

$\alpha, \beta \in \mathbb{R}, \vec{v}, \vec{w} \in \mathbb{R}^2$  1h test 3 UN 2012-13 | 14

(a)  $F(\alpha \vec{v} + \beta \vec{w}) \stackrel{?}{=} \alpha F(\vec{v}) + \beta F(\vec{w})$

$\Leftrightarrow F(\alpha \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} + \beta \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}) \stackrel{?}{=} \alpha F(\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}) + \beta F(\begin{pmatrix} w_1 \\ w_2 \end{pmatrix})$

$\Leftrightarrow F(\begin{pmatrix} \alpha v_1 + \beta w_1 \\ \alpha v_2 + \beta w_2 \end{pmatrix}) \stackrel{?}{=} \alpha F(\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}) + \beta F(\begin{pmatrix} w_1 \\ w_2 \end{pmatrix})$

$\Leftrightarrow \begin{pmatrix} 2(\alpha v_1 + \beta w_1) \\ -(\alpha v_2 + \beta w_2) \end{pmatrix} \stackrel{?}{=} \alpha \begin{pmatrix} 2v_1 \\ -v_2 \end{pmatrix} + \beta \begin{pmatrix} 2w_1 \\ -w_2 \end{pmatrix}$

$\Leftrightarrow \begin{pmatrix} 2\alpha v_1 + 2\beta w_1 \\ -\alpha v_2 - \beta w_2 \end{pmatrix} \stackrel{?}{=} \begin{pmatrix} \alpha 2v_1 + \beta 2w_1 \\ \alpha(-v_2) + \beta(-w_2) \end{pmatrix}$  oui,  $F$  est linéaire (6)

(b)  $K(0,0) = (0,1) \neq (0,0)$  donc  $K$  pas linéaire (3)

(c)  $\alpha = \beta = 1$   
 $\vec{v} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \vec{w} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \left\{ \begin{array}{l} G(\alpha \vec{v} + \beta \vec{w}) \stackrel{?}{=} \alpha G(\vec{v}) + \beta G(\vec{w}) \\ \Leftrightarrow G(1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} + 1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}) \stackrel{?}{=} 1 G(\begin{pmatrix} 0 \\ 1 \end{pmatrix}) + 1 G(\begin{pmatrix} 0 \\ 1 \end{pmatrix}) \\ \Leftrightarrow G(\begin{pmatrix} 0 \\ 2 \end{pmatrix}) \stackrel{?}{=} G(\begin{pmatrix} 0 \\ 1 \end{pmatrix}) + G(\begin{pmatrix} 0 \\ 1 \end{pmatrix}) \\ \Leftrightarrow \begin{pmatrix} 2 \cdot 0 - 3 \cdot 2 \\ 0^2 + 2^2 \end{pmatrix} \stackrel{?}{=} \begin{pmatrix} -3 \\ 1 \end{pmatrix} + \begin{pmatrix} -3 \\ 1 \end{pmatrix} \\ \Leftrightarrow \begin{pmatrix} -6 \\ 4 \end{pmatrix} \stackrel{?}{=} \begin{pmatrix} -6 \\ 2 \end{pmatrix} \end{array} \right.$  non,  $G$  pas linéaire (5)