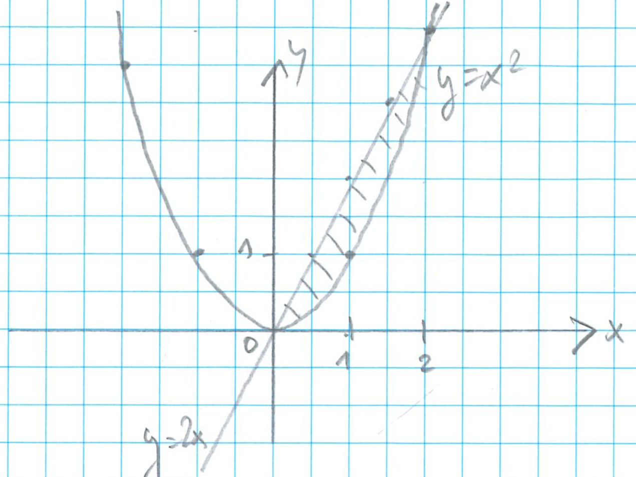
ex 36

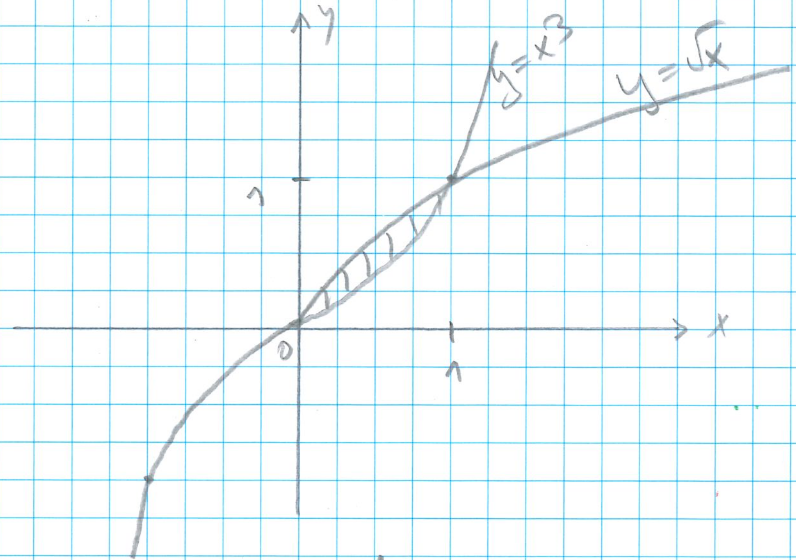
a)



$$V=\pi\int_0^2 (2x)^2\,dx - \pi\int_0^2 (x^2)^2\,dx \qquad \left[\triangle\!\!! \; V\neq \pi\int_0^2 (2x-x^2)^2\,dx \;!!!! \right]$$

$$=\pi\cdot 4\frac{x^3}{3}\Big|_0^2 - \pi\cdot\frac{x^5}{5}\Big|_0^2 = \pi\cdot 4\cdot\frac{8}{3} - \pi\cdot\frac{32}{5} = \pi\left(\frac{32}{3}-\frac{32}{5}\right) = \pi\frac{64}{15}$$

b)



$$V=\pi\int_0^1 (\sqrt{x})^2\,dx - \pi\int_0^1 (x^3)^2\,dx = \pi\int_0^1 x\,dx - \pi\int_0^1 x^6\,dx$$

$$=\pi\frac{x^2}{2}\Big|_0^1 - \pi\frac{x^7}{7}\Big|_0^1 = \pi\cdot\frac{1}{2} - \pi\cdot\frac{1}{7} = \frac{5\pi}{14}$$

ex34

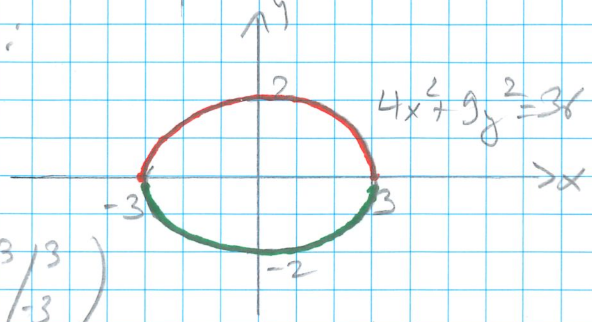
a) $V = \pi \int_0^2 2^2\,dx = 4\pi \int_0^2 1\,dx = 4\pi\, x \Big|_0^2 = 8\pi$

b) $V = \pi \int_0^5 (2x^2)^2\,dx = \pi \int_0^5 4x^4\,dx = \pi \cdot 4 \cdot \frac{x^5}{5} \Big|_0^5 = \pi \cdot 4 \cdot 625$
$= 2500\pi$

ex35

$4x^2 + 9y^2 = 36 \Leftrightarrow 9y^2 = 36 - 4x^2 \Leftrightarrow y^2 = 4 - \frac{4}{9}x^2 \Leftrightarrow y = \pm\sqrt{4 - \frac{4}{9}x^2}$

On choisit la branche supérieure de l'ellipse : $y = f(x) = +\sqrt{4 - \frac{4}{9}x^2}$ qu'on fait tourner autour de Ox entre -3 et 3 pour obtenir l'ellipsoïde :



$V = \pi \int_{-3}^{3} \left(\sqrt{4 - \frac{4}{9}x^2}\right)^2 dx$

$= \pi \int_{-3}^{3} 4 - \frac{4}{9}x^2\,dx = \pi\left(4x - \frac{4}{9}\frac{x^3}{3} \Big|_{-3}^{3}\right)$

$= \pi\left(\left(12 - \frac{4}{9}\cdot 9\right) - \left(-12 + \frac{4}{9}\cdot 9\right)\right) = \pi(24 - 8) = 16\pi$