ex40 a) $\int x\cos(2x)\,dx$ $\quad f(x)=x \quad g'(x)=\cos(2x)$
$f'(x)=1 \quad g(x)=\frac{1}{2}\sin(2x)$

$$\int x\cos(2x) = x\cdot\frac{1}{2}\sin(2x)-\int 1\cdot\frac{1}{2}\sin(2x)\,dx = \frac{x}{2}\sin(2x)-\frac{1}{2}\left[-\frac{1}{2}\cos(2x)\right]$$
$$= \frac{x}{2}\sin(2x)+\frac{1}{4}\cos(2x)+C$$

b) $$\int x^2\sin(x)\,dx = -x^2\cos(x)+2\int x\cos(x)\,dx = -x^2\cos(x)+2\left[x\sin(x)-\int\sin(x)\right]$$

$\left[\begin{array}{l} f(x)=x^2 \Rightarrow f'(x)=2x \\ g'(x)=\sin(x)\Rightarrow g(x)=-\cos(x)\end{array}\right]$ $\qquad$ $\left[\begin{array}{l} f(x)=x \Rightarrow f'(x)=1 \\ g'(x)=\cos(x)\Rightarrow g(x)=\sin(x)\end{array}\right]$

$$\int x^2\sin(x)\,dx = -x^2\cos(x)+2x\sin(x)+2\cos(x)+C$$

c) $\int \sin(x)\cos(x)\,dx$

$f'(x)=\sin(x) \Rightarrow f(x)=-\cos(x)$
$g(x)=\cos(x) \Rightarrow g'(x)=-\sin(x)$

$$\int\sin(x)\cos(x)\,dx = -\cos(x)\cdot\cos(x)-\int\cos(x)\sin(x)+C$$
$$\Leftrightarrow\ 2\int\sin(x)\cos(x)\,dx = -(\cos^2(x))+C$$
$$\int\sin(x)\cos(x)\,dx = -\frac{\cos^2(x)}{2}+cte$$

ex41 a) $\int_0^{\pi/2} x^2\sin(x)\,dx$ $\quad \left[\begin{array}{l} f(x)=x^2 \Rightarrow f'(x)=2x \\ g'(x)=\sin(x)\Rightarrow g(x)=-\cos(x)\end{array}\right]$

$$= -x^2\cos(x)\Big|_0^{\pi/2}+\int_0^{\pi/2}2x\cos(x)\,dx$$
$$= -\left(\frac{\pi}{2}\right)^2\cdot 0-0\cdot\cos(0)+2\int_0^{\pi/2}x\cos(x)\,dx$$

$\left[\begin{array}{l} f(x)=x \Rightarrow f'(x)=1 \\ g'(x)=\cos(x)\Rightarrow g(x)=\sin(x)\end{array}\right]$

$$= 2\left[x\sin(x)\Big|_0^{\pi/2}-\int_0^{\pi/2}1\cdot\sin(x)\,dx\right]$$
$$= 2\left[\left(\frac{\pi}{2}\,1-0\right)+\cos(x)\Big|_0^{\pi/2}\right]$$
$$= 2\left[\frac{\pi}{2}+\cos\left(\frac{\pi}{2}\right)-\cos(0)\right] = 2\left[\frac{\pi}{2}+0-1\right]=\pi-2$$

ex41 b) $f(x) = \int_0^{\frac{\pi}{2}} x^2 \cos(x)\, dx$

$f'(x) = \cos(x) \to f(x) = \sin(x)$

$g(x) = x^2 \to g'(x) = 2x$

$\Rightarrow F(x) = \sin(x) \cdot x^2 \Big|_0^{\frac{\pi}{2}} - \underbrace{\int_0^{\frac{\pi}{2}} \sin(x) \cdot 2x \, dx}$

$\begin{bmatrix} f'(x) = \sin(x) \to f(x) = -\cos(x) \\ g(x) = 2x \to g'(x) = 2 \end{bmatrix}$

$\to -\cos(x) \cdot 2x \Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} -2\cos(x)$

$= -2\sin(x)\Big|_0^{\frac{\pi}{2}}$

$$= \left(\frac{\pi}{2}\right)^2 + 0 + 2 \cdot 0 - 2 = \frac{\pi^2}{4} - 2$$

ex41 c) $I = \int_0^{\pi/2} \cos^4(x)\, dx = \int_0^{\pi/2} \cos(x) \cdot \cos^3(x)\, dx$

posons : $\begin{bmatrix} f'(x) = \cos(x) & f(x) = \sin(x) \\ g(x) = \cos^3(x) & g'(x) = 3\cos^2(x)(-\sin(x)) \end{bmatrix}$

$I = \sin(x)\cos^3(x)\Big|_0^{\pi/2} + 3\int_0^{\pi/2} \sin^2(x)\cos^2(x)\, dx$

$= 3\int_0^{\pi/2} (1-\cos^2(x))\cos^2(x)\, dx = 3\int_0^{\pi/2} \cos^2(x)\, dx - 3\int_0^{\pi/2} \cos^4(x)\, dx$

càd: $I = 3\int_0^{\pi/2} \cos^2(x)\, dx - 3 \cdot I \iff 4I = 3\int_0^{\pi/2} \cos^2(x)\, dx$

$\iff I = \frac{3}{4} \quad \frac{1}{2}(x + \sin(x)\cos(x))\Big|_0^{\pi/2} = \frac{3}{8} \cdot \frac{\pi}{2} = \frac{3\pi}{16}$

ex 41 d)

$$\int_0^\pi (3t^2-4)\cos(t)\,dt.$$

$$\left[\begin{array}{l} 1)\ f'(t)=(3t^2-4)\to f(t)=\frac{4}{9}\left(3\frac{t^3}{3}\right)-4t. \\ \quad g(t)=\cos(t)\to g'(t)=-\sin(t). \\ 2)\ f'(t)=\cos(t)\to f(t)=\sin(t). \\ \quad g(t)=(3t^2-4)\to g'(t)=6t. \end{array}\right]$$

dans le 2):

$$\int_0^\pi (3t^2-4)\cdot\cos(t)\,dt = (3t^2-4)\cdot\sin(t)\Big|_0^\pi - \underbrace{\int_0^\pi \sin(t)\cdot 6t\,dt}$$

$$\int_0^\pi \sin(t)\cdot 6t\,dt$$

$$\left[\begin{array}{l} 1)\ f'(t)=\sin(t)\to f(t)=-\cos(t). \\ \quad g(t)=6t\to g'(t)=6. \\ 2)\ f'(t)=6t\to f(t)=6\frac{t^2}{2}. \\ \quad g(t)=\sin(t)\to g'(t)=\cos(t). \end{array}\right]$$

dans le 1):

$$\int_0^\pi \sin(t)\cdot 6t\,dt = -\cos(t)\cdot 6t\Big|_0^\pi - \int_0^\pi -6\cos(t)$$

$$= 6\pi-(1-1)=6\pi.$$

donc

$$\int_0^\pi (3t^2-4)\cdot\cos(t)\,dt = (3t^2-4)\cdot\sin(t)\Big|_0^\pi - \int_0^\pi \sin(t)\cdot 6t = (3\pi^2-4)\cdot\sin\pi-(6\pi)=0-6\pi = -6\pi.$$

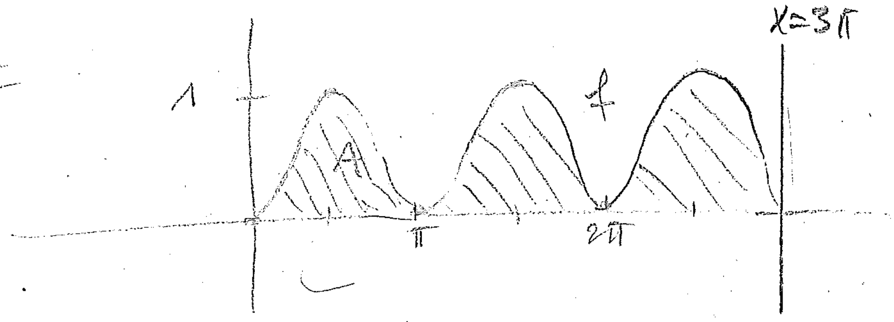
ex 41 e) $I=\int_0^1 x(1-x)^n\,dx$ posons : $f'(x)=(1-x)^n$ $f(x)=-(1-x)^{n+1}\cdot\frac{1}{n+1}$

$g(x)=x$ $g'(x)=1$

$$I=\left[-\frac{x(1-x)^{n+1}}{n+1}\right]\Bigg|_0^1+\int_0^1(1-x)^{n+1}\cdot\frac{1}{n+1}=\frac{1}{n+1}\int_0^1(1-x)^{n+1}=-\frac{1}{n+1}\cdot\frac{1}{n+2}(1-x)^{n+2}\Big|_0^1$$

$$=\frac{1}{(n+1)(n+2)}$$

ex42



$$I = \int_0^{3\pi} \sin^2(x)\,dx \qquad \begin{array}{l} f(x) = \sin(x) \\ f'(x) = \cos(x) \end{array} \qquad \begin{array}{l} g'(x) = \sin(x) \\ g(x) = -\cos(x) \end{array}$$

$$I = -\sin(x)\cos(x)\Big|_0^{3\pi} + \int_0^{3\pi} \cos^2(x)\,dx$$

$$= -\sin(3\pi)\cos(3\pi) + \int_0^{3\pi} 1 - \sin^2(x)\,dx$$

$$= 0 + x\Big|_0^{3\pi} - \underbrace{\int_0^{3\pi} \sin^2(x)\,dx}_{I}$$

donc $2I = 3\pi$

$\Leftrightarrow I = 3\pi/2$

MA 4 ex. 43 Corrigés

a) $\int_{\pi/4}^{\pi/2} x \sin(2x)\, dx = ?$

$f(x) = x \Rightarrow f'(x) = 1$
$g'(x) = \sin 2x \Rightarrow g(x) = \frac{-\cos 2x}{2}$

$$\int_{\pi/4}^{\pi/2} (x \sin 2x)\, dx = -\frac{x}{2} \cos 2x \Big|_{\pi/4}^{\pi/2} + \int_{\pi/4}^{\pi/2} \left(\frac{1}{2} \cos 2x\right) dx$$

$$= +\frac{\pi}{4} - 0 + \frac{1}{2} \int_{\pi/4}^{\pi/2} (\cos 2x)\, dx$$

$$= +\frac{\pi}{4} + \frac{1}{2} \left(\frac{+\sin 2x}{2} \Big|_{\pi/4}^{\pi/2}\right)$$

$$= \frac{1}{4}(\pi - 1)$$

b) $\int_{-\pi/2}^{\pi/3} x \cos\left(\frac{x}{2}\right) dx$

$f(x) = x \Rightarrow f'(x) = 1$
$g'(x) = \cos\left(\frac{x}{2}\right) \Rightarrow g(x) = 2\sin\left(\frac{x}{2}\right)$

$$= 2x \sin\left(\frac{x}{2}\right)\Big|_{-\pi/2}^{\pi/3} - \int_{-\pi/2}^{\pi/3} 2 \sin\left(\frac{x}{2}\right) dx$$

$$= 2\frac{\pi}{3} \sin\left(\frac{\pi}{6}\right) + 2\frac{\pi}{2} \sin\left(-\frac{\pi}{4}\right) + 4\cos\left(\frac{x}{2}\right)\Big|_{-\pi/2}^{\pi/3}$$

$$= 2\frac{\pi}{3} \cdot \frac{1}{2} + 2\frac{\pi}{2}\left(-\frac{\sqrt{2}}{2}\right) + 4\cos\left(\frac{\pi}{6}\right) - 4\cos\left(-\frac{\pi}{4}\right)$$

$$= \frac{\pi}{3} - \frac{\pi\sqrt{2}}{2} + 4\frac{\sqrt{3}}{2} - 4\left(\frac{\sqrt{2}}{2}\right) = \sqrt{2}\left(-\frac{\pi}{2} - 2\right) + \frac{\pi}{3} + 4\frac{\sqrt{3}}{2}$$

c) $\int_{0}^{\pi/2} x \sin\left(\frac{2x}{3}\right) dx$

$f(x) = x \Rightarrow f'(x) = 1$
$g'(x) = \sin\left(\frac{2x}{3}\right) \Rightarrow g(x) = -\frac{3}{2}\cos\left(\frac{2x}{3}\right)$

$$= -\frac{3}{2}x \cos\left(\frac{2x}{3}\right)\Big|_0^{\pi/2} + \int_0^{\pi/2} \frac{3}{2} \cos\left(\frac{2x}{3}\right) dx$$

$$= -\frac{3}{2} \cdot \frac{\pi}{2} \cdot \cos\left(\frac{\pi}{3}\right) + \frac{3}{2} \cdot \frac{3}{2} \sin\left(\frac{2x}{3}\right)\Big|_0^{\pi/2}$$

$$= -\frac{3\pi}{4} \cdot \frac{1}{2} + \frac{9}{4} \sin\left(\frac{\pi}{3}\right) = -\frac{3\pi}{8} + \frac{9}{4}\frac{\sqrt{3}}{2}$$

d) $\int_0 x \sin(x)\, dx$ $\quad f(x) = x \Rightarrow f'(x) = 1$

$g'(x) = \sin(x) \Rightarrow g(x) = -\cos(x)$

$$= -x\cos(x)\Big|_0^{\pi/2} + \int_0^{\pi/2} \cos(x)\, dx$$

$$= \underbrace{-\frac{\pi}{2}\cos\left(\frac{\pi}{2}\right) + 0\cos(0)}_{=0} + \sin(x)\Big|_0^{\pi/2}$$

$$= \sin\left(\frac{\pi}{2}\right) - 0 = 1$$

e) $\int_0^{\pi/2} x\cos(x)\, dx = x\sin(x)\Big|_0^{\pi/2} - \int_0^{\pi/2} \sin(x)\, dx = x\sin(x) + \cos(x)\Big|_0^{\pi/2}$

$f(x) = x \Rightarrow f'(x) = 1$

$g'(x) = \cos(x) \Rightarrow g(x) = \sin(x)$

$$= \left[\frac{\pi}{2}\cdot 1 + 0\right] - \left[0 + 1\right]$$

$$= \frac{\pi}{2} - 1$$

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f) $I = \int_0^{\pi/2} x\sin^3(x)\, dx$ $\quad$ posons $f'(x) = ???$

$\hookrightarrow I = \int_0^{\pi/2} x\sin(x)\sin^2(x)\, dx = \int_0^{\pi/2} x\sin(x)(1-\cos^2(x))\, dx = \underbrace{\int_0^{\pi/2} x\sin(x)\, dx}_{I_1} - \underbrace{\int_0^{\pi/2} x\sin(x)\cos^2(x)\, dx}_{I_2}$

I_1 : par parties, ex 2 d) : $I_1 = 1$

I_2 : posons $\begin{bmatrix} f'(x) = \sin(x)\cos^2(x) & f(x) = -\frac{1}{3}\cos^3(x) \\ g(x) = x & g'(x) = 1 \end{bmatrix}$

$$I_2 = \left[-\frac{x}{3}\cos^3(x)\right]\Big|_0^{\pi/2} + \frac{1}{3}\int_0 \cos^3(x) = \frac{1}{3}\int_0^{\pi/2} \cos(x)\cos^2(x)\, dx = \frac{1}{3}\int_0^{\pi/2} \cos(x)\left[1-\sin^2(x)\right] dx$$

$$= \frac{1}{3}\int_0^{\pi/2} \cos(x)\, dx - \frac{1}{3}\int_0^{\pi/2} \cos(x)\sin^2(x)\, dx = \frac{1}{3}\sin(x)\Big|_0^{\pi/2} - \frac{1}{3}\frac{1}{3}\sin^3(x)\Big|_0^{\pi/2} = \frac{1}{3} - \frac{1}{9} = \frac{2}{9}$$

D'où, enfin !, $I = 1 - \frac{2}{9} = \frac{7}{9}$

ex 43 g) $I = \int_0^{\pi/2} 2x\cos^2(x)\sin(x)\,dx$ posons $f'(x) = \sin(x)\cos^2(x)$ $f(x) = -\frac{1}{3}\cos^3(x)$

$g(x) = 2x$ $g'(x) = 2$

$$I = \left(-\frac{2}{3}x\cos^3(x)\right)\Big|_0^{\pi/2} + \underbrace{\frac{2}{3}\int_0^{\pi/2}\cos^3(x)\,dx}_{= \frac{2}{3}\text{ ; cf. ex 6 f)}} = \frac{2}{3}\cdot\frac{2}{3} = \frac{4}{9}$$

h) $I = \int_{-\pi/2}^{0} x^2\cos^3(x)\,dx$

$$I = \int_{-\pi/2}^{0} x^2\cos(x)(1-\sin^2(x))\,dx = \underbrace{\int_{-\pi/2}^{0} x^2\cos(x)\,dx}_{I_1} - \underbrace{\int_{-\pi/2}^{0} x^2\cos(x)\sin^2(x)\,dx}_{I_2}$$

I_1 : posons $f'(x) = \cos(x)$ $f(x) = \sin(x)$

$g(x) = x^2$ $g'(x) = 2x$

$$I_1 = \left(x^2\sin(x)\right)\Big|_{-\pi/2}^{0} - \int_{-\pi/2}^{0} 2x\sin(x)\,dx = +\frac{\pi^2}{4} - 2\int_{-\pi/2}^{0} x\sin(x)\,dx$$

$$= \frac{\pi^2}{4} - 2\left(-x\cos(x) + \sin(x)\right)\Big|_{-\pi/2}^{0} = \frac{\pi^2}{4} - 2(0+1) = \frac{\pi^2}{4} - 2$$

I_2 : posons $f'(x) = \cos(x)\sin^2(x)$ $f(x) = \frac{1}{3}\sin^3(x)$

$g(x) = x^2$ $g'(x) = 2x$

$$I_2 = \frac{1}{3}x^2\sin^3(x)\Big|_{-\pi/2}^{0} - \frac{2}{3}\int_{-\pi/2}^{0} x\sin^3(x)\,dx = +\frac{1}{3}\frac{\pi^2}{4} - \frac{2}{3}\left[\frac{7}{9}\right]$$

cf. ex2 f) et $\begin{cases} \sin^3(x)\cdot x \text{ est paire} \\ \text{donc } \int_{-\pi/2}^{0}\sin^3(x)x\,dx = \int_0^{\pi/2}\sin^3(x)x\,dx \end{cases}$

d'où : $I = \frac{\pi^2}{4} - 2 - \frac{\pi^2}{12} + \frac{14}{27} = \frac{\pi^2}{6} - \frac{40}{27}$

ex 44 a) $I=\int_{-\sqrt{3}/3}^{\sqrt{3}/3} \sqrt{1-3x^2}\,dx$

on pose : $3x^2 = \sin^2(t)$

$\hookrightarrow x = \frac{1}{\sqrt{3}}\sin(t) \Rightarrow dx = \frac{1}{\sqrt{3}}\cos(t)\,dt$

$\hookrightarrow x = -\frac{\sqrt{3}}{3} = -\frac{1}{\sqrt{3}} \rightarrow -\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}\sin(t) \Leftrightarrow \sin(t) = -1 \; ; \; t = -\pi/2$

$x = \frac{\sqrt{3}}{3} = +\frac{1}{\sqrt{3}} \Rightarrow \ldots \; t = \pi/2$

donc $I = \int_{-\pi/2}^{\pi/2} \sqrt{1-\sin^2(t)} \cdot \frac{1}{\sqrt{3}}\cos(t)\,dt = \frac{1}{\sqrt{3}} \int_{-\pi/2}^{\pi/2} \sqrt{\cos^2(t)} \cdot \cos(t)\,dt$

$= \frac{1}{\sqrt{3}} \int_{-\pi/2}^{\pi/2} \cos^2(t)\,dt \underset{\text{déjà vu}}{=} \frac{1}{\sqrt{3}}\left(\frac{1}{2}\left[t + \sin(t)\cos(t)\right]\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\right)$

$= \frac{1}{2\sqrt{3}}\left(\left(\frac{\pi}{2}+0\right) - \left(-\frac{\pi}{2}+0\right)\right) = \frac{\pi}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\pi\sqrt{3}}{6}$

b) $I = \int_0^2 \frac{x}{\sqrt{x+1}}\,dx$

on pose : $t = \sqrt{x+1}$ $(x > -1)$

$\hookrightarrow x = t^2 - 1 \Rightarrow dx = 2t\,dt$

$\hookrightarrow x = 0 : t = 1$

$x = 2 : t = \sqrt{3}$

$I = \int_1^{\sqrt{3}} \frac{t^2-1}{\cancel{t}} \cdot 2\cancel{t}\,dt = 2\left(\frac{t^3}{3} - t\right)\Big|_1^{\sqrt{3}} = 2\left(\left(\frac{3\sqrt{3}}{3} - \sqrt{3}\right) - \left(\frac{1}{3} - 1\right)\right) = \frac{4}{3}$

c) $I = \int_0^a x^2\sqrt{a^2-x^2}\,dx$ $(a>0)$

on pose : $x^2 = a^2\sin^2(t)$

$\hookrightarrow x = a\sin(t) \Rightarrow dx = a\cos(t)$

$\hookrightarrow t = \frac{1}{a}\arcsin(x)$

$x = 0 : \sin(t) = 0, \; t = 0$

$x = a : \sin(t) = 1, \; t = \pi/2$

$I = \int_0^{\pi/2} a^2\sin^2(t) \cdot \sqrt{a^2 - a^2\sin^2(t)} \cdot a\cos(t)\,dt$

$= a^3 \int_0^{\pi/2} \sin^2(t) \cdot a\cos(t) \cdot \cos(t)\,dt = a^4 \int_0^{\pi/2} \sin^2(t)\cos^2(t)\,dt$

$= a^4 \int_0^{\pi/2} \sin^2(t)\left(1 - \sin^2(t)\right)dt = a^4\left[\int_0^{\pi/2} \sin^2(t)\,dt - \int_0^{\pi/2} \sin^4(t)\,dt\right]$

$\underset{\text{vu}}{\overset{\text{déjà}}{=}} a^4\left[\frac{1}{2}\left(t - \sin(t)\cos(t)\right)\Big|_0^{\pi/2} - \left\{-\frac{1}{4}\sin^3(x)\cos(x) + \frac{3}{8}x - \frac{3}{4}\sin(x)\cos(x)\right\}\Big|_0^{\pi/2}\right]$

$= a^4\left[\frac{1}{2}\frac{\pi}{2} - \left\{\frac{3}{8}\frac{\pi}{2}\right\}\right] = a^4\left(\frac{\pi}{4} - \frac{3\pi}{16}\right) = a^4\left(\frac{\pi}{16}\right) = \frac{\pi a^4}{16}$

b) alternative : $I = \int_0^2 \frac{x}{\sqrt{x+1}} dx$ | on pose $t = x+1 \quad (x > -1)$
$\hookrightarrow x = t-1 \Rightarrow dx = dt$
$\hookrightarrow x = 0 : t = 1$
$x = 2 : t = 3$

$$I = \int_1^3 \frac{t-1}{\sqrt{t}} dt = \int_1^3 (t-1)t^{-1/2} dt = \int_1^3 t^{1/2} - t^{-1/2} dt$$

$$= \left(\frac{2}{3} t^{3/2}\right)\Big|_1^3 - \left(2t^{1/2}\right)\Big|_1^3 = \frac{2}{3}\left(3^{3/2} - 1\right) - 2\left(3^{1/2} - 1\right)$$

$$= \frac{2}{3}\left(\sqrt{3^2 \cdot 3} - 1\right) - 2\left(\sqrt{3} - 1\right) = \frac{2}{\cancel{3}} \cdot \cancel{3}\sqrt{3} - \frac{2}{3} - 2\sqrt{3} + 2 = 2 - \frac{2}{3} = \frac{4}{3}$$

$F(x) = \int \sin^4(x) dx$: on pose $f(x) = \sin^3(x) \quad g'(x) = \sin(x)$
$f'(x) = 3\sin^2(x)\cos(x) \quad g(x) = -\cos(x)$

d'où $F(x) = -\sin^3(x)\cos(x) + 3\int \sin^2(x) dx \cos^2(x) dx$

$$= -\sin^3(x)\cos(x) + 3\int \sin^2(x)\left(1 - \sin^2(x)\right) dx$$

$$= -\sin^3(x)\cos(x) + 3\int \sin^2(x) - 3\underbrace{\int \sin^4(x) dx}_{F(x)}$$

$$\Leftrightarrow 4F(x) = -\sin^3(x)\cos(x) + 3\int \sin^2(x) dx$$

$$\overset{\text{déjà vu}}{=} -\sin^3(x)\cos(x) + 3 \cdot \left[\frac{1}{2}\left(x - \sin(x)\cos(x)\right)\right]$$

$$\Leftrightarrow F(x) = -\frac{1}{4}\sin^3(x)\cos(x) + \frac{3}{8}x - \frac{3}{4}\sin(x)\cos(x) + C$$

d) $I = \int_0^a \sqrt{a^2 - x^2}\, dx$ $(a>0)$; on pose $x^2 = a^2 \sin^2(t)$
$\hookrightarrow x = a\sin(t) \Rightarrow dx = a\cos(t)dt$
$\hookrightarrow x=0 : \sin(t)=0 \to t=0$
$x = a : a = a\sin(t) \Rightarrow \sin(t) = 1 \to t = \pi/2$

$$I = \int_0^{\pi/2} \sqrt{a^2 - a^2\sin^2(t)} \cdot a\cos(t)dt = \int_0^{\pi/2} a\cos(t) a\cos(t) dt$$

$$= a^2 \int_0^{\pi/2} \cos^2(t)dt \overset{\text{déjà vu}}{=} a^2 \, \frac{1}{2}\left(t + \sin(t)\cos(t)\right)\Big|_0^{\pi/2} = \frac{a^2}{2}\cdot\frac{\pi}{2} = \frac{\pi a^2}{4}$$

ex 45 a) $I = \int_0^{\pi} (x-1)\cos(x)\, dx$ $\quad f(x) = x-1 \quad g'(x) = \cos(x)$
$\quad f'(x) = 1 \quad g(x) = \sin(x)$

donc $I = (x-1)\sin(x)\Big|_0^{\pi} - \int_0^{\pi} 1\cdot\sin(x)dx = 0 + \cos(x)\Big|_0^{\pi} = (-1)-(1) = -2$

b) $I = \int_0^{\pi} 3\sin(3x)\, dx = -\cos(3x)\Big|_0^{\pi} = -\cos(3\pi) + \cos(0) = -(-1)+1 = 2$

c) $I = \int_0^{\pi} 3x\cdot\sin(3x)\, dx$ $\quad f(x) = x \quad g'(x) = 3\sin(3x)$
$\quad f'(x) = 1 \quad g(x) = -\cos(3x)$.

donc $I = -x\cdot\cos(3x)\Big|_0^{\pi} + \int_0^{\pi} \cos(3x)\, dx$

$$= -\pi\cdot(-1) - 0 + \frac{1}{3}\sin(3x)\Big|_0^{\pi} = \pi - (0-0) = \pi$$

d) $I = \int_0^a \sqrt{1 - \frac{x^2}{a^2}}\, dx = \frac{1}{a}\int_0^a \sqrt{a^2 - x^2}\, dx \overset{\text{ex 44}}{=} \frac{1}{a}\cdot\left(\frac{\pi a^2}{4}\right) = \frac{\pi a}{4}$

e) $I = \int_0^3 x\sqrt{1+x}\, dx$; on pose $t = 1+x$
$\hookrightarrow x = t-1 \Rightarrow dx = dt$
$\hookrightarrow x=0 : t=1$; $x=3 : t=4$

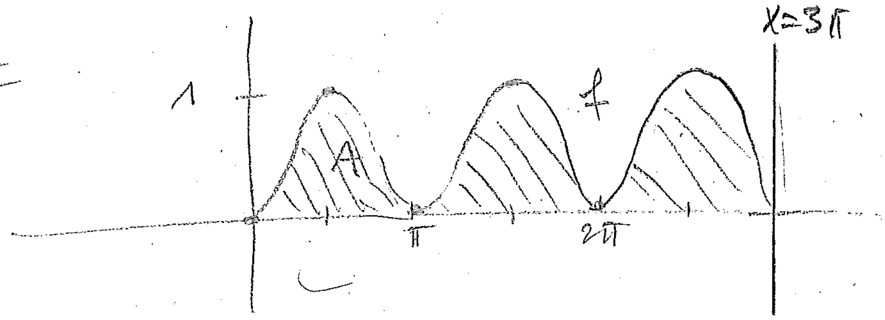
$$I = \int_1^4 (t-1)\cdot\sqrt{t}\, dt = \int_1^4 t\sqrt{t} - \sqrt{t}\, dt = \int_1^4 t^{3/2} dt - \int_1^4 t^{1/2} dt$$

$$= \frac{2}{5} t^{5/2}\Big|_1^4 - \frac{2}{3} t^{3/2}\Big|_1^4 = \frac{2}{5}\left(4^{5/2} - 1\right) - \frac{2}{3}\left(4^{3/2} - 1\right)$$

$$= \frac{2}{5}\left(\sqrt{4}^5 - 1\right) - \frac{2}{3}\left(\sqrt{4}^3 - 1\right) = \frac{2}{5}(32-1) - \frac{2}{3}(8-1)$$

$$= \frac{2}{5}\cdot 31 - \frac{2}{3}\cdot 7 = \frac{62}{5} - \frac{14}{3} = \frac{186 - 70}{15} = \frac{116}{15}$$

ex42



$$I = \int_0^{3\pi} \sin^2(x)\,dx \qquad f(x) = \sin(x) \qquad g'(x) = \sin(x)$$
$$f'(x) = \cos(x) \qquad g(x) = -\cos(x)$$

$$I = -\sin(x)\cos(x)\Big|_0^{3\pi} + \int_0^{3\pi} \cos^2(x)\,dx$$

$$= -\sin(3\pi)\cos(3\pi) + \int_0^{3\pi} 1 - \sin^2(x)\,dx$$

$$= 0 + x\Big|_0^{3\pi} - \underbrace{\int_0^{3\pi} \sin^2(x)\,dx}_{I}$$

donc $2I = 3\pi$

$\Leftrightarrow I = 3\pi/2$

ex46

a) $I = -3\int \frac{1}{4\sqrt{1-x^2}}\,dx = -\frac{3}{4}\int \frac{1}{\sqrt{1-x^2}}\,dx = -\frac{3}{4}\arcsin(x)$

b) $I = \frac{1}{3}\int (x^3+2)^{-1/4}\cdot 3x^2\,dx = \frac{1}{3}\,\frac{(x^3+2)^{3/4}}{3/4} = \frac{4}{3}\cdot\frac{1}{3}\sqrt[4]{(x^3+2)^3}$

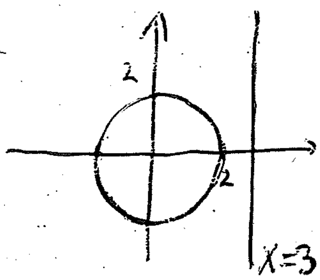
c) $I = \frac{3}{4}\arcsin(x)$ [cf a)]

(ou $-\frac{3}{4}\arccos(x)$!)

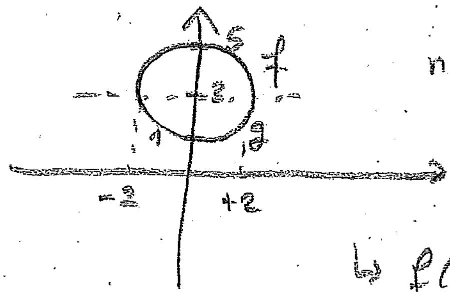
d) $I = \int \frac{x^2+2x+1-1}{(x+1)^2}\,dx = \int \frac{(x+1)^2}{(x+1)^2}\,dx - \int \frac{1}{(x+1)^2}\,dx$

$= \int 1\,dx - \int (x+1)^{-2}\,dx = x - \frac{(x+1)^{-1}}{-1} = x + \frac{1}{x+1}$

ex 47



$\Rightarrow$



nouveau cercle :
$x^2+(y-3)^2 = 4$
$(y-3)^2 = 4-x^2$
$y = \pm\sqrt{4-x^2}+3$
↳ $f(x) = \sqrt{4-x^2}+3$
$g(x) = -\sqrt{4-x^2}+3$

$$V = \pi\int_{-2}^{2} f^2(x)\,dx - \pi\int_{-2}^{2} g^2(x)\,dx = \pi\left[\int_{-2}^{2} \left(\sqrt{4-x^2}+3\right)^2 - \left(-\sqrt{4-x^2}+3\right)^2 dx\right]$$

$$= \pi\int_{-2}^{2} \left(\cancel{(4-x^2)} + 6\sqrt{\ } + \cancel{9} - \cancel{(4-x^2)} + 6\sqrt{\ } - \cancel{9}\right) dx = 12\pi\int_{-2}^{2}\sqrt{4-x^2}\,dx$$

$$= 12\pi\left[\frac{x}{2}\sqrt{4-x^2} + \frac{2^2}{2}\arcsin\left(\frac{x}{2}\right)\right]_{-2}^{2} = 12\pi\cdot 2\cdot\left(\arcsin 1 - \arcsin(-1)\right)$$

$$= 24\pi\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) = 24\pi^2$$

cf. table p. 77
(int. par substitution : poser $x = 2\sin t$)