MA40F.02 Travail écrit n°2 Corrigé

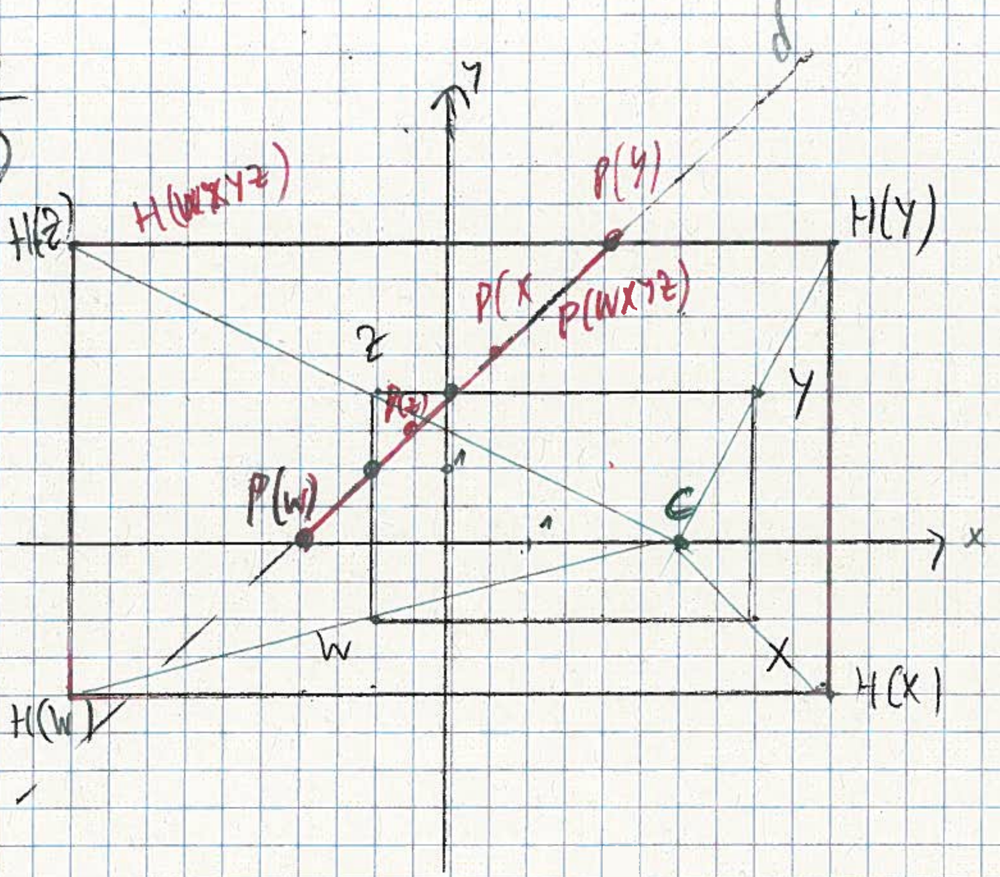
total : 61,5

ex 1

/7

a) b)

2) 2



c) $H(\vec{0}) = (6;0) \neq \vec{0}$, donc H pas linéaire
$P(\vec{0}) = (-1;1) \neq \vec{0}$, " P " "

3

$\left[\frac{1}{5}\right]$

ex2

$\alpha, \beta \in \mathbb{R}\ ;\ \vec{v}, \vec{w} \in \mathbb{R}^2.$

a) $F(\alpha\vec{v}+\beta\vec{w}) \overset{?}{=} \alpha F(\vec{v})+\beta F(\vec{w})\quad \forall \alpha,\beta \in \mathbb{R},\ \forall \vec{v},\vec{w} \in \mathbb{R}$

$\Leftrightarrow F\left(\alpha\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}+\beta\begin{pmatrix} w_1 \\ w_2 \end{pmatrix}\right) \overset{?}{=} \alpha F\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}+\beta F\begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$

$\Leftrightarrow F\begin{pmatrix} \alpha v_1+\beta w_1 \\ \alpha v_2+\beta w_2 \end{pmatrix} \overset{?}{=} \alpha F\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}+\beta F\begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$

$\Leftrightarrow \begin{pmatrix} 2(\alpha v_1+\beta w_1)+\alpha v_2+\beta w_2 \\ -3(\alpha v_2+\beta w_2) \end{pmatrix} \overset{?}{=} \alpha\begin{pmatrix} 2v_1+w_1 \\ -3v_2 \end{pmatrix}+\beta\begin{pmatrix} 2w_1+w_2 \\ -3w_2 \end{pmatrix}$

$\Leftrightarrow \begin{pmatrix} 2\alpha v_1+2\beta w_1+\alpha v_2+\beta w_2 \\ -3\alpha v_2-3\beta w_2 \end{pmatrix} \overset{?}{=} \begin{pmatrix} \alpha 2v_1+\alpha w_1+\beta 2w_1+\beta w_2 \\ \alpha(-3)v_2+\beta(-3)w_2 \end{pmatrix}$ OK

donc F est linéaire (4)

b) $K(\vec{0}) = K(0;0) = (0;1) \neq (0;0)$ donc K pas linéaire (2)

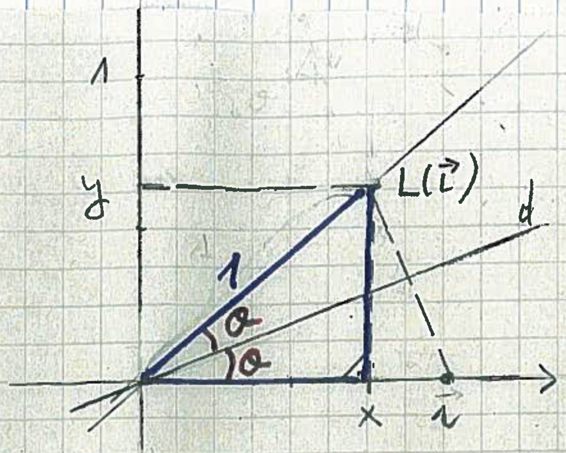
c) $G(2\cdot(0;1)) \overset{?}{=} 2\cdot G(0;1)$

$\Leftrightarrow G(0;2) \overset{?}{=} 2\cdot G(0;1)$

$\Leftrightarrow (6;-4) \overset{?}{=} 2\cdot(3;-1)$

$\Leftrightarrow (6;-4) \overset{?}{=} (6;-2)$ non, donc G pas linéaire (3)

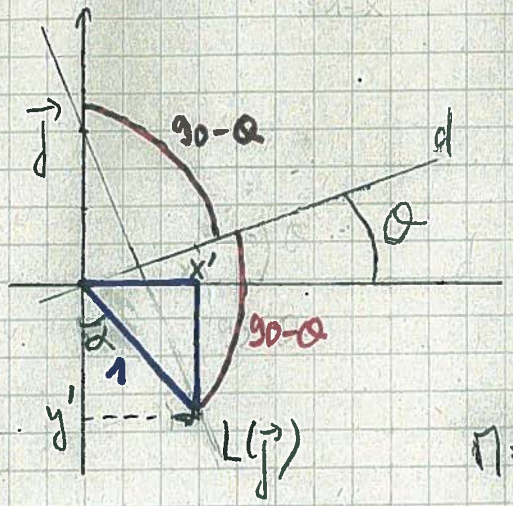
[8] ex3



On observe dans le triangle bleu :

$\sin(2\theta) = \frac{y}{1} = y$

$\cos(2\theta) = \frac{x}{1} = x$



On a : $\alpha + (90-\theta) = 90+\theta$ d'où $\alpha = 2\theta$

$y' = -\cos(2\theta)$

$x' = \sin(2\theta)$

$$M = \begin{pmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{pmatrix}$$

[/7]

ex 4

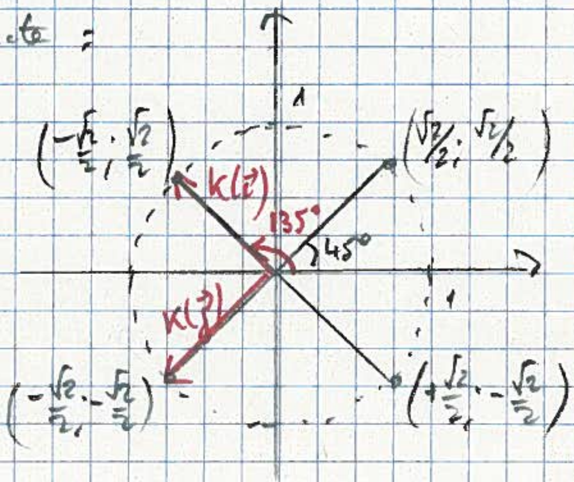
(a) $\left.\begin{aligned} F(\vec{i}) &= -\vec{i} \\ F(\vec{j}) &= -\vec{j} \\ F(\vec{k}) &= -\vec{k} \end{aligned}\right\}$ homothétie de centre $O(0;0;0)$ et de rapport -1

(3)

(b) on reconnaît une valeur exacte :

$\vec{i}$ et $\vec{j}$ ont effectué une rotation de $135°$

$\Rightarrow$ K : rotation de centre $O(0;0)$ et d'angle $135°$



(4)

[/12]

ex 5

a) Matrice de F : $F(\vec{i}) = F\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}2\\0\end{pmatrix}$

$F(\vec{j}) = F\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\-1\end{pmatrix}$ $\Rightarrow M_F = \begin{pmatrix}2 & 1\\0 & -1\end{pmatrix}$

$$F \circ L \begin{pmatrix}15\\29\end{pmatrix} = M_F \cdot M_L \begin{pmatrix}15\\29\end{pmatrix} = \begin{pmatrix}2 & 1\\0 & -1\end{pmatrix} \cdot \begin{pmatrix}0 & 0\\-1 & 2\end{pmatrix} \begin{pmatrix}15\\29\end{pmatrix}$$

$$= \begin{pmatrix}2 & 1\\0 & -1\end{pmatrix} \begin{pmatrix}0 \cdot 15 + 0 \cdot 29\\(-1) \cdot 15 + 2 \cdot 29\end{pmatrix} = \begin{pmatrix}2 & 1\\0 & -1\end{pmatrix} \begin{pmatrix}0\\43\end{pmatrix}$$

$$= \begin{pmatrix}43\\-43\end{pmatrix}$$

(5)

b) Inverse de F : $\det M_F = 2(-1) - 0 \cdot 1 = -2$

$$(M_F)^{-1} = \frac{1}{-2} \begin{pmatrix}-1 & -1\\0 & 2\end{pmatrix} = \begin{pmatrix}1/2 & 1/2\\0 & -1\end{pmatrix}$$

$$F^{-1}\begin{pmatrix}12\\-13\end{pmatrix} = \begin{pmatrix}1/2 & 1/2\\0 & -1\end{pmatrix}\begin{pmatrix}12\\-13\end{pmatrix} = \begin{pmatrix}-1/2\\13\end{pmatrix}$$

(4)

c) $\det M_L = 0 \cdot 2 - 0(-1) = 0$ donc $(M_L)^{-1}$ n'existe pas

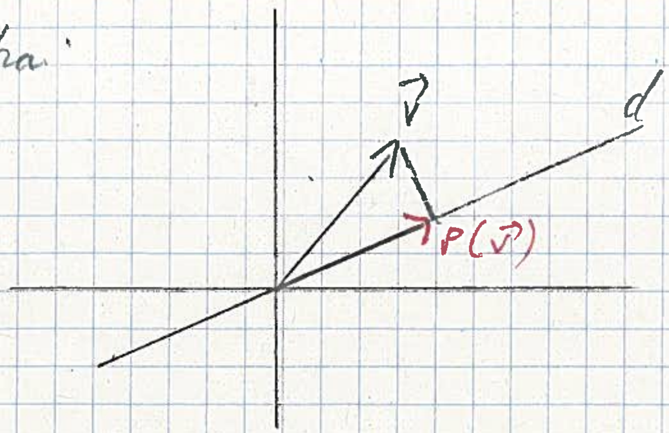
donc L^{-1} n'existe pas

(3)

ex 6

[/12]

(a) Vrai



Comme $P(\vec{v}) \in d$ on a $P(P(\vec{v})) = P(\vec{v})$

et donc $P \circ P(\vec{v}) = P(\vec{v})$

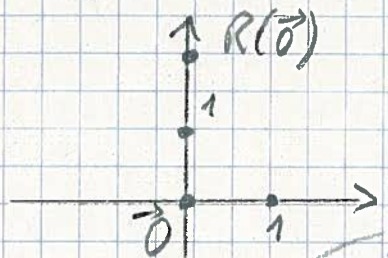
1+3

(b) Faux

c-ex: Rotation de 180° autour de $(0;1)$

$R(\vec{0}) = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \neq \vec{0}$

donc R pas linéaire



1+3

(c) Faux

c-ex: appl. L de l'ex 5a) !

1+3

ex 7

[16]

(a) $F(\vec{i}) = F\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

(b) $F(\vec{j}) = F\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ $\rightarrow M_F = \begin{pmatrix} 2 & 1 \\ 1 & -1 \\ 1 & 1 \end{pmatrix}$ 4

(c) $F\begin{pmatrix} -2 \\ 1 \end{pmatrix} = \underbrace{\begin{pmatrix} 2 & 1 \\ 1 & -1 \\ 1 & 1 \end{pmatrix}}_{3\times 2} \underbrace{\begin{pmatrix} -2 \\ 1 \end{pmatrix}}_{2\times 1} = \underbrace{\begin{pmatrix} 2(-2) + 1\cdot 1 \\ 1(-2) + (-1)\cdot 1 \\ 1(-2) + 1\cdot 1 \end{pmatrix}}_{3\times 1} = \begin{pmatrix} -3 \\ -3 \\ -1 \end{pmatrix}$

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