4N - 4Ra1DF02 - Corrigé travail su' n°2

ex 1 $f(x) = x\ln^2(x)$

/14 (a) $f'(x) = 1\cdot \ln^2(x) + x\left([\ln(x)]^2\right)'$

$= \ln^2(x) + x\cdot 2\ln(x)\cdot(\ln(x))'$

$= \ln^2(x) + \cancel{x}\cdot 2\cdot \ln(x)\cdot \frac{1}{\cancel{x}}$

$= \ln^2(x) + 2\ln(x)$

$= \ln(x)\left[\ln(x)+2\right]$ 2

(b) On veut $f'(x)=0 \Leftrightarrow \ln(x)(\ln(x)+2)=0$

$\ln(x)=0$ ou $\ln(x)+2=0$

$x=1$ $\qquad \ln(x)=-2$

$f(1)=0$ $\boxed{I_1(1;0)}$ $\qquad x=e^{-2}=\frac{1}{e^2}$ 4

$f(e^{-2})=e^{-2}(-2)^2=\frac{4}{e^2}$

$\boxed{I_2\left(\frac{1}{e^2};\frac{4}{e^2}\right)}$

(c) $t: y=ax+b$ où $a=f'(e)$

$= \ln(e)[\ln(e)+2]$

$= 1[1+2]$

$= 3$ 2

donc $y=3x+b$

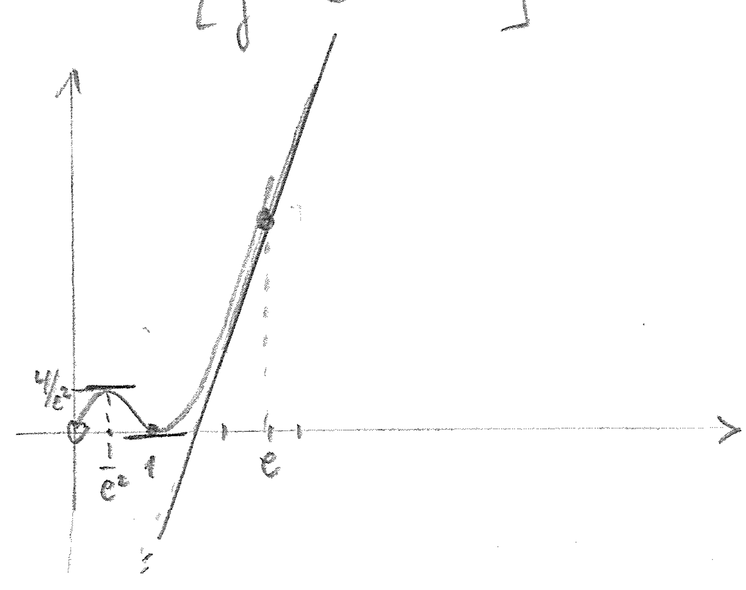
$(e;f(e))\in t \Leftrightarrow f(e)=3\cdot e+b$

$\Leftrightarrow e\cdot\underbrace{\ln^2(e)}_{1}=3e+b$

$\Leftrightarrow b=-2e$ 2

donc $\boxed{y=3x-2e}$

(d)



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ex 2 [/8]

$\mathcal{Z}_f = \{-4; 0; 1\}$ et $f(x) = x(x-1)(x+4) = x^3+3x^2-4x$

$$A = \int_{-4}^{0} f(x)\,dx - \int_{0}^{1} f(x)\,dx \quad (2)$$

$$= \int_{-4}^{0} x^3+3x^2-4x\,dx - \int_{0}^{1} x^3+3x^2-4x\,dx \quad (2)$$

$$= \left(\frac{x^4}{4}+\cancel{3}\frac{x^3}{\cancel{3}}-\overset{2}{\cancel{4}}\frac{x^2}{\cancel{2}}\right)\Big|_{-4}^{0} - \left(\frac{x^4}{4}+\cancel{3}\frac{x^3}{\cancel{3}}-\overset{2}{\cancel{4}}\frac{x^2}{\cancel{2}}\right)\Big|_{0}^{1} \quad (2)$$

$$= \left(0-[\cancel{64}-\cancel{64}-32]\right)-\left(\left[\frac{1}{4}+1-2\right]-0\right) \quad (2)$$

$$= 32+\frac{3}{4} = \frac{131}{4}$$

ex 3 [/15]

- $f(x) = 3x\,e^{-2x^2} = \frac{3}{-4}\left[e^{-2x^2}\cdot(-4x)\right]$

$$\Rightarrow F(x) = -\frac{3}{4}\,e^{-2x^2}+C \quad (3)$$

- $F(0)=2 \iff -\frac{3}{4}\underbrace{e^{-2\cdot 0}}_{=1}+C = 2 \iff C = 2+\frac{3}{4} = \frac{11}{4}$

$$\Rightarrow F(x) = -\frac{3}{4}e^{-2x^2}+\frac{11}{4} \quad (2)$$

ex 4 [/8]

a) $\int(3x-1)\cos\left(\frac{x}{2}\right)dx =$

$$\left[\begin{array}{ll} f'(x) = \cos\left(\frac{x}{2}\right) & \Rightarrow f(x) = 2\sin\left(\frac{x}{2}\right) \\ g(x) = 3x-1 & \Rightarrow g'(x) = 3 \end{array}\right]$$

$$= 2\sin\left(\frac{x}{2}\right)(3x-1) - \int 3\cdot 2\sin\left(\frac{x}{2}\right)dx \;(+C)$$

$$= 2(3x-1)\sin\left(\frac{x}{2}\right) - 6\int \sin\left(\frac{x}{2}\right)dx \;(+C)$$

$$= 2(3x-1)\sin\left(\frac{x}{2}\right) - 6\left[-\cos\left(\frac{x}{2}\right)\cdot 2\right] \;(+C)$$

$$= 2(3x-1)\sin\left(\frac{x}{2}\right) + 12\cos\left(\frac{x}{2}\right) + C \quad (5)$$

b) $\underline{I} = 2(3x-1)\sin\left(\frac{x}{2}\right)+12\cos\left(\frac{x}{2}\right)\Big|_{0}^{2\pi}$

$$= \left[2(6\pi-1)\underline{\sin(\pi)}+12\cos(\pi)\right]-\left[0+12\cos(0)\right] = -24$$

ex 5

[/12]

c) $f(x)=\frac{1}{\sqrt{-x}}-\frac{3}{x}=(-x)^{-1/2}-3\cdot\frac{1}{x}=-\left[(-x)^{-1/2}(-1)\right]-3\frac{1}{x}$

$\Rightarrow F(x)=-\left[\frac{(-x)^{1/2}}{1/2}\right]-3\ln|x|$

$=-2\sqrt{-x}-3\ln|x|$ (4)

$I=F(x)\Big|_{-e^2}^{-1}=\left(-2\sqrt{1}-3\underbrace{\ln|-1|}_{=0}\right)-\left(-2\sqrt{e^2}-3\ln|e^2|\right)$

$=-2+2\sqrt{e^2}+3\ln(e^2)=-2+2e+3\cdot 2=4+2e$

a) $f(x)=\frac{x+2}{x^2+4x+1}=\frac{1}{2}\left(\frac{2x+4}{x^2+4x+1}\right)$

$\Rightarrow F(x)=\frac{1}{2}\ln|x^2+4x+1|$ (4)

$I=F(x)\Big|_{-1}^{0}=\frac{1}{2}\underbrace{\ln|1|}_{=0}-\frac{1}{2}\ln|-2|=-\frac{1}{2}\ln 2$

b) $f(x)=\frac{x+2}{(x^2+4x+1)^2}=(x^2+4x+1)^{-2}\cdot(x+2)$

$=\frac{1}{2}\left[(x^2+4x+1)^{-2}\cdot(2x+4)\right]$

$\Rightarrow F(x)=\frac{1}{2}\left[\frac{(x^2+4x+1)^{-1}}{-1}\right]=-\frac{1}{2}\,\frac{1}{x^2+4x+1}$ (4)

$I=F(x)\Big|_{-1}^{0}=-\frac{1}{2}\cdot 1-\left(-\frac{1}{2}\cdot\frac{1}{-2}\right)=-\frac{1}{2}-\frac{1}{4}=-\frac{3}{4}$