1\end{pmatrix}$ vect normal de d_1 et vect dir. de d_3

$M(x;y) \in d_3 \Leftrightarrow \vec{DM} = \lambda \vec{n} \Leftrightarrow \begin{pmatrix}x-1\\y-1\end{pmatrix} = \lambda \begin{pmatrix}1\\1\end{pmatrix} \Leftrightarrow \begin{cases} x-1 = \lambda \\ y-1 = \lambda \end{cases} \begin{vmatrix} 1 \\ -1 \end{vmatrix}$

$[x - y = 0]$

ou

$M(x;y) \in d_3 \Leftrightarrow \vec{DM} \cdot \vec{v} = 0 \Leftrightarrow \begin{pmatrix}x-1\\y-1\end{pmatrix} \cdot \begin{pmatrix}-1\\1\end{pmatrix} = 0 \Leftrightarrow -(x-1) + y - 1 = 0$

$\Leftrightarrow [-x + y = 0]$