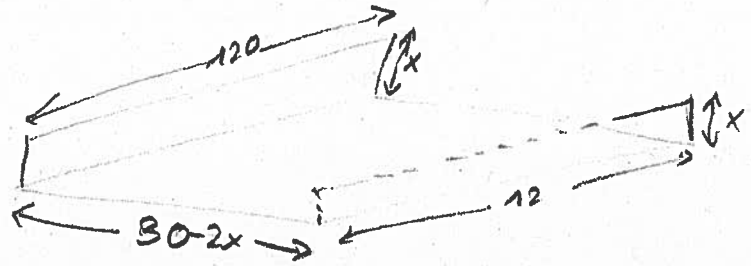
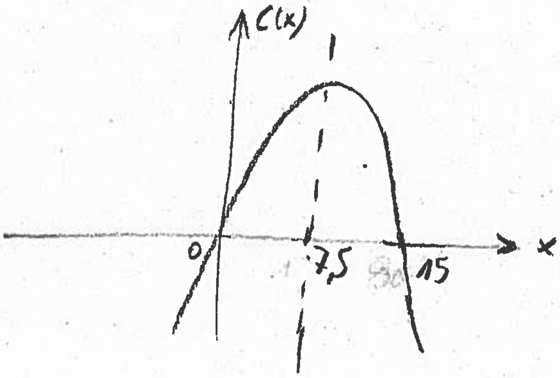
3I La1 Dt02 — Corrigé du travail 3D n°1

ex1 [/13]



a) $C(x) = (30-2x)\cdot x \cdot 120$ (4) b) $D_{app} =]0;15[$ (2)

c)



(2)

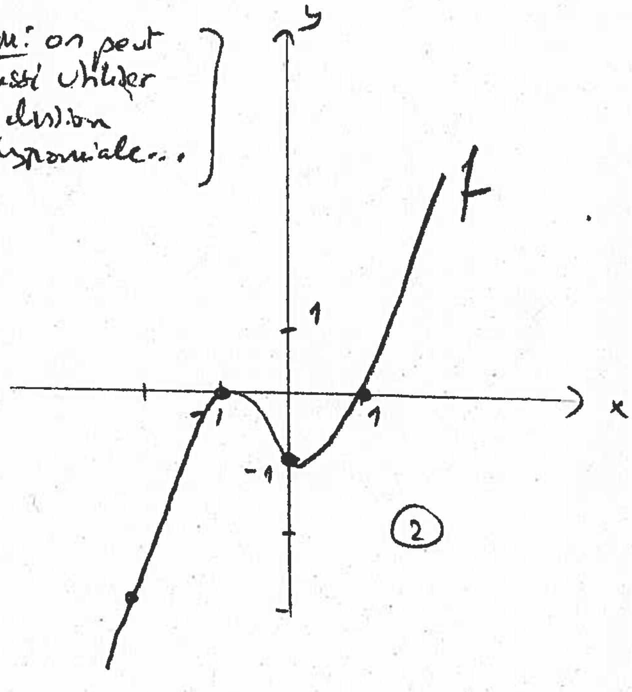
d) La contenance $C(x)$ maximale est atteinte pour $x = 7,5$ cm (3)

e) Cette contenance vaut alors $C(7,5) = (30-15)\cdot 7,5 \cdot 120$
$= 13500$ cm² (2)

ex2 [/13]

a) $f(x) = x^3 + x^2 - x - 1$
$= x^2(x+1) - (x+1)$
$= (x+1)(x^2-1)$
$= (x+1)(x+1)(x-1)$ (3)

[rem: on peut aussi utiliser la division polynomiale...]



(2)

$D_f = \mathbb{R}$ (0,5)

$Z_f = \{\pm 1\}$ (2)

$f(0) = -1$ (0,5)

$f(2) = 8+4-2-1 = 9$
$f(-2) = -8+4+2-1 = -3$ (1)

b) non! par exemple, $f(\frac{1}{2}) = -9/8$
$f(-\frac{1}{2}) = -7/8$
} il n'y a donc pas de symétrie d'axe $x=0$

(4)

[/21]

ex 3

a) $k(x) = -\ln(x+1) - 1$

D_k : problème si $x+1 \leq 0$
$x \leq -1$

$D_k =]-1 ; +\infty[$ 2

Z_k : $-\ln(x+1) - 1 = 0$

$\Leftrightarrow -\ln(x+1) = 1$

$\Leftrightarrow \ln(x+1) = -1$

$\Leftrightarrow x+1 = e^{-1}$

$\Leftrightarrow x = e^{-1} - 1 = \frac{1}{e} - 1 \cong -0{,}63$

$Z_k = \left\{ \frac{1}{e} - 1 \right\} \cong \{-0{,}63\}$ 3

$k(0) = -\ln(1) - 1 = 0 - 1 = -1$ 1

$k(1) = -\ln(2) - 1 \cong -1{,}69$ 1

b)

x	$k(x)$
$-0{,}5$	$-0{,}31$
$-0{,}6$	$-0{,}08$
$-0{,}7$	$0{,}20$
$-0{,}8$	$0{,}61$
$-0{,}9$	$1{,}30$
$-0{,}99$	$3{,}61$
$-0{,}999$	$5{,}91$
$-0{,}9999$	$8{,}21$

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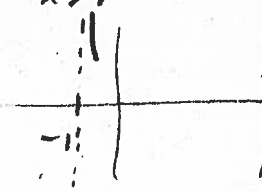
c) $\lim_{x \to -1^+} k(x) = +\infty$



mais on pourrait se tromper car les valeurs n'augmentent pas très vite...

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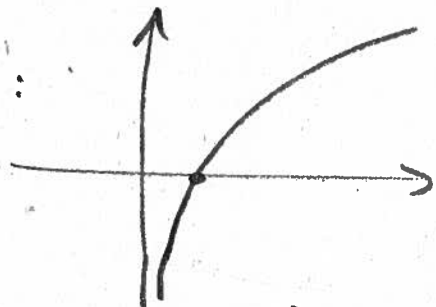
d) On connaît $\ln(x)$:



qui a $\lim_{x \to 0^+} \ln(x) = -\infty$

donc, comme k est construite à partir de ln, cela confirme la conjecture

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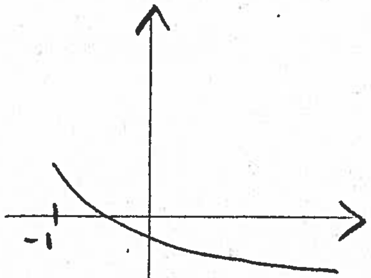
e)



le comportement près de $x = -1$ n'est très bien représenté...

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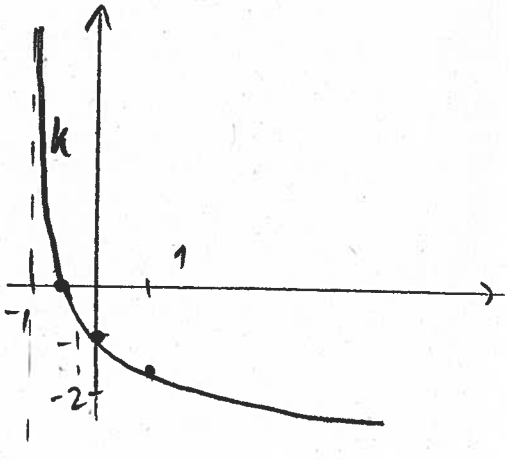
f)



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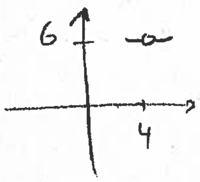
/20 Ex 4

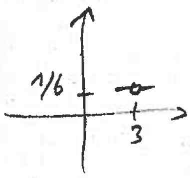


a) $\lim\limits_{x\to 4} \dfrac{4-x}{\sqrt{13-x}-3}$ type "$\frac{0}{0}$"

$= \lim\limits_{x\to 4} \dfrac{(4-x)(\sqrt{13-x}+3)}{(\sqrt{13-x}-3)(\sqrt{13-x}+3)} = \lim\limits_{x\to 4} \dfrac{(4-x)(\sqrt{13-x}+3)}{(13-x)-9}$

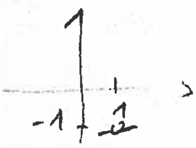
$= \lim\limits_{x\to 4} \dfrac{\cancel{(4-x)}(\sqrt{13-x}+3)}{\cancel{(4-x)}} = \sqrt{13-4}+3 = \sqrt{9}+3 = 6$ 4

b) $\lim\limits_{x\to 0} \dfrac{\sqrt{x^2+1}+\sqrt{2}}{\sqrt{x^2+x}-\sqrt{x+1}} = \dfrac{\sqrt{1}+\sqrt{2}}{\sqrt{0}-\sqrt{1}} = -(1+\sqrt{2})$ 3



c) $\lim\limits_{x\to 3} \dfrac{x^2-5x+6}{2x^2-6x}$ type "$\frac{0}{0}$" 3

$= \lim\limits_{x\to 3} \dfrac{\cancel{(x-3)}(x-2)}{2x\cancel{(x-3)}} = \dfrac{3-2}{6} = \dfrac{1}{6}$

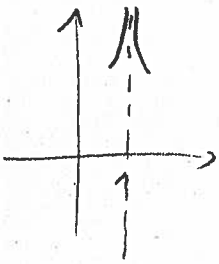


d) $\lim\limits_{x\to 1} \dfrac{x-1}{1-x}$ type "$\frac{0}{0}$" 3

$= \lim\limits_{x\to 1} \dfrac{\cancel{x-1}}{-\cancel{(x-1)}} = \lim\limits_{x\to 1} \dfrac{1}{-1} = \lim\limits_{x\to 1} -1 = -1$

e) $\lim\limits_{x\to 1} \dfrac{1-x}{(1-x)^3}$ type "$\frac{0}{0}$"

$= \lim\limits_{x\to 1} \dfrac{\cancel{1-x}}{(1-x)^{\cancel{3}}} = \lim\limits_{x\to 1} \dfrac{1}{(1-x)^2}$ type "$\frac{1}{0}$"



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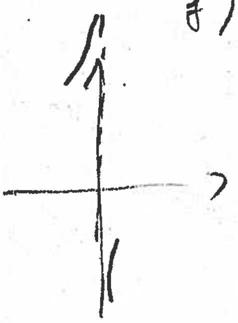
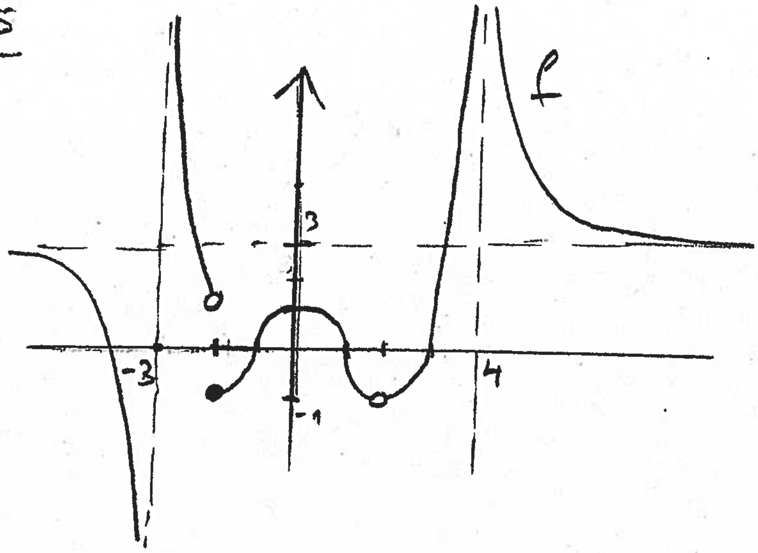
$\left.\begin{array}{l} \lim\limits_{x\to 1^+} \dfrac{1}{(1-x)^2} = \dfrac{1}{(0^-)^2} = +\infty \\ \lim\limits_{x\to 1^-} \dfrac{1}{(1-x)^2} = \dfrac{1}{(0^+)^2} = +\infty \end{array}\right\}$ donc $\lim\limits_{x\to 1} f(x) = +\infty$

f) $\lim\limits_{x\to 0} \dfrac{x^2-5x+6}{2x^2-6x}$ type "$\frac{1}{0}$"



$\left.\begin{array}{l} \lim\limits_{x\to 0^+} \dfrac{\cancel{(x-3)}(x-2)}{2x\cancel{(x-3)}} = \dfrac{-2}{2\cdot 0^+} = -\infty \\ \lim\limits_{x\to 0^-} \dfrac{\cancel{(x-3)}(x-2)}{2x\cancel{(x-3)}} = \dfrac{-2}{2 0^-} = +\infty \end{array}\right\}$ donc $\lim\limits_{x\to 1} f(x) \not\exists$ 3 ex 5

[/8]



a) $\lim\limits_{x\to 4} f(x) = +\infty$

b) $\lim\limits_{x\to -3} f(x) \nexists$

c) $\lim\limits_{x\to 0} f(x) = 1$

d) $\lim\limits_{x\to 2} f(x) = -1$

e) $\lim\limits_{x\to -1} f(x) = 0$ $\left(\lim\limits_{x\to -2} f(x) \nexists \right.$

f) $\lim\limits_{x\to -1^+} f(x) = 0$ $\left(\lim\limits_{x\to -2^+} f(x) = -1 \right.$

g) $\lim\limits_{x\to 1^-} f(x) = 0$ $\left(\lim\limits_{x\to -2^-} f(x) = 1 \right.$

h) $\lim\limits_{x\to +\infty} f(x) = 3$