<u>ex10</u> $f(x) = \dfrac{x^4+x^3-x^2-x}{1-x}$

a) D_f : pb si $1-x=0$
$\Leftrightarrow x=1$ $\qquad D_f = \mathbb{R} \setminus \{1\}$

Z_f : $x^4+x^3-x^2-x=0$
$\Leftrightarrow x^3(x+1)-x(x+1)=0$
$\Leftrightarrow x(x+1)(x^2-1)=0$
$\Leftrightarrow x(x+1)(x-1)(x+1)=0$

$x=0$ <u>ou</u> $x=-1$ <u>ou</u> $x=1$ $\qquad Z_f = \{-1;0;1\}$

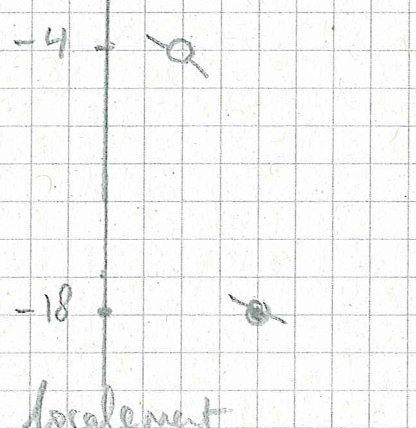
b) $f(1{,}9) \cong -15{,}98$ $\qquad f(2{,}1) \cong -20{,}18$ $\qquad$ (cf "tablo" de la calculatrice ou geogebra)
$f(1{,}99) \cong -17{,}8$ $\qquad f(2{,}01) \cong -18{,}21$
$f(1{,}999) \cong -17{,}98$ $\qquad f(2{,}001) \cong -18{,}02$

c) Conjecture : $\lim\limits_{x \to 2} f(x) = -18$

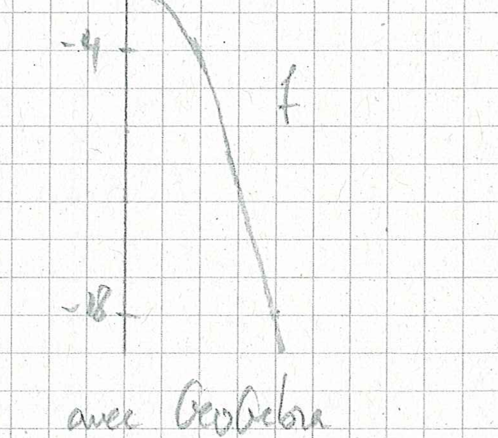
d) $f(0{,}9) \cong -3{,}25$ $\qquad f(1{,}1) \cong -4{,}85$
$f(0{,}99) \cong -3{,}92$ $\qquad f(1{,}01) \cong -4{,}08$
$f(0{,}999) \cong -3{,}99$ $\qquad f(1{,}001) \cong -4{,}01$

e) Conjecture : $\lim\limits_{x \to 1} f(x) = -4$

f)



localement



avec GeoGebra
ne montre pas le pb à $x=1$!!!

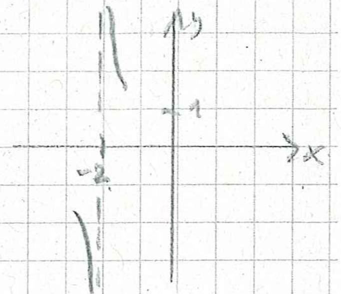
ex13

a) $\lim\limits_{x\to -2} f(x) = "\frac{2}{0}"$: type $\frac{1}{0}$

$\lim\limits_{x\to -2^+} f(x) = \lim\limits_{x\to -2^+} \frac{\cancel{2}\,1}{\cancel{2}(x+2)} = \frac{1}{0^+} = +\infty$

$\lim\limits_{x\to -2^-} f(x) = \frac{1}{0^-} = -\infty$

donc $\lim\limits_{x\to -2} f(x) \nexists$

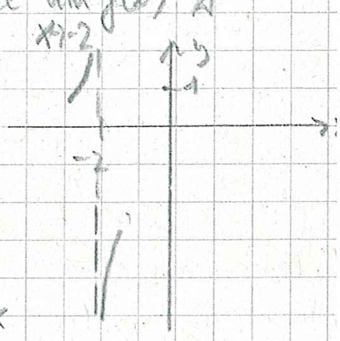


b) $\lim\limits_{x\to -2} \frac{-2}{2x+4} = "\frac{-2}{0}"$: type $\frac{1}{0}$

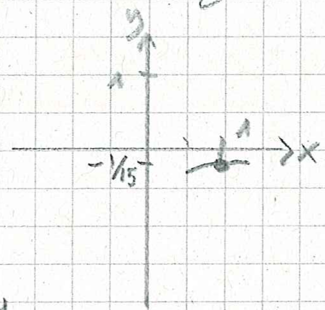
$\lim\limits_{x\to -2^+} f(x) = \lim\limits_{x\to -2^+} \frac{-\cancel{2}\,1}{\cancel{2}(x+2)} = \frac{-1}{0^+} = -\infty$

$\lim\limits_{x\to -2^-} f(x) = \frac{-1}{0^-} = +\infty$

donc $\lim\limits_{x\to -2} f(x) \nexists$



c) $\lim\limits_{x\to 1} f(x) = \frac{2-1}{1-16} = \frac{1}{-15}$

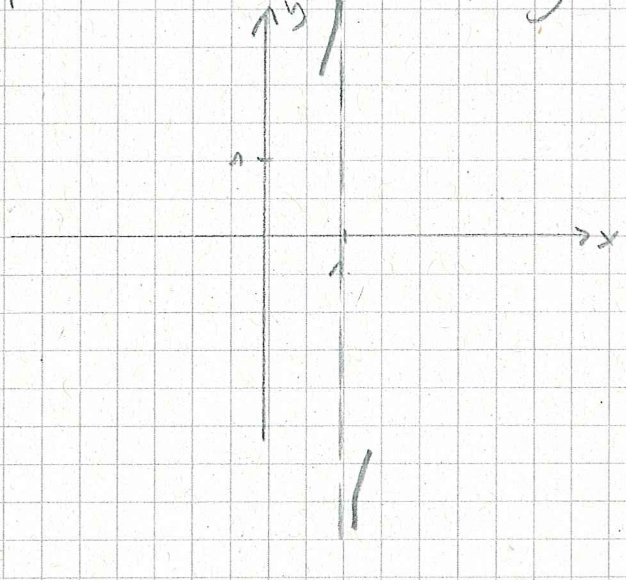


d) $\lim\limits_{x\to 1} f(x) = "\frac{1}{0}"$: type $\frac{1}{0}$

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} \frac{x-2}{(x+4)(x-1)} = \frac{-1}{5\cdot 1^+} = -\infty$

$\lim\limits_{x\to 1^-} f(x) = \frac{-1}{5\cdot 1^-} = +\infty$

$\lim\limits_{x\to 1} f(x) \nexists$



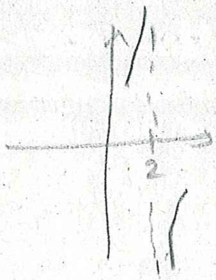
ex 13

e) $\lim\limits_{x\to 2} \frac{3x-7}{x-2}$ type "$\frac{1}{0}$"

$\lim\limits_{x\to 2^-} f(x) = \frac{3\cdot 2-7}{2^- -2} = \frac{-1}{0^-} = +\infty$

$\lim\limits_{x\to 2^+} f(x) = \frac{3\cdot 2-7}{2^+ -2} = \frac{-1}{0^+} = -\infty$

$\Big\}$ $\lim\limits_{x\to 2} f(x) \nexists$

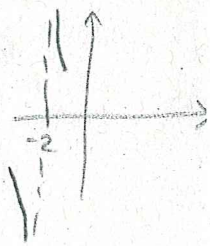


f) $\lim\limits_{x\to -2} \frac{-3x+2}{x+2}$ type "$\frac{1}{0}$"

$\lim\limits_{x\to -2^-} f(x) = \lim\limits_{x\to -2^-} \frac{-3(-2)+2}{-2^- +2} = \frac{8}{0^-} = -\infty$

$\lim\limits_{x\to -2^+} f(x) = \lim\limits_{x\to -2^+} \frac{-3(-2)+2}{-2^+ +2} = \frac{8}{0^+} = +\infty$

$\Big\}$ $\lim\limits_{x\to -2} f(x) \nexists$



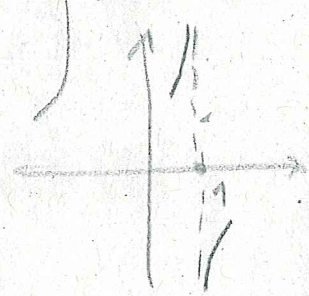
g) $\lim\limits_{x\to 1} \frac{3x}{1-x^2}$ type "$\frac{1}{0}$"

$\lim\limits_{x\to 1^+} f(x) = \underbrace{\lim\limits_{x\to 1^+} \frac{3x}{(1-x)(1+x)}} = \frac{3\cdot 1}{0^-\cdot 2} = -\infty$

⚠ on factorise le plus possible le dénominateur pour contrôler le signe

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} \frac{3x}{(1-x)(1+x)} = \frac{3}{0^+\cdot 2} = +\infty$

$\Big\}$ $\lim\limits_{x\to 1} f(x) \nexists$

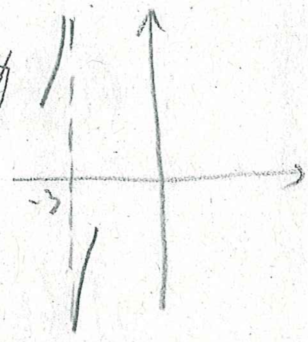


h) $\lim\limits_{x\to -3} \frac{5x}{(x+3)^3}$ type "$\frac{1}{0}$"

$\lim\limits_{x\to -3^+} f(x) = \frac{5(-3)}{(0^+)^3} = \frac{-15}{0^+} = -\infty$

$\lim\limits_{x\to -3^-} f(x) = \frac{5(-3)}{(0^-)^3} = \frac{-15}{0^-} = +\infty$

$\Big\}$ $\lim\limits_{x\to -3} f(x) \nexists$

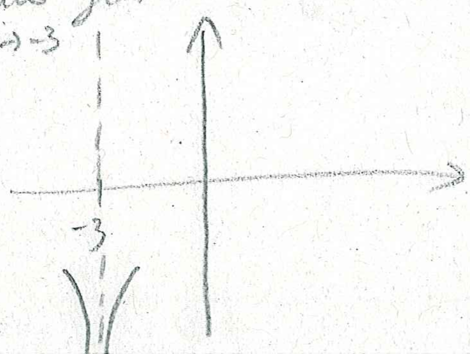


i) $\lim\limits_{x\to -3} \frac{5x}{(x+3)^2}$ type "$\frac{1}{0}$"

$\lim\limits_{x\to -3^+} f(x) = \lim\limits_{x\to -3^+} \frac{5(-3)}{(0^+)^2} = -\infty$

$\lim\limits_{x\to -3^-} f(x) = \lim\limits_{x\to -3^-} \frac{5(-3)}{(0^-)^2} = -\infty$

$\Big\}$ $\lim\limits_{x\to -3} f(x) = -\infty$



ex14

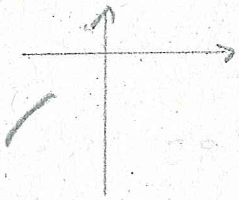
a) $f(x)=\frac{1}{x-2}$ ou $f(x)=\frac{2}{x-2}$ ...

b) $f(x)=\frac{1}{(x-2)^2}$ ou $f(x)=\frac{1}{(x-2)^4}$ ...

c) $f(x)=-\frac{1}{(x-2)^2}$ ou $f(x)=-\frac{1}{(x-2)^4}$ ...

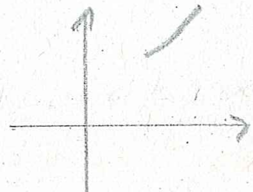
ex15

a) $\lim\limits_{x\to-\infty}(x^3-2x^2+5)$ type "$\infty-\infty$"



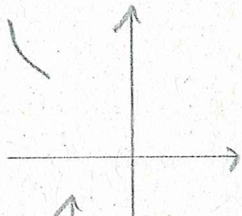
$=\lim\limits_{x\to-\infty} x^3\left(1-\frac{2}{x}+\frac{5}{x^3}\right)=(-\infty)^3\cdot\left(1-\frac{2}{-\infty}+\frac{5}{(-\infty)^3}\right)=(-\infty)\cdot 1$
$=-\infty$

b) $\lim\limits_{x\to+\infty}(x^3-2x+5)$ type "$+\infty-\infty$"



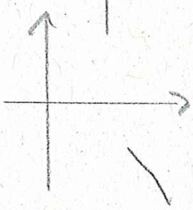
$=\lim\limits_{x\to+\infty} x^3\left(1-\frac{2}{x^2}+\frac{5}{x^3}\right)=(+\infty)^3\left(1-\frac{2}{+\infty}+\frac{5}{(+\infty)^3}\right)=(+\infty)\cdot 1$
$=+\infty$

c) $\lim\limits_{x\to-\infty}(x^4+2x)$ type "$(+\infty)+(-\infty)$"



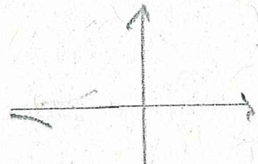
$=\lim\limits_{x\to-\infty} x^4\left(1+\frac{2}{x^3}\right)=(-\infty)^4\left(1-\frac{2}{(\infty)^3}\right)=(+\infty)\cdot(1-0)=+\infty$

d) $\lim\limits_{x\to+\infty}(x^3-x^4)$ type "$\infty-\infty$"



$=\lim\limits_{x\to+\infty} x^4\left(\frac{1}{x}-1\right)=(+\infty)^4\left(\frac{1}{\infty}-1\right)=(+\infty)(0-1)=-\infty$

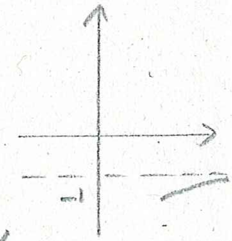
e) $\lim\limits_{x\to-\infty}\frac{-3}{x^2+5}=\frac{-3}{(-\infty)^2+5}=\frac{-3}{(+\infty)+5}=\frac{-3}{+\infty}=0$
(toujours <0)



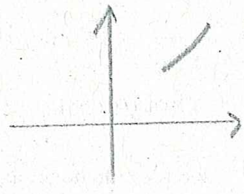
f) $\lim\limits_{x\to+\infty}\frac{3x^2+5}{1-3x^2}$ type "$\frac{\infty}{\infty}$"

$=\lim\limits_{x\to+\infty}\frac{\cancel{x^2}(3+5/x^2)}{\cancel{x^2}(1/x^2-3)}=\frac{3+0}{0-3}=-1$

ou

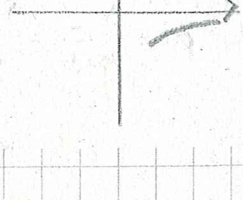


g) $\lim\limits_{x\to+\infty}\frac{x^2+x+1}{x}$ type "$\frac{\infty}{\infty}$"



$=\lim\limits_{x\to+\infty}\frac{x^{\cancel{2}}(1+1/x+1/x^2)}{\cancel{x}}=(+\infty)[1+0+0]=+\infty$

h) $\lim\limits_{x\to+\infty}\frac{3-x^2}{x^3}$ type "$\frac{\infty}{\infty}$" : $\lim\limits_{x\to+\infty}\frac{\cancel{x^2}\left(\frac{3}{x^2}-1\right)}{x^{\cancel{3}}}=\frac{\frac{3}{+\infty}-1}{+\infty}$
$=\frac{0-1}{+\infty}=\frac{-1}{+\infty}=0$



ex 16

$$\frac{f(x)}{g(x)} = \frac{x^2+2x+1}{\underbrace{x^3+2x^2-x-2}_{\text{forme développée}}} = \frac{(x+1)^2}{x^2(x+2)-(x+2)} = \frac{(x+1)^2}{(x+2)(x^2-1)}$$

$$= \frac{(x+1)^2}{\underbrace{(x+2)(x-1)(x+1)}_{\text{forme factorisée}}} = \frac{x+1}{\underbrace{(x+2)(x-1)}_{\text{si } x \neq -1}}$$

a) $\lim\limits_{x\to 1} \frac{f(x)}{g(x)} = \text{"}\frac{2}{0}\text{"} : \text{type "}\frac{1}{0}\text{"}$

$\lim\limits_{x\to 1^+} \frac{f(x)}{g(x)} = \lim\limits_{x\to 1^+} \frac{x+1}{(x+2)(x-1)} = \frac{2}{3\cdot 0^+} = +\infty$

$\lim\limits_{x\to 1^-} \frac{f(x)}{g(x)} = \frac{2}{3\cdot 0^-} = -\infty$

$\Rightarrow \lim\limits_{x\to 1} \frac{f(x)}{g(x)} \nexists$

c) $\lim\limits_{x\to+\infty} \frac{f(x)}{g(x)} = \lim\limits_{x\to+\infty} \frac{\cancel{x^2}(1+2/x+1/x^2)}{\cancel{x^3}_{x}(1+2/x-1/x^2-2/x^3)} = \frac{1+0+0}{+\infty(1+0-0-0)} = 0$

b) $\lim\limits_{x\to 1} \frac{g(x)}{f(x)} = \frac{0}{2} = 0$

d) $\lim\limits_{x\to-\infty} \frac{g(x)}{f(x)} = \lim\limits_{x\to-\infty} \frac{{}^{x}\cancel{x^3}(1+2/x-1/x^2-2/x^3)}{\cancel{x^2}(1+2/x+1/x^2)} = \frac{-\infty\cdot 1}{1} = -\infty$

e) $\lim\limits_{x\to+\infty} f(x)-g(x) = \lim\limits_{x\to+\infty} (x^2+2x+1)-(x^3+2x^2-x-2) = \lim\limits_{x\to+\infty}(-x^3+3x^2+3x+3)$

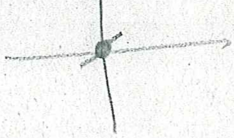
$= \lim\limits_{x\to+\infty} x^3(-1+3/x+3/x^2+3/x^3) = (+\infty)^3\cdot(-1) = (+\infty)\cdot(-1) = -\infty$

f) $\ldots = \left(\lim\limits_{x\to+\infty} \frac{f(x)}{g(x)}\right)^3 \overset{(c)}{=} 0^3 = 0$

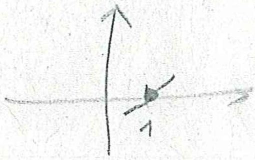
Ma3 Ch1 Exercices corrigés

ex 17

(a) $\lim\limits_{x\to 0} \frac{x^2}{x+2} \overset{\text{"direct"}}{=} \frac{0}{2} = 0$

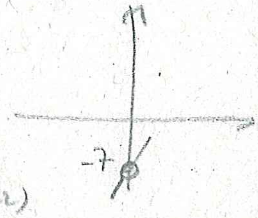


b) $\lim\limits_{x\to 1} \frac{x^2-1}{x^2+3x+5} \overset{\text{"direct"}}{=} \frac{0}{9} = 0$

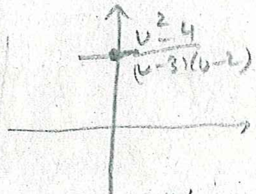


c) $\lim\limits_{x\to 0} \frac{2\cancel{x}}{\cancel{x}} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 0} 2 \overset{\text{"direct"}}{=} 2$

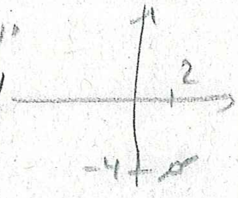
d) $\lim\limits_{x\to 0} \frac{5x^2-7x}{x} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 0} \frac{\cancel{x}(5x-7)}{\cancel{x}} = \lim\limits_{x\to 0} 5x-7 \overset{\text{"direct"}}{=} -7$



e) $\lim\limits_{x\to 2} \underbrace{\frac{u^2-4}{u^2-5u+6}}_{\text{constante par rapport à } x} \overset{\text{"direct"}}{=} \frac{u^2-4}{(u-3)(u-2)}$ $\begin{pmatrix}\text{pour} \\ u \neq 2 \\ u \neq 3\end{pmatrix}$



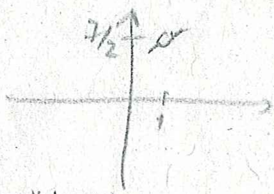
f) $\lim\limits_{u\to 2} \frac{u^2-4}{u^2-5u+6} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{u\to 2} \frac{\cancel{(u-2)}(u+2)}{(u-3)\cancel{(u-2)}} = \lim\limits_{u\to 2} \frac{u+2}{u-3} \overset{\text{"direct"}}{=} -4$



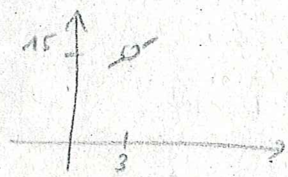
g) $\lim\limits_{x\to 1} \frac{x^3+3x^2-2x-2}{x^2-1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 1} \frac{\cancel{(x-1)}(x^2+4x+2)}{\cancel{(x-1)}(x+1)}$

↑ fact intuitive ou div. polyn par $(x-1)$

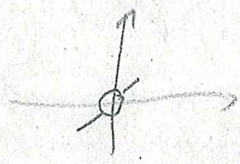
$= \lim\limits_{x\to 1} \frac{x^2+4x+2}{x+1} \overset{\text{"direct"}}{=} \frac{7}{2}$



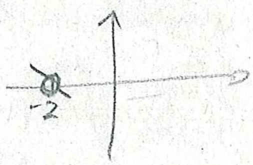
h) $\lim\limits_{x\to 3} \frac{x^3-27}{x-3} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 3} \frac{\cancel{(x-3)}(x^2+3x+9)}{\cancel{(x-3)}} = \lim\limits_{x\to 3} x^2+3x+9 \overset{\text{"direct"}}{=} 15$



i) $\lim\limits_{x\to 0} \frac{x^3}{x^2} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to 0} x \overset{\text{"direct"}}{=} 0$

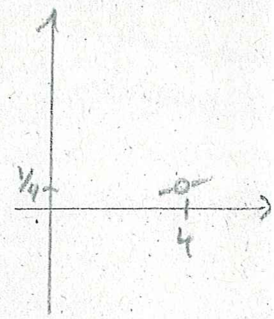


j) $\lim\limits_{x\to -2} \frac{x^2+4x+4}{x^2-x-6} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{x\to -2} \frac{(x+2)^{\cancel{2}}}{(x-3)\cancel{(x+2)}} = \lim\limits_{x\to -2} \frac{x+2}{x-3} \overset{\text{"direct"}}{=} \frac{0}{-5} = 0$



ex 18. a) $\lim\limits_{x\to 4} \dfrac{2-\sqrt{x}}{4-x}$ type $\frac{0}{0}$ avec $\sqrt{\ }$

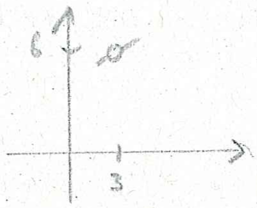
$= \lim\limits_{x\to 4} \dfrac{2-\sqrt{x}}{4-x}\cdot\dfrac{2+\sqrt{x}}{2+\sqrt{x}} = \lim\limits_{x\to 4} \dfrac{\cancel{4-x}}{\cancel{(4-x)}(2+\sqrt{x})} = \dfrac{1}{2+\sqrt{4}} = \dfrac{1}{4}$



b) $\lim\limits_{x\to 3} \dfrac{x-3}{\sqrt{x+6}-3}$ type "$\frac{0}{0}$" avec $\sqrt{\ }$

$= \lim\limits_{x\to 3} \dfrac{(x-3)}{(\sqrt{x+6}-3)}\dfrac{(\sqrt{x+6}+3)}{(\sqrt{x+6}+3)} = \lim\limits_{x\to 3} \dfrac{(x-3)(\sqrt{x+6}+3)}{(x+6)-9} = \lim\limits_{x\to 3} \dfrac{\cancel{(x-3)}(\sqrt{x+6}+3)}{\cancel{(x-3)}}$

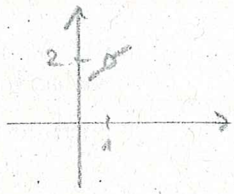
$= \lim\limits_{x\to 3}(\sqrt{x+6}+3) = \sqrt{9}+3 = 6$



c) $\lim\limits_{x\to 1} \dfrac{1-x}{\sqrt{12-3x}-3}$ type "$\frac{0}{0}$" avec $\sqrt{\ }$

$= \lim\limits_{x\to 1} \dfrac{(1-x)}{(\sqrt{12-3x}-3)}\dfrac{(\sqrt{12-3x}+3)}{(\sqrt{12-3x}+3)} = \lim\limits_{x\to 1} \dfrac{(1-x)(\sqrt{12-3x}+3)}{(12-3x)-9} = \lim\limits_{x\to 1} \dfrac{(1-x)(\sqrt{12-3x}+3)}{3-3x}$

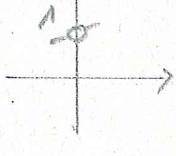
$= \lim\limits_{x\to 1} \dfrac{\cancel{(1-x)}(\sqrt{12-3x}+3)}{3\cancel{(1-x)}} = \dfrac{\sqrt{12-3}+3}{3} = \dfrac{6}{3} = 2$



d) $\lim\limits_{x\to 0} \dfrac{\sqrt{x+1}-\sqrt{x^2-x+1}}{x}$ type "$\frac{0}{0}$" avec $\sqrt{\ }$

$= \lim\limits_{x\to 0} \dfrac{(\sqrt{x+1}-\sqrt{x^2-x+1})}{x}\cdot\dfrac{(\sqrt{x+1}+\sqrt{x^2-x+1})}{(\sqrt{x+1}+\sqrt{x^2-x+1})} = \lim\limits_{x\to 0} \dfrac{(x+1)-(x^2-x+1)}{x(\sqrt{x+1}+\sqrt{x^2-x+1})}$

$= \lim\limits_{x\to 0} \dfrac{-x^2+2x}{x\cdot(\sqrt{x+1}+\sqrt{x^2-x+1})} = \lim\limits_{x\to 0} \dfrac{\cancel{x}(-x+2)}{\cancel{x}(\sqrt{x+1}+\sqrt{x^2-x+1})} = \dfrac{2}{\sqrt{1}+\sqrt{1}} = 1$



e) $\lim\limits_{x\to 2} \dfrac{x-\sqrt{x+2}}{\sqrt{4x+1}-3}$ type "$\frac{0}{0}$" avec $\sqrt{\ }$

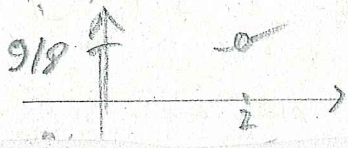
$= \lim\limits_{x\to 2} \dfrac{(x-\sqrt{x+2})}{(\sqrt{4x+1}-3)}\dfrac{(x+\sqrt{x+2})}{(x+\sqrt{x+2})} = \lim\limits_{x\to 2} \dfrac{x^2-(x+2)}{(\sqrt{4x+1}-3)(x+\sqrt{x+2})}$

$= \lim\limits_{x\to 2} \dfrac{(x-2)(x+1)}{(\sqrt{4x+1}-3)(x+\sqrt{x+2})}$ toujours pas de simplification possible ...

$= \lim\limits_{x\to 2} \dfrac{(x-2)(x+1)}{(x+\sqrt{x+2})(\sqrt{4x+1}-3)}\cdot\dfrac{(\sqrt{4x+1}+3)}{(\sqrt{4x+1}+3)} = \lim\limits_{x\to 2} \dfrac{(x-2)(x+1)(\sqrt{4x+1}+3)}{(x+\sqrt{x+2})[(4x+1)-9]}$

$= \lim\limits_{x\to 2} \dfrac{(x-2)(x+1)(\sqrt{4x+1}+3)}{(x+\sqrt{x+2})(4x-8)} = \lim\limits_{x\to 2} \dfrac{\cancel{(x-2)}(x+1)(\sqrt{4x+1}+3)}{(x+\sqrt{x+2})\,4\cancel{(x-2)}}$

$= \dfrac{3\cdot(\sqrt{9}+3)}{(2+\sqrt{4})\cdot 4} = \dfrac{3\cdot \cancel{6}^{3}}{4\cdot \cancel{4}_{2}} = \dfrac{9}{8}$



ex 19*

Soit $f(x) = 3x - \sqrt{x^2 - x + 1}$

D_f : pb si $x^2 - x + 1 < 0$

$\Delta = (-1)^2 - 4 \cdot 1 \cdot 1 < 0$

donc pas de zéro

$a = 1 > 0$

$D_f = \mathbb{R}$

on peut considérer les limites à $+\infty$ et à $-\infty$!

a) $\lim_{x \to +\infty} f(x) = 3(+\infty) - \sqrt{(\infty)^2 - \infty + 1}$

indétermination du type "$\infty - \infty$"

$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \left(3x - \sqrt{x^2(1 - 1/x + 1/x^2)}\right)$

$= \lim_{x \to +\infty} \left(3x - |x| \sqrt{1 - 1/x + 1/x^2}\right)$ car $\sqrt{x^2} = |x|$ et non $\sqrt{x^2} = x$!

$= \lim_{x \to +\infty} \left(3x - x\sqrt{1 - 1/x + 1/x^2}\right)$

$= \lim_{x \to +\infty} x\left[3 - \sqrt{1 - 1/x + 1/x^2}\right)$ on a raison à factoriser !

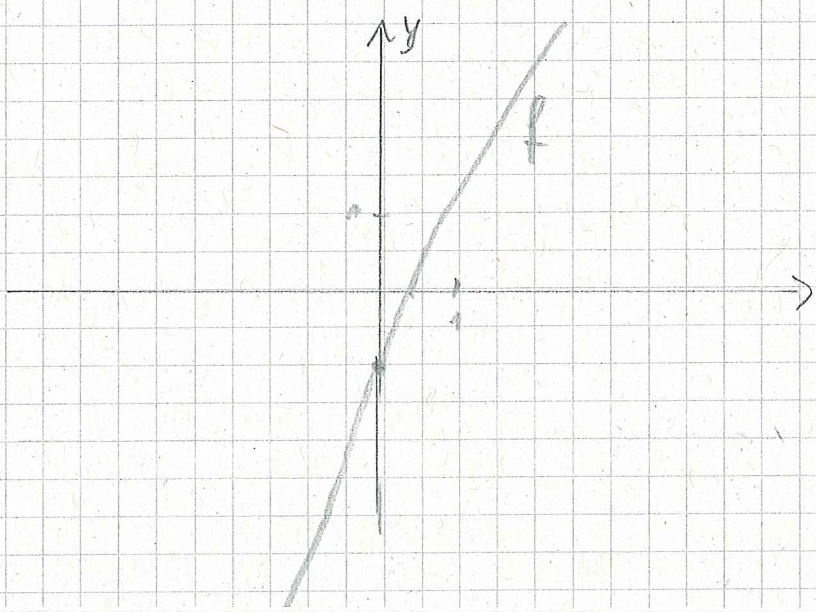
$= +\infty \left[3 - \sqrt{1 - 0 + 0}\right] = +\infty \cdot 2 = +\infty$

b) $\lim_{x \to +\infty} f(x) = \dots$

$= \lim_{x \to -\infty} \left(3x - |x|\sqrt{\dots}\right)$

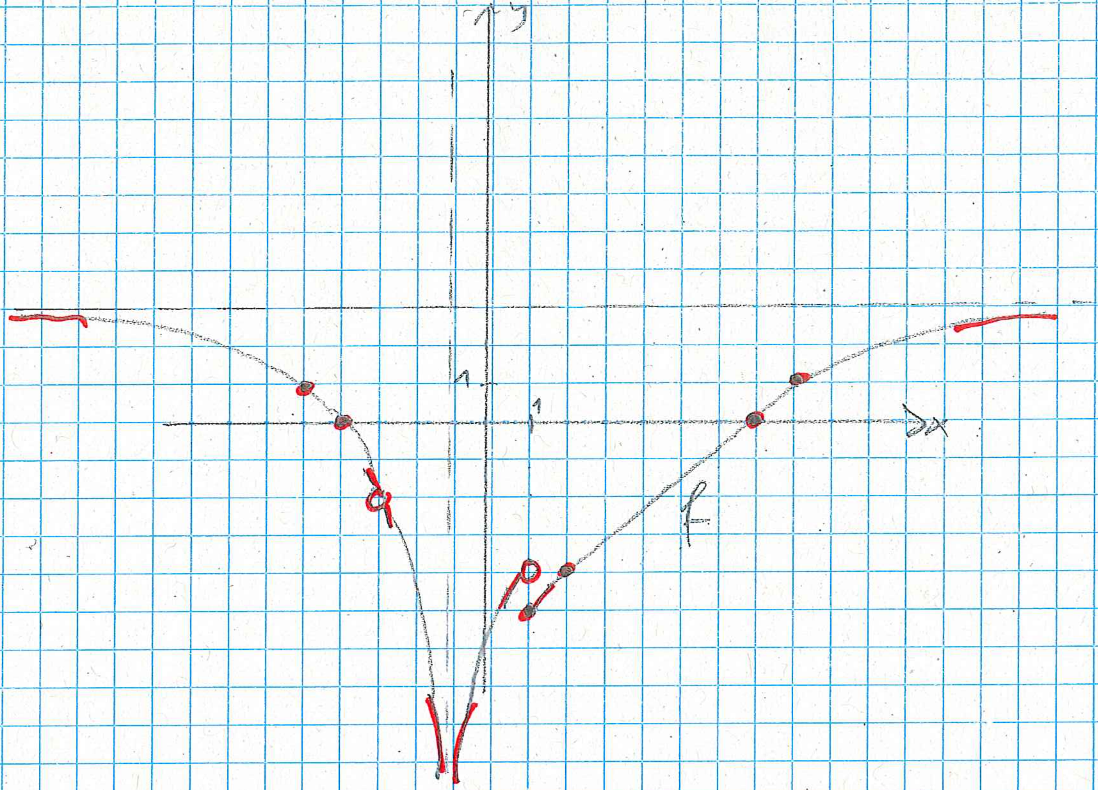
$= \lim_{x \to -\infty} \left(3x - (-x)\sqrt{\dots}\right)$ définition de $|\ |$

$= \dots = (-\infty) \cdot \left[3 + \sqrt{1 - 0 + 0}\right] = -\infty$



avec Geogebra

ex 20



ex 21

a) $\nexists$ d) 1 g) 0

b) 1 e) $\nexists$ h) $-\infty$

c) 3 f) 5

ex 22

a) $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} \frac{x-1}{x^2(x-2)} = \frac{-1}{(0^-)^2\cdot(-2)} = \frac{-1}{0^+(-2)} = +\infty$

$\lim\limits_{x\to 0^+} f(x) = \ldots = \frac{-1}{(0^+)^2(-2)} = +\infty$ d'où $\lim\limits_{x\to 0} f(x) = +\infty$

$\lim\limits_{x\to 2^+} f(x) = \ldots = \frac{1}{4\cdot 0^+} = +\infty$

$\lim\limits_{x\to 2^-} f(x) = \ldots = \frac{1}{4\cdot 0^-} = -\infty$ d'où $\lim\limits_{x\to 2} f(x) \nexists$

b) $\lim\limits_{x\to +\infty} \frac{x-1}{x^3-2x^2} = \lim\limits_{x\to +\infty} \frac{\cancel{x}(1-1/x)}{\cancel{x}\,x^2(1-2/x)} = \frac{1}{(+\infty)^2\cdot 1} = 0$

c) Par ex : $g(x) = 3x^2$ (tout $g(x) = 3x^2+bx+c$)

d) Par ex : $h(x) = 2(x-2)$ (tout $h(x) = a(x-2)$)

e) Par ex : $k(x) = x^3$

ex 25

a) Faux ; c-ex : $f(x) = \frac{-1}{x^2}$, $g(x) = \frac{1}{x^2} + 1$ et $a = 0$

on a : $\lim_{x\to 0} f(x) = -\infty$, $\lim_{x\to 0} g(x) = +\infty$ et $\lim_{x\to 0} [f(x)+g(x)] = 1$

b) $\lim_{x\to -\infty} (x^2+7) = \lim_{x\to -\infty} x^2(1+7/x^2) = (-\infty)^2(1+0) = +\infty$

$\lim_{x\to +\infty} \frac{x^2+7}{7-x} = \lim_{x\to +\infty} \frac{\cancel{x}\, x^{\cancel{2}}(1+7/x^2)}{\cancel{x}(-1+7/x)} = \frac{+\infty(1+0)}{-1+0} = +\infty$, donc c'est vrai

c) On a $\lim_{x\to 2} f(x)$; posons $x = 2+h$ càd $h = x-2$

on obtient : $\lim_{2+h\to 2} f(2+h) = \lim_{h\to 0} f(2+h)$

c'est vrai

d) Faux ; c-ex : $\lim_{x\to +\infty} x = +\infty$, on prend $f(x) = x$

$\lim_{x\to +\infty} \frac{1}{x^2} = 0$ et $\frac{1}{x^2} > 0$ pour $x \neq 0$

on prend $g(x) = \begin{cases} \frac{1}{x^2} & \text{si } x \neq 0 \\ 1 & \text{si } x = 0 \end{cases}$

d'où $\lim_{x\to +\infty} \frac{f(x)}{g(x)} = \lim_{x\to +\infty} \frac{x}{x^2} = \lim_{x\to +\infty} \frac{1}{x} \not\exists$

Exercices de calculs de limites en vrac

Ex 24

a) $\lim\limits_{x\to 5} \dfrac{x-5}{\sqrt{2x-1}-3}$: type "$\frac{0}{0}$"

$\lim\limits_{x\to 5} \dfrac{(x-5)(\sqrt{2x-1}+3)}{(\sqrt{2x-1}-3)(\sqrt{2x-1}+3)} = \lim\limits_{x\to 5} \dfrac{(x-5)(\sqrt{2x-1}+3)}{(2x-1)-9}$

$= \lim\limits_{x\to 5} \dfrac{\cancel{(x-5)}(\sqrt{2x-1}+3)}{2\cancel{(x-5)}} = \dfrac{\sqrt{2\cdot 5-1}+3}{2} = \dfrac{6}{2} = 3$

b) $\lim\limits_{x\to 0} \dfrac{\sqrt{x^2+3}-\sqrt{x^2+1}}{x^2-1} = \dfrac{\sqrt{0^2+3}-\sqrt{0^2+1}}{0^2-1} = \dfrac{\sqrt{9}-\sqrt{1}}{-1} = -2$

c) $\lim\limits_{x\to 2} \dfrac{-9x}{(4-x^2)^3}$: type "$\frac{1}{0}$"

$\lim\limits_{x\to 2^+} \dfrac{-9x}{[(2-x)(2+x)]^3} = \dfrac{-18}{[0^-\cdot 4]^3} = \dfrac{-18}{(0^-)^3} = \dfrac{-18}{0^-} = +\infty$

$\lim\limits_{x\to 2^-} \dfrac{-9x}{[(2-x)(2+x)]^3} = \dfrac{-18}{[0^+\cdot 4]^3} = \dfrac{-18}{(0^+)^3} = \dfrac{-18}{0^+} = -\infty$

donc $\lim\limits_{x\to 2} f(x) \not\exists$

d) $\lim\limits_{x\to 1} \dfrac{x^2-3x+2}{(x^2-1)^2}$: type "$\frac{0}{0}$"

$\lim\limits_{x\to 1} \dfrac{(x-2)\cancel{(x-1)}}{(x-1)^{\cancel{2}}(x+1)^2} = \lim\limits_{x\to 1} \dfrac{x-2}{(x-1)(x+1)^2}$: type "$\frac{1}{0}$"

$\lim\limits_{x\to 1^+} \dfrac{x-2}{(x-1)(x+1)^2} = \dfrac{-1}{0^+\cdot 2^2} = -\infty$

$\lim\limits_{x\to 1^-} \dfrac{x-2}{(x-1)(x+1)^2} = \dfrac{-1}{0^-\cdot 2^2} = +\infty$

donc $\lim\limits_{x\to 1} f(x) \not\exists$

e) $\lim\limits_{x \to a} \dfrac{x-a}{x-a}$: type "$\frac{0}{0}$"

$$\lim_{x \to a} \frac{\cancel{x-a}}{\cancel{x-a}} = \lim_{x \to a} \frac{1}{1} = 1$$

f) $\lim\limits_{x \to a} \dfrac{\sqrt{x}-\sqrt{a}}{x-a}$: type "$\frac{0}{0}$" avec $\sqrt{\ }$

$$\lim_{x \to a} \frac{(\sqrt{x}-\sqrt{a})}{(x-a)} \frac{(\sqrt{x}+\sqrt{a})}{(\sqrt{x}+\sqrt{a})} = \lim_{x \to a} \frac{\cancel{(x-a)}}{\cancel{(x-a)}(\sqrt{x}+\sqrt{a})} = \frac{1}{\sqrt{a}+\sqrt{a}} = \frac{1}{2\sqrt{a}}$$

(pour $a > 0$)

g) $\lim\limits_{x \to a} \dfrac{x^2-a^2}{x-a}$: type "$\frac{0}{0}$"

$$\lim_{x \to a} \frac{x^2-a^2}{x-a} = \lim_{x \to a} \frac{\cancel{(x-a)}(x+a)}{\cancel{(x-a)}} = \lim_{x \to a} (x+a) = a+a = 2a$$

h) $\lim\limits_{x \to a} \dfrac{x^3-a^3}{x-a}$: type "$\frac{0}{0}$"

$$\lim_{x \to a} \frac{x^3-a^3}{x-a} = \lim_{x \to a} \frac{\cancel{(x-a)}(x^2+ax+a^2)}{\cancel{x-a}}$$

, car

$$\begin{array}{rr|l} x^3 & -a^3 & x-a \\ -(x^3-ax^2) & & x^2+ax+a^2 \\ \hline ax^2 & -a^3 & \\ -(ax^2-a^2x) & & \\ \hline a^2x & -a^3 & \\ -(a^2x & -a^3) & \\ \hline & 0 & \end{array}$$

$$= \lim_{x \to a} (x^2+ax+a^2)$$

$$= a^2 + a \cdot a + a^2 = 3a^2$$

i) $\lim\limits_{x \to a} \dfrac{\frac{1}{x}-\frac{1}{a}}{x-a}$; type "$\frac{0}{0}$"

$$\lim_{x \to a} \frac{\frac{1}{x}-\frac{1}{a}}{x-a} = \lim_{x \to a} \frac{\frac{a-x}{xa}}{x-a} = \lim_{x \to a} \frac{a-x}{x \cdot a} \cdot \frac{1}{x-a}$$

$$= \lim_{x \to a} \frac{-\cancel{(x-a)}}{x \cdot a} \cdot \frac{1}{\cancel{(x-a)}} = \lim_{x \to a} -\frac{1}{x \cdot a} = -\frac{1}{a \cdot a} = -\frac{1}{a^2}$$

(pour $a \neq 0$)

j) $\lim_{x\to 2} \frac{x^2-3x+2}{x^2-4x+4}$ type "$\frac{0}{0}$"

$= \lim_{x\to 2} \frac{\cancel{(x-2)}(x-1)}{(x-2)^{\cancel{2}}} = \lim_{x\to 2} \frac{x-1}{x-2}$ type "$\frac{1}{0}$"

$\Rightarrow \begin{cases} \lim_{x\to 2^+} \frac{x-1}{x-2} = \frac{1}{0^+} = +\infty \\ \lim_{x\to 2^-} \frac{x-1}{x-2} = \frac{1}{0^-} = -\infty \end{cases}$ donc $\lim_{x\to 2} f(x) \nexists$

k) $\lim_{x\to 3} \frac{x^2-5x+6}{2x^2-6x}$ type "$\frac{0}{0}$"

$= \lim_{x\to 3} \frac{\cancel{(x-3)}(x-2)}{2x\cancel{(x-3)}} = \frac{1}{6}$

---

l) $\lim_{x\to \infty} \frac{(x-1)^2}{x^2+3x}$ type "$\frac{\infty}{\infty}$"

$= \lim_{x\to\infty} \frac{x^2-2x+1}{x^2+3x} = \lim_{x\to\infty} \frac{\cancel{x^2}(1-2/x+1/x^2)}{\cancel{x^2}(1+3/x)} = \frac{1-0+0}{1+0} = 1$

m) $\lim_{x\to 1} \frac{\sqrt{x^2+x}-\sqrt{2}}{x-1}$ type "$\frac{0}{0}$" avec $\sqrt{\ }$

$= \lim_{x\to 1} \frac{(\sqrt{x^2+x}-\sqrt{2})}{(x-1)} \frac{(\sqrt{x^2+x}+\sqrt{2})}{(\sqrt{x^2+x}+\sqrt{2})} = \lim_{x\to 1} \frac{(x^2+x)-(2)}{(x-1)(\sqrt{x^2+x}+\sqrt{2})}$

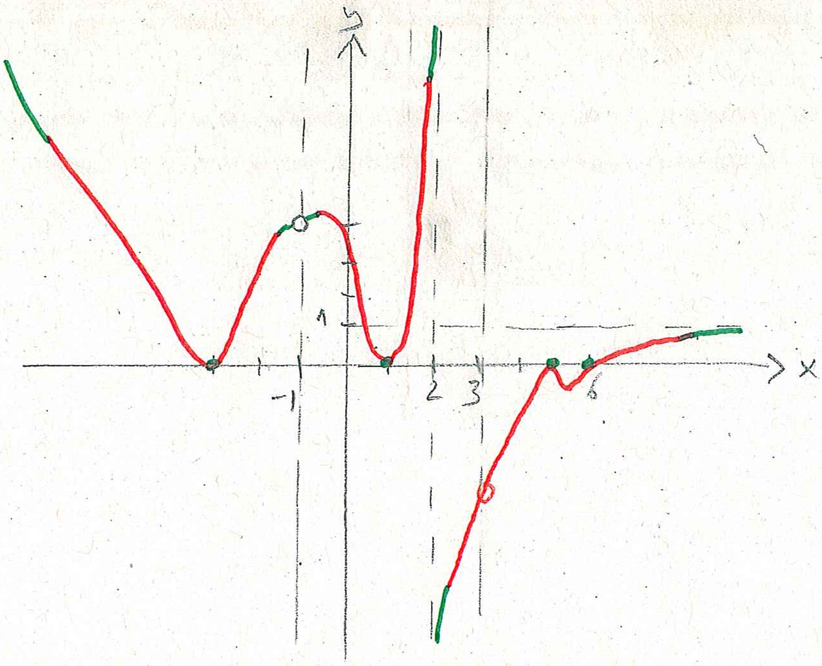
$= \lim_{x\to 1} \frac{(x+2)\cancel{(x-1)}}{\cancel{(x-1)}(\sqrt{x^2+x}+\sqrt{2})} = \frac{3}{\sqrt{2}+\sqrt{2}} = \frac{3}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2\cdot 2} = \frac{3\sqrt{2}}{4}$

n) $\lim_{x\to 3} \frac{x^2+2x-15}{x^2+8x+15} = \frac{9+6-15}{9+24+15} = \frac{0}{48} = 0$

o) $\lim_{x\to -3} \frac{x+1}{(x+3)^2}$ type "$\frac{1}{0}$"

$\Rightarrow \begin{cases} \lim_{x\to -3^+} \frac{x+1}{(x+3)^2} = \frac{-2}{(0^+)^2} = \frac{-2}{0^+} = -\infty \\ \lim_{x\to -3^-} \frac{x+1}{(x+3)^2} = \frac{-2}{(0^-)^2} = \frac{-2}{0^+} = -\infty \end{cases}$ donc $\lim_{x\to -3} f(x) = -\infty$

ex 25



en vert ce qui est demandé

on complète avec une proposition en rouge (plusieurs possibilités)