4N.- LIPO1D603 - Corrigé travail 90' n°2

ex1 $f(x) = 2x \ln^3(x)$

/14

a) $f'(x) = 2\ln^3(x) + 2x\left([\ln(x)]^3\right)'$

$= 2\ln^3(x) + 2x \cdot 3(\ln(x))^2 \cdot [\ln(x)]'$

$= 2\ln^3(x) + 6\cancel{x}\ln^2(x) \cdot \frac{1}{\cancel{x}}$

$= 2\ln^3(x) + 6\ln^2(x)$

$= 2\ln^2(x)(\ln(x) + 3)$ (2)

c) $t: y = ax + b$ où $a = f'(e)$

$= 2\underbrace{\ln^2(e)}_{=1}(\underbrace{\ln(e)}_{=1} + 3)$ (2)

$= 8$

donc $y = 8x + b$

$(e; f(e)) \in t \Leftrightarrow f(e) = 8 \cdot e + b \Leftrightarrow 2e\underbrace{\ln^3(e)}_{=1} = 8e + b$

$\Leftrightarrow b = -6e$ (2)

donc $\boxed{y = 8x - 6e}$

b) $f'(x) = 0 \Leftrightarrow 2\ln^2(x)(\ln(x) + 3) = 0$

$\ln^2(x) = 0$ ou $\ln(x) + 3 = 0$ (4)

$\ln(x) = 0$ $\qquad$ $\ln(x) = -3$

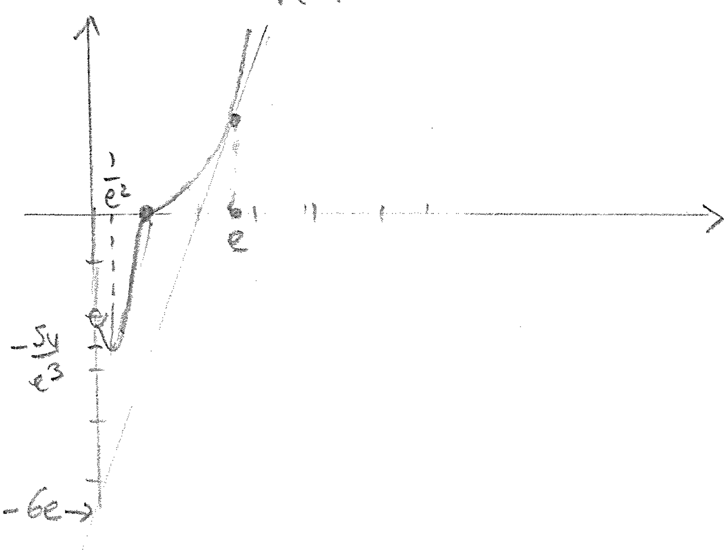
$x = 1$ $\qquad$ $x = e^{-3} = \frac{1}{e^3}$

$f(1) = 0$ $\qquad$ $f\left(\frac{1}{e^3}\right) = \frac{2}{e^3}\ln^3(e^{-3}) = \frac{2}{e^3}(-3)^3$

$I_1(1; 0)$ $\qquad$ $= -54/e^3 \simeq -2{,}71$

$I_2\left(\frac{1}{e^3}; \frac{-54}{e^3}\right)$

d)



(4)

ex 2 a) $f(x) = g(x) \Leftrightarrow \frac{(x-2)^2}{2} = x+2$

[/7]

$\Leftrightarrow x^2 - 4x + 4 = 2x + 4$

$\Leftrightarrow x^2 - 6x = 0$

$\Leftrightarrow x(x-6) = 0$

$x = 0 \quad \vee \quad x = 6$

$g(0) = 2 \qquad g(6) = 8$

$I_1(0;2) \qquad I_2(6;8)$ (3)

b) $A = \int_0^6 g(x) - f(x)\,dx = \int_0^6 (x+2) - \frac{(x-2)^2}{2}\,dx$

$= \int_0^6 x+2 - \left(\frac{x^2 - 4x + 4}{2}\right) dx = \int_0^6 \frac{2x + \cancel{4} - x^2 + 4x - \cancel{4}}{2}\,dx$ (2)

$= \int_0^6 \frac{6x - x^2}{2}\,dx = \frac{1}{2}\left(6\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^6 = \frac{1}{2}\left(3x^2 - \frac{x^3}{3}\right)\Big|_0^6$

$= \frac{1}{2}(108 - 72) - \frac{1}{2}(0-0) = 18$ (2)

ex 3 • $f(x) = -3x\,e^{-2x^2+2} = \frac{-3}{-4}\left[e^{-2x^2+2} \cdot (-4x)\right]$

[/5] $\hookrightarrow F(x) = \frac{3}{4}\left[e^{-2x^2+2}\right] + C$ (3)

• $F(1) = 2 \Leftrightarrow \frac{3}{4}e^0 + C = 2 \Leftrightarrow C = 2 - \frac{3}{4} = \frac{5}{4}$

• $F(x) = \frac{3}{4}\left[e^{-2x^2+2}\right] + \frac{5}{4}$ (2)

ex 4

[/8] a) $\int (1-2x)\sin\left(\frac{x}{3}\right)dx =$

$$\left[\begin{array}{l} f'(x)=\sin\left(\frac{x}{3}\right) \Rightarrow f(x)=-\cos\left(\frac{x}{3}\right)\cdot 3 \\ g(x)=1-2x \Rightarrow g'(x)=-2 \end{array}\right.$$

$$= -\cos\left(\frac{x}{3}\right)\cdot 3(1-2x) - \int (-2)\left(-3\cos\left(\frac{x}{3}\right)\right)dx$$

$$= -3(1-2x)\cos\left(\frac{x}{3}\right) - 6\int \cos\left(\frac{x}{3}\right)dx$$

$$= -3(1-2x)\cos\left(\frac{x}{3}\right) - 6\left[\sin\left(\frac{x}{3}\right)\cdot 3\right] + C \qquad (5)$$

b) $$I = -3(1-2x)\cos\left(\frac{x}{3}\right) - 18\sin\left(\frac{x}{3}\right)\Big|_0^{3\pi}$$

$$= \left[-3(1-6\pi)\underbrace{\cos\pi}_{=-1} - \underbrace{18\sin\pi}_{=0}\right] - \left[-3(1-0)\underbrace{\cos(0)}_{1} + \underbrace{18\sin 0}_{=0}\right]$$

$$= (-3+18\pi)\cdot(-1) + 3 \qquad (3)$$

$$= 6 - 18\pi$$

ex5 a) $f(x) = \frac{x+2}{x^2+4x+1} = \frac{1}{2}\left(\frac{2x+2}{x^2+4x+1}\right)$

[/12] $\Rightarrow F(x) = \frac{1}{2}\ln|x^2+4x+1|$ (4)

$$I = F(x)\Big|_{-1}^{0} = \frac{1}{2}\underbrace{\ln|1|}_{=0} - \frac{1}{2}\ln|-2| = -\frac{1}{2}\ln(2)$$

b) $f(x) = (3x+6)(x^2+4x+1)^3 = \frac{3}{2}\left[(x^2+4x+1)^3\cdot(2x+4)\right]$

$\Rightarrow F(x) = \frac{3}{2}\frac{(x^2+4x+1)^4}{4} = \frac{3}{8}(x^2+4x+1)^4$ (4)

$$I = F(x)\Big|_{-1}^{0} = \frac{3}{8}1 - \frac{3}{8}(-2)^4 = \frac{3}{8} - \frac{3}{8}\cdot 16^2 = \frac{3}{8} - 6 = -\frac{45}{8}$$

c) $f(x) = (x+2)(x^2+4x+1)^{-1/2} = \frac{1}{2}\left[(x^2+4x+1)^{-1/2}(2x+4)\right]$

$\Rightarrow F(x) = \frac{1}{2}\frac{(x^2+4x+1)^{1/2}}{1/2} = \sqrt{x^2+4x+1}$

(4)

$$I = F(x)\Big|_0^1 = \sqrt{6} - \sqrt{1} = \sqrt{6} - 1$$

ex 6

(a) faux $\ln(x) = \int_1^x \frac{1}{t}\,dt \Rightarrow \ln'(x) = \frac{1}{x} > 0$

donc par Cor AF, $\ln(x)$ est str. $\nearrow$ sur $\mathbb{R}_*^+$!

(b) vrai $F'(x) = \frac{1}{x}$ donc F est dérivable, donc continue

(c) faux contre-exemple :

$$\ln(3-2) \overset{?}{=} \frac{\ln(3)}{\ln(2)} \Leftrightarrow \underset{\overset{\parallel}{0}}{\ln(1)} \overset{?}{=} \underset{\overset{\neq}{0}}{\frac{\ln(3)}{\ln(2)}}$$