4ΠA Av Travail 9U' n°2 Corrigé

ex1 [/9]

(a) $f(x) = 2(2x+2)\cdot(x^2+2x+2)^{-2} = 2\left[(x^2+2x+2)^{-2}\cdot(2x+2)\right]$ /3

$\Rightarrow F(x) = 2\left[\dfrac{(x^2+2x+2)^{-1}}{-1}\right] = \dfrac{-2}{x^2+2x+2}$

(b) $f(x) = \dfrac{(e^x+e^{-x})'}{e^x+e^{-x}} = \ln(e^x+e^{-x})$ /3

(c) $f(x) = 4^{1/2x} = e^{\ln(4^{1/2x})} = e^{\frac{1}{2}x\ln(4)}$

$= \dfrac{1}{\frac{1}{2}\ln(4)}\left[\exp\left(\tfrac{1}{2}\ln(4)x\right)\cdot\tfrac{1}{2}\ln(4)\right]$

$\Rightarrow F(x) = \dfrac{2}{\ln(4)}\left[\exp\left(\tfrac{1}{2}\ln(4)x\right)\right] = \dfrac{2}{\ln 2^2}\cdot e^{\frac{1}{2}x\ln(4)}$ /3

$= \dfrac{2}{2\ln 2}\, e^{\ln 4^{1/2x}} = \dfrac{1}{\ln 2}\, 4^{\frac{1}{2}x}$

ex2 [/10]

(a) $I = \displaystyle\int_{-1}^{1} \frac{1}{\sqrt{4-x^2}}\,dx$ on pose [cf table] $x = 2\sin(y)$

$dx = 2\cos(y)\,dy$

$x=1:\ \sin(y) = 1/2 \Rightarrow y = \pi/6$

$x=-1:\ \sin(y) = -1/2 \Rightarrow y = -\pi/6$

$I = \displaystyle\int_{-\pi/6}^{\pi/6} \frac{1}{\sqrt{4-4\sin^2(y)}}\cdot 2\cos(y)\,dy =$

$= \displaystyle\int_{-\pi/6}^{\pi/6} \frac{\cancel{2}\cos(y)}{\cancel{2}\sqrt{1-\sin^2(y)}}\,dy = \int_{-\pi/6}^{\pi/6} \frac{\cos(y)}{|\cos(y)|}\,dy = \int_{-\pi/6}^{\pi/6} \frac{\cos(y)}{\cos(y)}\,dy$

$\cos(y) > 0$ si $y \in [-\pi/6; \pi/6]$

$= \displaystyle\int_{-\pi/6}^{\pi/6} 1\,dy = y\Big|_{-\pi/6}^{\pi/6} = \frac{\pi}{6} - \left(-\frac{\pi}{6}\right) = \underline{\frac{\pi}{3}}$ /6

b) $J = \displaystyle\int_0^{e-1} \frac{-2}{x-e} + e\,dx = -2\int_0^{e-1} \frac{1}{x-e}\,dx + \int_0^{e-1} e\,dx$

$= -2\ln|x-e|\Big|_0^{e-1} + ex\Big|_0^{e-1}$

$= -2\left[\ln|-1| - \ln|-e|\right] + \left[e(e-1) - 0\right]$

$= -2\left(\ln(1) - \ln(e)\right) + e(e-1)$ /4

$= -2(-1) + e^2 - e = e^2 - e + 2$

ex 3

(1/7)

(a) $I + J = \int_0^{\pi/2} \frac{\cos(x) + \sin(x)}{\cos(x) + \sin(x)} dx = \int_0^{\pi/2} 1\, dx = x \Big|_0^{\pi/2} = \pi/2$ /2

$I - J = \int_0^{\pi/2} \frac{\cos(x) - \sin(x)}{\cos(x) + \sin(x)} dx = \ln|\cos(x) + \sin(x)| \Big|_0^{\pi/2}$ /3

$= \ln(1) - \ln(1) = 0$

(b) $\begin{cases} I + J = \pi/2 \\ I - J = 0 \end{cases}$

$2I = \pi/2$

$I = \pi/4$, donc $J = \pi/4$ /2

ex 4

(a) $I_0 = \int_0^1 x^0 e^x dx = \int_0^1 e^x dx = e^x \Big|_0^1 = e^1 - e^0 = e - 1$ /1

$I_1 = \int_0^1 x^1 e^x dx = \int_0^1 x e^x dx$ $\quad f(x) = x \quad g'(x) = e^x$
$\quad f'(x) = 1 \quad g(x) = e^x$

$= x e^x \Big|_0^1 - \int_0^1 1 e^x dx$

$= (1e - 0) - e^x \Big|_0^1 = e - [e - 1] = 1$ /3

b) $I_n = \int_0^1 x^n e^x dx$ $\quad f(x) = x^n \quad g'(x) = e^x$
$\quad f'(x) = n x^{n-1} \quad g(x) = e^x$

$I_n = x^n e^x \Big|_0^1 - n \int_0^1 x^{n-1} e^x dx = (e - 0) - n I_{n-1} = e - n I_{n-1}$ /4

$\forall n \geq 1$

c) $I_2 = e - n I_1 = e - 2 \cdot 1 = e - 2$ /1

d) $I = \int_0^1 x^3 e^x dx + 2 \int_0^1 x^2 e^x dx - 2 \int_0^1 x e^x dx = I_3 + 2I_2 - 2I_1$

$= e - 3I_2 + 2I_2 - 2I_1$

$= e - I_2 - 2I_1 = e - (e-2) - 2 \cdot 1$

$= 0$

/4

ex5 (a) $z = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$

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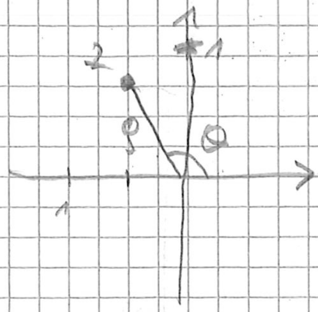

- $\rho^2 = \left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$

  $\rho = 1$

- $\theta \text{ "=" } \arctan\left(\frac{\sqrt{3}/2}{-1/2}\right)$

  $= \arctan(-\sqrt{3}) = \pi/3$

  ⚠ à π près

  donc $\theta = 2\pi/3$

/3

donc $z = 1 \cdot \left[\cos(2\pi/3) + i\sin(2\pi/3)\right]$

(b) $w = \frac{1+i}{1-\sqrt{3}i} \cdot \frac{(1+\sqrt{3}i)}{(1+\sqrt{3}i)} = \frac{(1-\sqrt{3}) + (\sqrt{3}+1)i}{1+3} = \frac{1-\sqrt{3}}{4} + \frac{\sqrt{3}+1}{4} i$

/3

(c) $z + 2\bar{z} + i = 0$; on pose $z = x + yi$

$x + yi + 2(x - yi) + i = 0$

$\Leftrightarrow 3x + (1-y)i = 0$

$\Leftrightarrow \begin{cases} x = 0 \\ 1 - y = 0 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ y = 1 \end{cases}$ $z = i$ $S = \{i\}$ /3