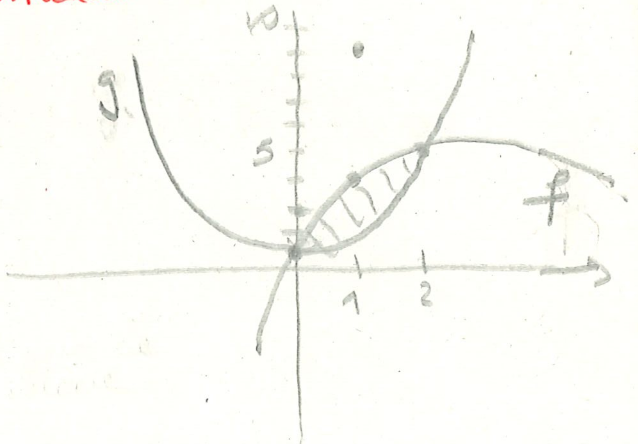
ex31 a) $f(x) = -x^2+4x+1$

$g(x) = x^2+1$

① on représente la situation



$f(x)=g(x) \Leftrightarrow -x^2+4x+1 = x^2+1$

$\Leftrightarrow 2x^2-4x=0$

$\Leftrightarrow 2x(x-2)=0$

$S=\{0;2\}$

② on pose le pb en termes d'intégrale

$A = \int_0^2 f(x)dx - \int_0^2 g(x)\,dx$

③ on résoud

$= \int_0^2 f(x)-g(x)\,dx$

$= \int_0^2 (-x^2+4x+1)-(x^2+1)\,dx = \int_0^2 -2x^2+4x\,dx$

$= \left[-2\frac{x^3}{3}+4\frac{x^2}{2}\right]_0^2 = \left(-2\frac{2^3}{3}+4\frac{2^2}{2}\right) - (0+0)$

$= -2\frac{8}{3}+\cancel{4}^{2}\frac{4}{\cancel{2}} = -\frac{16}{3}+8 = \frac{-16+24}{3} = 8/3$

b) $f(x) = x^3-x^2-2x+2$

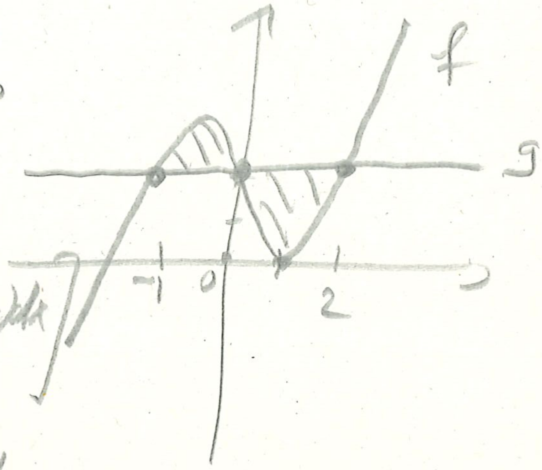
$g(x)=2$

$f(x)=g(x) \Leftrightarrow x^3-x^2-2x+\cancel{2}=\cancel{2}$

$\Leftrightarrow x(x^2-x-2)=0$

$\Leftrightarrow x(x-2)(x+1)=0$

$S=\{-1;0;2\}$



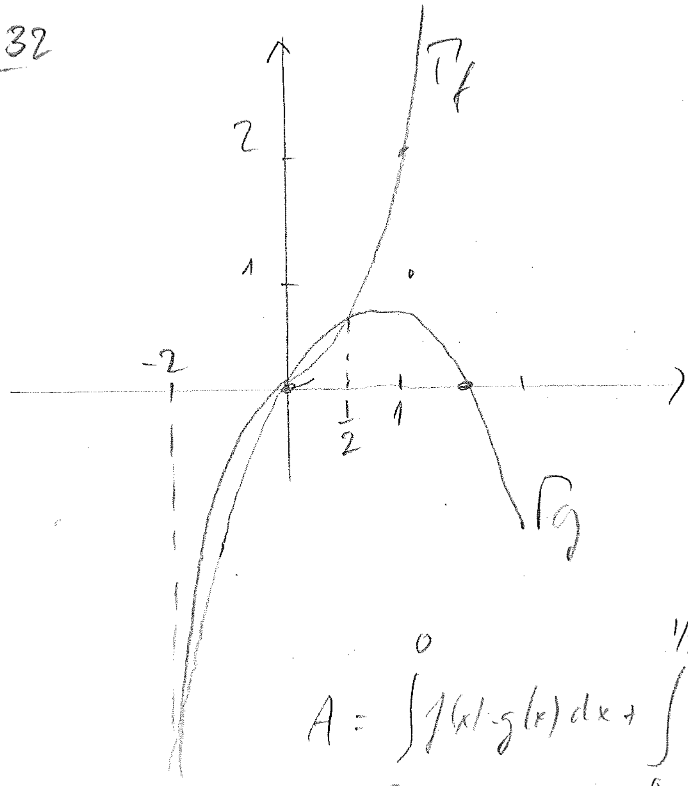
$A=\left[\int_{-1}^0 f(x)dx - \int_{-1}^0 g(x)dx\right]+\left[\int_0^2 g(x)dx - \int_0^2 f(x)dx\right]$

$=\int_{-1}^0 f(x)-g(x)\,dx + \int_0^2 g(x)-f(x)\,dx$

$=\int_{-1}^0 (x^3-x^2-2x)dx + \int_0^2(-x^3+x^2+2x)dx = \left(\frac{x^4}{4}-\frac{x^3}{3}-x^2\right)\Big|_{-1}^0 + \left(-\frac{x^4}{4}+\frac{x^3}{3}+x^2\right)\Big|_0^2$

$= 0-\left(+\frac{1}{4}+\frac{1}{3}-1\right)+\left(-\frac{16}{4}+\frac{8}{3}+4\right)-0 = -\left(\frac{7}{12}-1\right)+\left(\frac{8}{3}\right) = \frac{25}{12}+1 = \frac{37}{12}$

ex32



$f(x) = 2x^3$

$g(x) = x(2-3x)$

$\Gamma_f \cap \Gamma_g : 2x^3 = 2x - 3x^2$

$\Leftrightarrow 2x^3 + 3x^2 - 2x = 0$

$\Leftrightarrow x(\underbrace{2x^2+3x-2}) = 0$

$\Delta = 9 + 16 = 25$

$x_{1,2} = \frac{-3 \pm 5}{4} \rightarrow x_1 = \frac{1}{2}, \; x_2 = -2$

$$A = \int_{-2}^{0} f(x) - g(x)\,dx + \int_{0}^{1/2} g(x) - f(x)\,dx$$

$$= \int_{-2}^{0} 2x^3 + 3x^2 - 2x\,dx + \int_{0}^{1/2} 2x - 3x^2 - 2x^3\,dx$$

$$= \cancel{2}\frac{x^4}{\cancel{4}_2} + \cancel{3}\frac{x^3}{\cancel{3}} - \cancel{2}\frac{x^2}{\cancel{2}} \Big|_{-2}^{0} + x^2 - x^3 - \frac{x^4}{2} \Big|_{0}^{1/2}$$

$$= 0 - [8 - 8 - 4] + \left[\frac{1}{4} - \frac{1}{8} - \frac{1}{32}\right] - 0$$

$$= 4 + \frac{8 - 4 - 1}{32}$$

$$= 4 + \frac{3}{32}$$

$$= \frac{125}{32}$$