MAHN - Minitest n°2 - Corrigé

mot /1

/36

ex1 /16

a) $f(x) = 6x^3 \sin(3x^4-\pi) = \frac{1}{2}\left[\sin(3x^4-\pi)\cdot 12x^3\right]$ /4

$\Rightarrow F(x) = \frac{1}{2}\left[-\cos(3x^4-\pi)\right] = -\frac{1}{2}\cos(3x^4-\pi)$

b) $f(x) = 2x\left(x^8 - \frac{1}{4x^8}\right) = 2x^9 - \frac{2}{4x^7} = 2x^9 - \frac{2x^{-7}}{4} = 2x^9 - \frac{1}{2}x^{-7}$ /4

$\Rightarrow F(x) = 2\cdot\frac{x^{10}}{10} - \frac{1}{2}\left(\frac{x^{-6}}{-6}\right) = \frac{1}{5}x^{10} + \frac{1}{12x^6} \underline{+ C}$ /1

c) $f(x) = \frac{\pi}{\sqrt[3]{2x+1}} = \pi(2x+1)^{-1/3} = \pi\frac{1}{2}\left[(2x+1)^{-1/3}\cdot 2\right]$ /4

$\Rightarrow F(x) = \frac{\pi}{2}\left[\frac{(2x+1)^{2/3}}{2/3}\right] = \frac{\pi}{2}\cdot\frac{3}{2}\sqrt[3]{(2x+1)^2} \underline{+ C}$

$A(0;1) \in \Gamma_f \Leftrightarrow \frac{3\pi}{4}\sqrt[3]{1} + C = 1 \Leftrightarrow C = 1 - \frac{3\pi}{4}$ /3

$$\boxed{F(x) = \frac{3\pi}{4}\sqrt[3]{(2x+1)^2} + 1 - \frac{3\pi}{4}}$$

ex2 /16

a) $A = \int_0^{\pi} f(x)-g(x)\,dx + \int_{\pi}^{2\pi} g(x)-f(x)\,dx$ /3

b) $V = \pi\int_0^{\pi} f^2(x)\,dx - \pi\int_0^{\pi} g^2(x)\,dx + \pi\int_{\pi}^{2\pi} g^2(x)\,dx - \pi\int_{\pi}^{2\pi} f^2(x)\,dx$ /3

ex3 $\pi : 2x - y + z + 1 = 0$ /13

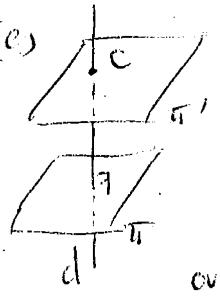
(a) $A(0;1;0) \in \pi$ (car $2\cdot 0 - 1 + 0 + 1 = 0$) /2

$B(0;0;-1) \in \pi$ (car $2\cdot 0 - 0 + (-1) + 1 = 0$)

(b) $\vec{AB} = \begin{pmatrix} 0-0 \\ 0-1 \\ -1-0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix}$ vect dir de π /1

$\|\vec{AB}\| = \sqrt{0^2+(-1)^2+(-1)^2} = \sqrt{2}$, donc $\frac{1}{\sqrt{2}}\vec{AB} = \begin{pmatrix} 0 \\ -1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$ vect dir unitaire de π /1 /1

(c)



$\vec{n}\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \perp \pi$ donc $\vec{n} \perp \pi'$

$P(x;y;z) \in \pi' \Leftrightarrow \vec{CP}\cdot\vec{n} = 0 \Leftrightarrow \begin{pmatrix} x+1 \\ y-0 \\ z-3 \end{pmatrix}\cdot\begin{pmatrix} +2 \\ -1 \\ 1 \end{pmatrix} = 0$

$\Leftrightarrow +2(x+1) - y + (z-3) = 0 \Leftrightarrow +2x - y + z - 1 = 0$ /4

ou $\pi' : +2x - y + z + d = 0$; $C(-1;0;3) \in \pi' \Leftrightarrow 2(-1) - 0 + 3 + d = 0 \Leftrightarrow d = -1$

donc $\pi' : 2x - y + z - 1 = 0$

(d) $\vec{n}$ vect dir de d : $P(x;y;z) \in d \Leftrightarrow \vec{CP} = \lambda\vec{n}$ $(\lambda \in \mathbb{R})$

$\Leftrightarrow \begin{pmatrix} x+1 \\ y-0 \\ z-3 \end{pmatrix} = \lambda\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \Leftrightarrow \begin{cases} x+1 = 2\lambda \\ y = -\lambda \\ z-3 = \lambda \end{cases} \Leftrightarrow \lambda = \left[\underbrace{\frac{x+1}{2} = \frac{y}{-1} = \frac{z-3}{1}}_{\text{2 éq. cart. de } d}\right]$ /4