$\boxed{6}$

a) $\left(\ln(4x-5)\right)' = \dfrac{(4x-5)'}{4x-5} = \dfrac{4}{4x-5}$

b) $\left(\ln\left(\sqrt{3-x^2}\right)\right)' = \dfrac{\left(\sqrt{3-x^2}\right)'}{\sqrt{3-x^2}} = \dfrac{\frac{1}{2\sqrt{3-x^2}}\cdot(3-x^2)'}{\sqrt{3-x^2}}$

$= \dfrac{1}{2\sqrt{3-x^2}\sqrt{3-x^2}}\cdot(3-x^2)' = \dfrac{1}{\cancel{2}(3-x^2)}\,(-\cancel{2}x)$

c) $\left(\ln(3x^5)\right)' = \dfrac{(3x^5)'}{3x^5} = \dfrac{15x^4}{3x^5} = \dfrac{5}{x}$

d) $\left[\ln\left((x^2+x-1)^3\right)\right]' = \dfrac{3(x^2+x-1)^2\cdot(2x+1)}{(x^2+x-1)^3}$

$= \dfrac{3(2x+1)}{x^2+x-1}$

e) $(x\ln(x)-x)' = 1\cdot\ln(x) + x\cdot\frac{1}{x} - 1 = \ln(x)$

f) $\left(\ln(\cos(x))\right)' = \dfrac{(\cos(x))'}{\cos(x)} = \dfrac{-\sin(x)}{\cos(x)}$

g) $\left(\dfrac{\ln(x)}{x}\right)' = \dfrac{\frac{1}{x}\cdot x - \ln(x)\cdot 1}{x^2} = \dfrac{1-\ln(x)}{x^2}$

h) $\left(\ln(\ln(x))\right)' = \dfrac{(\ln(x))'}{\ln(x)} = \dfrac{\frac{1}{x}}{\ln(x)} = \dfrac{1}{x\ln(x)}$

i) $(e^{5x})' = e^{5x}\cdot(5x)' = 5e^{5x}$

j) $(e^{x^3})' = e^{x^3}\cdot(x^3)' = 3x^2e^{x^3}$

k) $(e^{\sin(2x)})' = e^{\sin(2x)}\cdot[\sin(2x)]' = e^{\sin(2x)}\cdot[\cos(2x)\cdot(2x)']$

$= e^{\sin(2x)}\cdot\cos(2x)\cdot 2$

l) $(x^2e^x)' = (x^2)'e^x + x^2(e^x)' = 2xe^x + x^2e^x = e^x\cdot x\cdot(2+x)$

ex 7 $\left[\ln\left(\sqrt{x}\cdot\sqrt[3]{x+3}\cdot\sqrt[5]{3x-2}\right)\right]'$

$= \left[\ln(\sqrt{x}) + \ln(\sqrt[3]{x+3}) + \ln(\sqrt[5]{3x-2})\right]'$

$= \left[\ln x^{1/2} + \ln(x+3)^{1/3} + \ln(3x-2)^{1/5}\right]'$

$= \left[\frac{1}{2}\ln(x) + \frac{1}{3}\ln(x+3) + \frac{1}{5}\ln(3x-2)\right]'$

$= \frac{1}{2}\cdot\frac{1}{x} + \frac{1}{3}\,\frac{1}{x+3} + \frac{1}{5}\cdot\frac{3}{3x-2}$

ex 8 $f(x) = x^2 - 8\ln(x)$

$D_f = \mathbb{R}_+^*$

$f'(x) = 2x - 8\cdot\frac{1}{x} = \frac{2x^2-8}{x} = \frac{2(x^2-4)}{x}$

x	0			2		
$2(x^2-4)$	$-$	$-$	$-$	0	+	+
x	0	+	+	+	+	
$f'(x)$	//	$-$	$-$	0	+	
$f(x)$	//	$\searrow$		min	$\nearrow$	

minimum en $x=2$: $f(2) = 4 - 8\ln(2)$

maximum ? : pas de maximum $\left(\lim_{x\to\infty} f(x) = +\infty\right)$

ex 9 a) $f(x) = x \ln x$

$D_f = \mathbb{R}_*^+$ $\qquad Z_f = \{1\}$

$f'(x) = \ln x + 1$; zeros de f': $\ln x = -1$

$x = \frac{1}{e}$

x	0		$1/e$		$+\infty$
$f'(x)$		$-$	0	$+$	
$f(x)$		$\searrow$	m	$\nearrow$	

$f(1/e) = \frac{1}{e}(-1) = -\frac{1}{e}$

fac [$f''(x) = \frac{1}{x}$

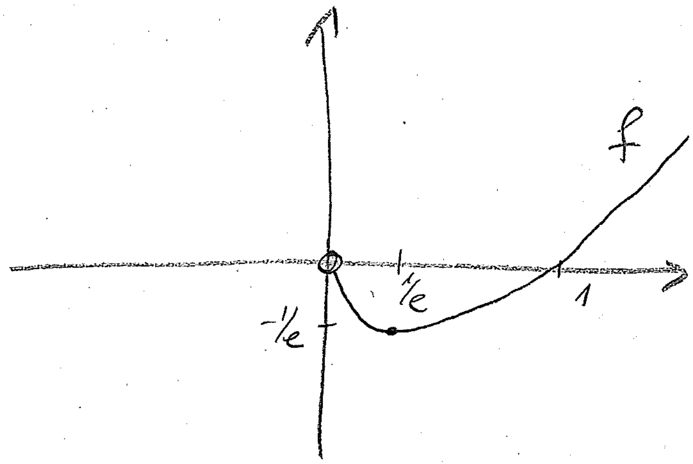
x	0		$+\infty$
$f''(x)$	$+$		$+$
$f(x)$		$\cup$	

]

asymptotes: horiz: $\lim\limits_{x \to +\infty} x \ln x = (+\infty)\cdot(+\infty) = +\infty$ pas d'as. horiz.

fac [obl: $\lim\limits_{x \to +\infty} \frac{f(x)}{x} = \lim\limits_{x \to +\infty} \ln x = +\infty$ pas d'as. obl.]

vert: $\lim\limits_{x \to 0^+} x \ln x = 0$ cf. table p. 86 [sans preuve]



ex 9

a) $f(x) = x \ln x$

$D_f = \mathbb{R}^+_*$ $\qquad Z_f = \{1\}$

$f'(x) = \ln x + 1$; zeros de f' : $\ln x = -1$

$x = \frac{1}{e}$

x	0		$1/e$		$+\infty$
$f'(x)$		$-$	0	$+$	
$f(x)$		$\searrow$	m	$\nearrow$	

$f(1/e) = \frac{1}{e}(-1) = -\frac{1}{e}$

fac [$f''(x) = \frac{1}{x}$

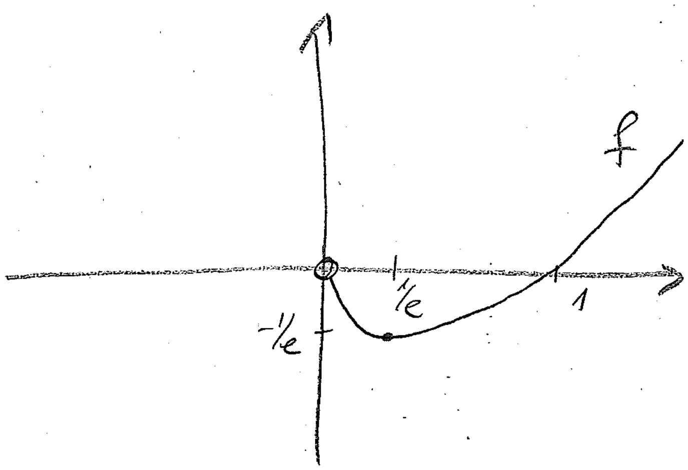
x	0			$+\infty$
$f''(x)$		$+$	$+$	
$f(x)$		$\smile$		

]

asymptotes: horiz: $\lim_{x \to +\infty} x \ln x = (+\infty)\cdot(+\infty) = +\infty$ pas d'as. horiz.

fac [obl: $\lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \ln x = +\infty$ pas d'as. obl.]

vert: $\lim_{x \to 0^+} x \ln x = 0$ cf. table num [sans preuve]



b) $f(x) = \dfrac{1+2\ln(x)}{x}$ $D_f = \mathbb{R}_*^+$; zéros $= \{e^{-1/2}\}$

$f'(x) = \dfrac{2\cdot\frac{1}{x}\cdot x - 1 - 2\ln x}{x^2} = \dfrac{1-2\ln x}{x^2}$

x	0		$e^{1/2}$		$+\infty$
$1-2\ln x$		+	0	−	
x^2		+	+	+	
$f'(x)$		+	0	−	
$f(x)$		↗	M	↘	

$f(e^{1/2}) = \dfrac{2}{e^{1/2}}$

$f''(x) = \dfrac{-2\frac{1}{x}x^2 - 2x(1-2\ln x)}{x^4}$

$= \dfrac{-4x + 4x\ln x}{x^4}$

$= \dfrac{4x(\ln x - 1)}{x^4}$

$= \dfrac{4(\ln x - 1)}{x^3}$

x	0		e		$+\infty$
$4(\ln x - 1)$		−	0	+	
x^3		+	+	+	
$f''(x)$		−	0	+	
$f(x)$		⌒		⌣	

$f(e) = \dfrac{3}{e}$

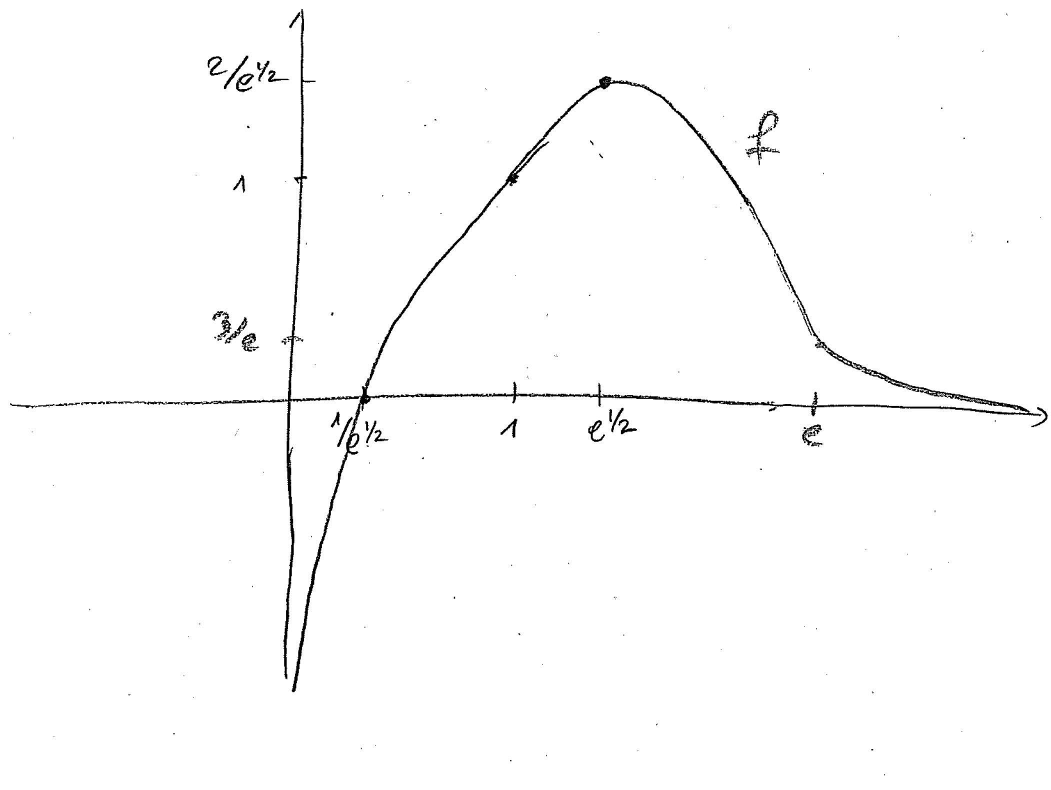
fac

as. horiz : $\lim\limits_{x\to+\infty} \dfrac{1+2\ln x}{x} = \underbrace{\lim\limits_{x\to+\infty} \dfrac{1}{x}}_{=0} + 2\underbrace{\lim\limits_{x\to+\infty} \dfrac{\ln x}{x}}_{=0 \text{ cf table num (sans preuve)}} = 0$

donc as. horiz $y = 0$

as. vert : $\lim\limits_{x\to 0^+} \dfrac{1+2\ln x}{x}$ du type $\lim\limits_{x\to 0^+} \dfrac{\ln x}{x} = -\infty$ (cf. table num) $-\infty$, car de signe $\dfrac{"-"}{"+"}$

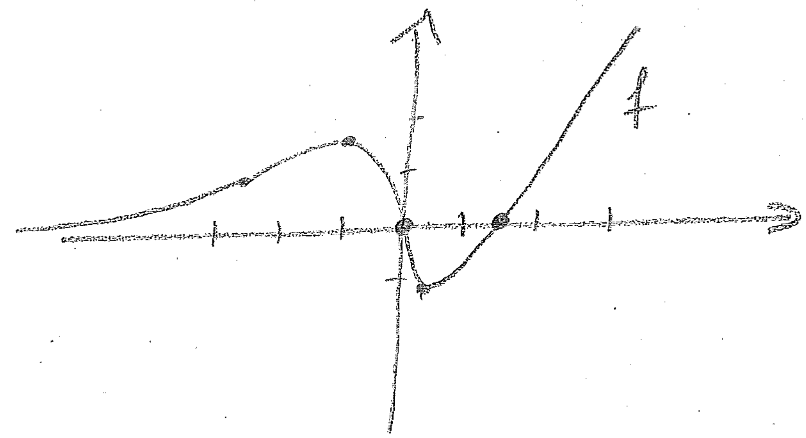
donc as vert à $x = 0^+$



- as. vert. ✓
- as. horiz.: à $+\infty$: $\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} e^x x^2\left(2-\frac{3}{x}\right) = +\infty$

à $-\infty$: $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{x^2(2-3/x)}{e^{-x}} = 0$ $\hookrightarrow$ tableau   as. horiz. à $-\infty$: $y=0$

uc [as. obl. $\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} e^x x\left(2-\frac{3}{x}\right) = +\infty$
$\hookrightarrow$ pas d'as. obl. à $+\infty$]



e) $f(x) = \frac{e^x+e^{-x}}{2}$. $D_f = \mathbb{R}$ zeros: $\frac{1}{2}\left(e^x+\frac{1}{e^x}\right)=0 \Leftrightarrow \frac{1}{2}\left(\frac{e^{2x}+1}{e^x}\right)=0$ $\not\exists$

- $f'(x) = \frac{1}{2}\left(e^x-e^{-x}\right) = \frac{1}{2}\left(e^x-\frac{1}{e^x}\right)$
$= \frac{1}{2}\left(\frac{e^{2x}-1}{e^x}\right)$

zeros de f': $e^{2x}=1$
$x=0$

x		0	
$f'(x)$	$-$	0	$+$
$f(x)$	$\searrow$	m	$\nearrow$

$f(0)=1$

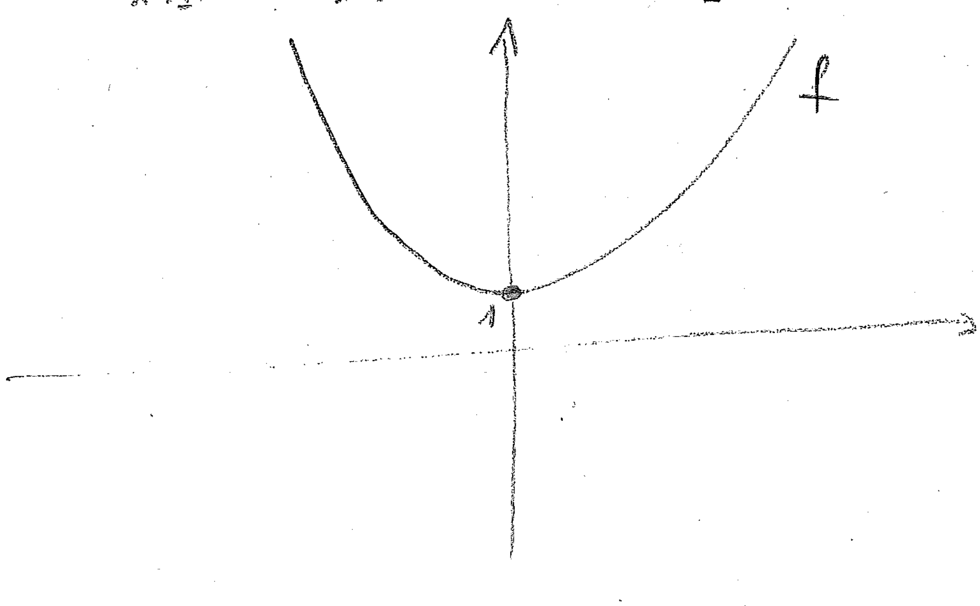
Jac [$f''(x) = \frac{1}{2}\left(e^x+e^{-x}\right) = f(x)$!

x			
$f''(x)$	$+$	$+$	$+$
$f(x)$	$\cup$		

]

- as. horiz.
$\lim_{x\to\pm\infty} \frac{1}{2}\left(e^x+\frac{1}{e^x}\right) = +\infty$

Jac [as. obl: $\lim_{x\to\pm\infty} \frac{f(x)}{x} = \lim_{x\to\pm\infty} \frac{1}{2}\left(\frac{e^x}{x}+\frac{1}{xe^x}\right) = +\infty$]



c) $f(x) = xe^x$

- $D_f = \mathbb{R}$ zeros $= \{0\}$
- $f'(x) = e^x + xe^x = e^x(x+1)$

  $f'(x) = 0 \Leftrightarrow x = -1$

  $f(-1) = -\frac{1}{e} \approx -0{,}37$

x	$-\infty$		-1		$+\infty$
e^x		+	+	+	
$x+1$		$-$	0	+	
$f'(x)$		$-$	0	+	
$f(x)$		↘	m	↗	

Inc [$f''(x) = e^x + e^x(x+1) = e^x(x+2)$

$f''(x) = 0 \Leftrightarrow x = -2$

$f(-2) = -\frac{2}{e^2} \approx 0{,}27$

x	$-\infty$		-2		$+\infty$
$x+2$		$-$	0	+	
e^x		+	+	+	
$f''(x)$		$-$	0	+	
$f(x)$		⌒		⌣	

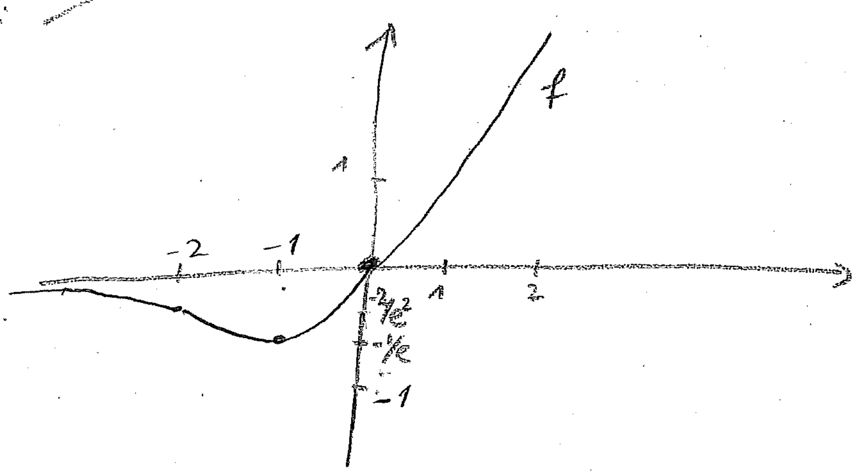
]

- as. horiz: $\lim_{x\to+\infty} xe^x = +\infty$

  $\lim_{x\to-\infty} xe^x = \lim_{x\to-\infty} \frac{x}{e^{-x}} = 0$ ↑ cf. tabela num   as. horiz à $-\infty$: $y=0$

Inc [as. obl: à $+\infty$: $\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} e^x = +\infty$: pas d'as. obl à $+\infty$]

- as. vert: —



d) $f(x) = e^x(2x^2 - 3x) = e^x \cdot x \cdot (2x-3)$

- $D_f = \mathbb{R}$ zeros $= \{0; \frac{3}{2}\}$
- $f'(x) = e^x(2x^2 - 3x) + e^x(4x-3)$

  $= e^x[2x^2 + x - 3]$

  $= e^x(2x-3)(x+1)$

  $f'(x) = 0 \Leftrightarrow x = \frac{3}{2}$ ou $x = -1$

x	$-\infty$		-1		$3/2$		$+\infty$
$2x^2+x-3$		+	0	$-$	0	+	
e^x		+	+	+	+	+	
$f'(x)$		+	0	$-$	0	+	
$f(x)$		↗	M	↘	m	↗	

$f(-1) = 5/e \approx 1{,}84$

$f(3/2) = 0$

Inc [$f''(x) = e^x[2x^2 + x - 3] + e^x[4x+1]$

$= e^x[2x^2 + 5x - 2]$

zeros de f'': $2x^2 + 5x - 2 = 0$

$\Delta = 41$

$x_1 = \frac{-5+\sqrt{41}}{4} \approx 0{,}35$

$x_2 = \frac{-5-\sqrt{41}}{4} \approx -2{,}85$]

x		$-2{,}85$		$0{,}35$	
$f''(x)$	+	0	$-$	0	+
$f(x)$	⌣		⌒		⌣

$f(x_2) \approx 1{,}43$ $f(x_1) \approx -1{,}14$

ex10 a) $f(x) = \ln(x) + \frac{1-x}{x}$ $\qquad D_f = \mathbb{R}^+_*$ $\qquad$ zeros : $\{1 ; \ldots ?\}$

$f'(x) = \frac{1}{x} + \frac{-x-(1-x)}{x^2} = \frac{x-1}{x^2}$

x	0		1		$+\infty$
$x-1$		$-$	0	$+$	
x^2		$+$	$+$	$+$	
$f'(x)$		$-$	0	$+$	
$f(x)$		$\searrow$	m	$\nearrow$	

$f(1) = 0$

Fac [$f''(x) = \frac{x^2 - 2x(x-1)}{x^4} = \frac{-x^2+2x}{x^4} = \frac{2-x}{x^3}$

x	0		2		$+\infty$
$2-x$		$+$	0	$-$	
x^3		$+$	$+$	$+$	
$f''(x)$		$+$	0	$-$	
$f(x)$		$\cup$		$\cap$	

$f(2) = \ln 2 - \frac{1}{2}$]

as. horiz : $\lim_{x \to +\infty} \left(\ln x + \frac{1-x}{x}\right) = +\infty - 1 = +\infty$ $\quad$ pas d'as horiz

Fac [as. obl. $\lim_{x \to +\infty} \frac{f(x)}{x} = \lim_{x \to +\infty} \left(\frac{\ln x}{x} + \frac{1-x}{x^2}\right) = 0$

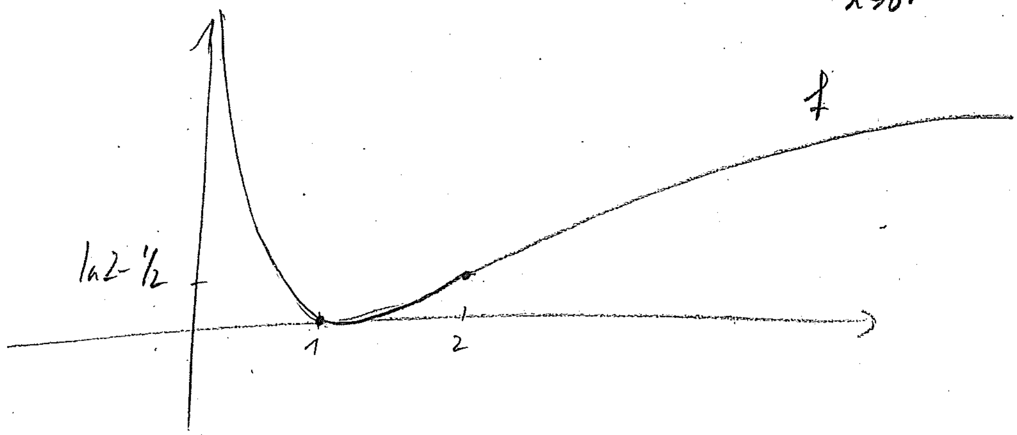
$= \underbrace{\lim_{x \to +\infty} \frac{\ln x}{x}}_{=0} + \underbrace{\lim_{x \to +\infty} \frac{1-x}{x^2}}_{=0}$

(cf. table p.86)

} pas d'as. obl.

$\lim_{x \to +\infty} f(x) - ax = \lim_{x \to +\infty} f(x) = +\infty$]

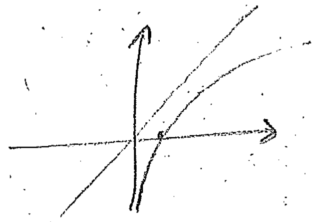
[as. vert : techniquement non accessible ; exploration numérique $\rightarrow$ $\lim_{x \to 0^+} f(x) = +\infty$]



b) $f(x) = x - \ln(x)$

$D = \mathbb{R}_+^*$

zéros : $x - \ln(x) = 0 \iff x = \ln(x)$??



on peut conjecturer que $Z_f = \emptyset$

$f'(x) = 1 - \frac{1}{x}$

$Z_{f'}$: $1 - \frac{1}{x} = 0 \iff \frac{x-1}{x} = 0 \quad Z_{f'} = \{1\}$

x	0		1	
$x-1$		$-$	0	$+$
x		$+$	$+$	$+$
$f'(x)$		$-$	0	$+$
$f(x)$		$\searrow$	min	$\nearrow$

$f(1) = 1 - 0 = 1$

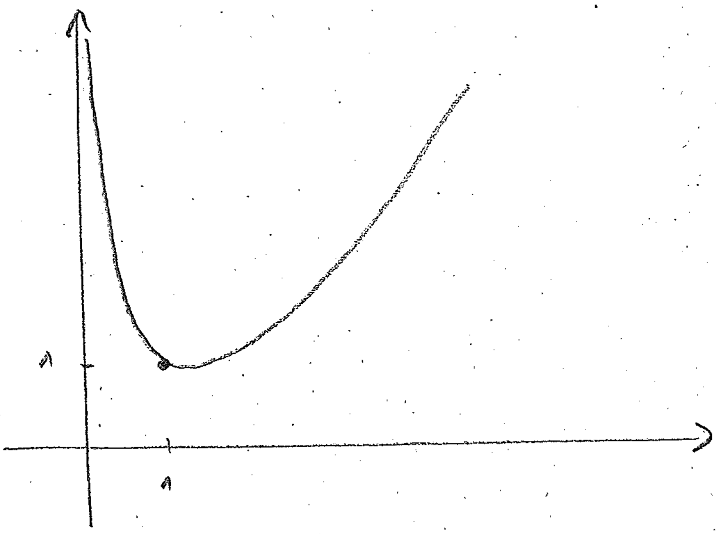
$(1;1)$ min

$f''(x) = \frac{1}{x^2}$

x	0	
$f''(x)$		$+ \; + \; + \; +$
$f(x)$		$\cup$

as. vert : $\lim_{x \to 0^+} x - \ln(x) = 0 - (-\infty) = +\infty$: $x = 0$ as. vert (à droite)

as. horiz : $\lim_{x \to +\infty} x - \ln(x)$: type "$\infty - \infty$" indéterminé ; cf tds de f'' ⇒ pas d'as. horiz.



ex 10 (a) $f(x) = e^{-\frac{x^2}{2}} = \frac{1}{e^{\frac{x^2}{2}}}$

$D_f = \mathbb{R}$

$Z_f = \emptyset$

$f(0) = e^0 = 1$

as. vert: $\emptyset$

as. horiz: $\lim_{x \to \pm\infty} \frac{1}{e^{x^2/2}} = \frac{1}{\infty} = 0$ $y = 0$ as. horiz de f

$f'(x) = e^{-\frac{x^2}{2}} \cdot \left(-\frac{x^2}{2}\right)' = e^{-\frac{x^2}{2}}(-x)$

zéros de f': $\underbrace{e^{-\frac{x^2}{2}}}_{\emptyset}\underbrace{(-x)}_{x=0} = 0$ $Z_{f'} = \{0\}$

tds $f'(x)$:

x		0	
e^{-x^2}	+	+	+
$-2x$	+	0	−
$f'(x)$	+	0	−
$f(x)$	↗	max	↘

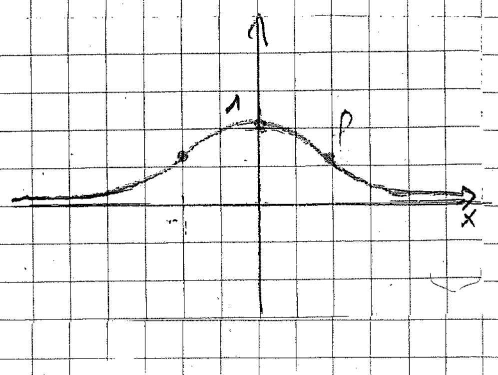
maximum en $(0;1)$

$f''(x) = e^{-\frac{x^2}{2}}(-1) + (-x)e^{-\frac{x^2}{2}}(-x) = e^{-\frac{x^2}{2}}\left[-1 + x^2\right]$

$Z_{f''} =$

x		-1		1	
$e^{-x^2/2}$	+	+	+	+	+
x^2-1	+	0	−	0	+
$f''(x)$	+	0	−	0	+
$f(x)$	∪	PI	∩	PI	∪

$f(-1) = e^{-1/2} = \frac{1}{\sqrt{e}} = f(1)$
$\cong 0{,}6$



(d) $f(x) = (x-2)^2 e^x$

$D_f = \mathbb{R}$

Z_f : $(x-2)^2 e^x = 0 \Leftrightarrow x = 2$

$f(0) = (-2)^2 e^0 = 4$

as. vert : $\emptyset$

as. horiz : $\lim_{x \to +\infty} (x-2)^2 e^x = (+\infty)\cdot(+\infty) = +\infty$

$\left(\lim_{x \to -\infty} (x-2)^2 e^x = \lim_{x \to -\infty} \frac{(x-2)^2}{e^{-x}} \right.$ type "$\frac{+\infty}{+\infty}$" indét.)

↳ utiliser limites données dans la table ou la calculatrice (table) pour voir que $\lim_{x \to -\infty} f(x) = 0$

$f'(x) = 2(x-2)e^x + (x-2)^2 e^x = e^x (x-2)[2 + x - 2]$

$= e^x (x-2) \cdot x$

x		0		2	
e^x	+	+	+	+	+
$x-2$	−	−	−	0	+
x	−	0	+	+	+
$f'(x)$	+	0	−	0	+
$f(x)$	↗	MAX	↘	min	↗

$(0;4)$ max (local)

$(2;0)$ min (local)

$f''(x) = [e^x (x-2)x]' = [e^x (x^2 - 2x)]'$

$= e^x (x^2 - 2x) + e^x (2x - 2) = e^x (x)(x-2) + e^x 2(x-1)$

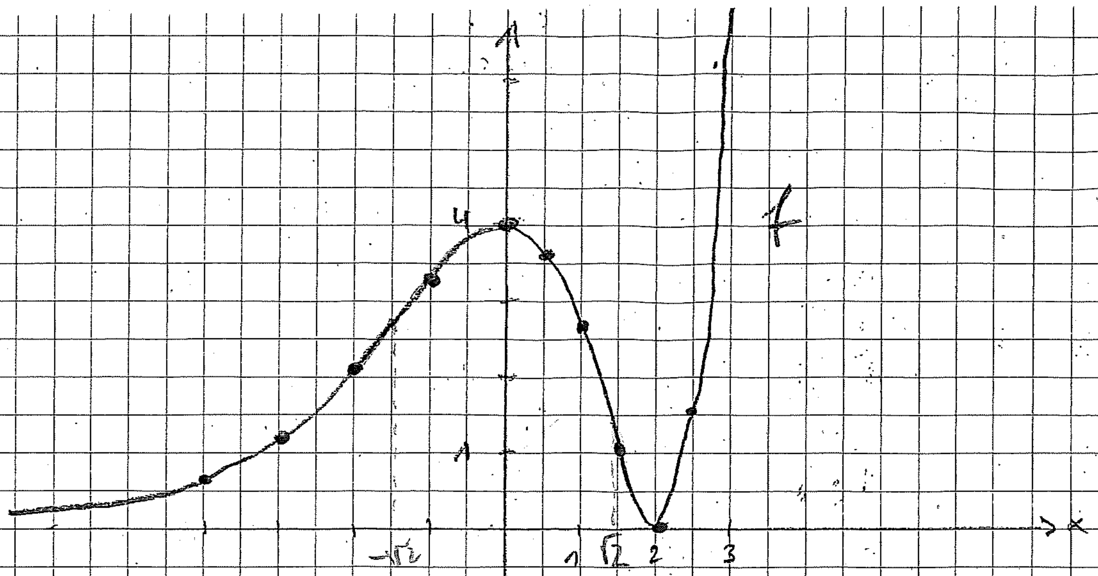
$= e^x [x(x-2) + 2(x-1)] = e^x (x^2 - 2)$

$Z_{f''} = \{\pm\sqrt{2}\}$

x		$-\sqrt{2}$		$\sqrt{2}$	
e^x	+	+	+	+	+
x^2-2	+	0	−	0	+
$f''(x)$	+	0	−	0	+
$f(x)$	∪	pt inf	∩	pt inf	∪

$f(\sqrt{2}) = (\sqrt{2}-2)^2 e^{\sqrt{2}} \approx$

$f(-\sqrt{2}) = (-\sqrt{2}-2)^2 e^{-\sqrt{2}} \approx$



On utilise la table de la calculatrice
pour aider la repr. graphique

(d) $f(x) = (x-2)^2 e^x$

$D_f = \mathbb{R}$

Z_f : $(x-2)^2 e^x = 0 \Leftrightarrow x = 2$

$f(0) = (-2)^2 e^0 = 4$

as. vert : $\emptyset$

as. horiz : $\lim_{x \to +\infty} (x-2)^2 e^x = (+\infty)\cdot(+\infty) = +\infty$

$\left(\lim_{x \to -\infty} (x-2)^2 e^x = \lim_{x \to -\infty} \frac{(x-2)^2}{e^{-x}} \right.$ type "$\frac{+\infty}{+\infty}$" indét. $\left.\right)$

↳ utiliser limites donnée dans la table ou la calculatrice (table) pour voir que $\lim_{x \to -\infty} f(x) = 0$

$f'(x) = 2(x-2)e^x + (x-2)^2 e^x = e^x(x-2)[2 + x - 2]$

$= e^x (x-2) \cdot x$

x		0		2	
e^x	+	+	+	+	+
$x-2$	−	−	−	0	+
x	−	0	+	+	+
$f'(x)$	+	0	−	0	+
$f(x)$	↗	MAX	↘	min	↗

$(0;4)$ max (local)

$(2;0)$ min (local)

$f''(x) = [e^x(x-2)x]' = [e^x(x^2-2x)]'$

$= e^x(x^2-2x) + e^x(2x-2) = e^x(x)(x-2) + e^x 2(x-1)$

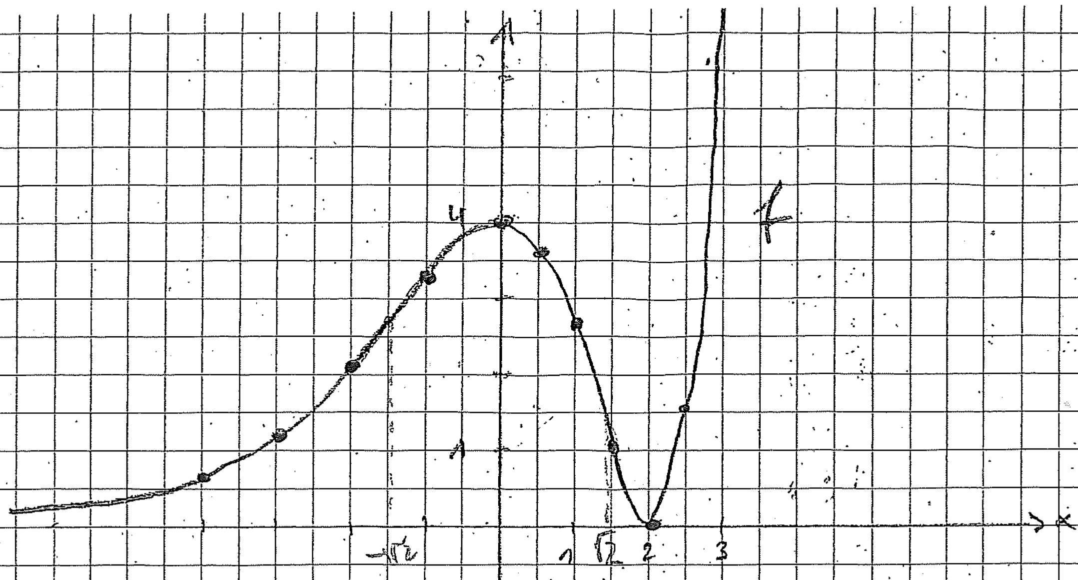
$= e^x[x(x-2) + 2(x-1)] = e^x(x^2-2)$ ≠

$Z_{f''} = \{\pm\sqrt{2}\}$

x		$-\sqrt{2}$		$\sqrt{2}$	
e^x	+	+	+	+	+
x^2-2	+	0	−	0	+
$f''(x)$	+	0	−	0	+
$f(x)$	∪	pt inf	∩	pt inf	∪

$f(\sqrt{2}) = (\sqrt{2}-2)^2 e^{\sqrt{2}} \simeq$

$f(-\sqrt{2}) = (-\sqrt{2}-2)^2 e^{-\sqrt{2}} \simeq$



On utilise la table de la calculatrice pour aider la repr. graphique

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a) $\int \frac{1}{x+2}\,dx = \ln|x+2| + cte$

b) $\int \frac{x}{x^2-1}\,dx = \frac{1}{2}\ln|x^2-1| + cte$

c) $\int \frac{x+2}{x+1}\,dx = \int \frac{x+1+1}{x+1}\,dx$

$= \int 1\,dx + \int \frac{1}{x+1}\,dx$

$= x + \ln|x+1| + cte$

d) $\int e^{-x}\,dx = -e^{-x} + cte$

e) $\int e^{3x}\,dx = \frac{1}{3}e^{3x} + cte$

f) $\int (e^x+1)^3 e^x\,dx = \frac{1}{4}(e^x+1)^4 + cte$

g) $\int \frac{1}{e^x+1}\,dx = \int \frac{e^x+1-e^x}{e^x+1}\,dx$

$= \int 1\,dx - \int \frac{e^x}{e^x+1}\,dx$

$= x - \ln|e^x+1| + cte$

13 a) $\int_{-1}^{1} \frac{1}{2}\left(\frac{1}{3+x} + \frac{1}{3-x}\right)dx$

$= \frac{1}{2}\int_{-1}^{1} \frac{1}{3+x}\,dx + \frac{1}{2}\int_{-1}^{1} \frac{1}{3-x}\,dx$

$= \frac{1}{2}\ln|3+x|\Big|_{-1}^{1} + \frac{1}{2}\left(-\ln|3-x|\right)\Big|_{-1}^{1}$

$= \frac{1}{2}(\ln 4 - \ln 2) - \frac{1}{2}(\ln 2 - \ln 4)$

$= \ln 4 - \ln 2 = \ln 2^2 - \ln 2$

$= 2\ln 2 - \ln 2 = \ln 2$

b) $\int_0^1 \frac{4x}{x^2-4}\,dx = 2\ln|x^2-4|\Big|_0^1$

$= 2(\ln 3 - \ln 4) \quad \left(= 2\ln(3/4)\right)$

c) $\int_2^3 5e^{2x+1}\,dx = 5e^{2x+1}\cdot\frac{1}{2}\Big|_2^3$

$= \frac{5e^7}{2} - \frac{5e^5}{2} = \frac{5}{2}e^5(e^2-1)$

d) $\int_{-1}^{2} 3x\,e^{x^2-1}\,dx = \frac{3}{2}e^{x^2-1}\Big|_{-1}^{2}$

$= \frac{3}{2}(e^3 - e^0) = \frac{3}{2}(e^3-1)$

e) $\int_{\pi/8}^{\pi/4} \frac{\cos(2x)}{\sin(2x)}\,dx = \frac{1}{2}\ln|\sin(2x)|\Big|_{\pi/8}^{\pi/4}$

$= \frac{1}{2}\ln\left|\sin\frac{\pi}{2}\right| - \frac{1}{2}\ln\left|\sin\frac{\pi}{4}\right| = \frac{1}{2}\ln(1) - \frac{1}{2}\ln\left(\frac{\sqrt{2}}{2}\right)$

$= 0 - \frac{1}{2}\ln\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{2}\left(\ln(1) - \ln(\sqrt{2})\right) = \frac{1}{2}\ln(\sqrt{2})$

$= \frac{1}{2}\ln(2^{1/2}) = \frac{1}{4}\ln(2)$

f) $\int_0^{\pi/4} tg(x)\,dx = \int_0^{\pi/4} \frac{\sin(x)}{\cos(x)}\,dx$

$= -\ln|\cos(x)|\Big|_0^{\pi/4} = -\left(\ln\left|\cos\left(\frac{\pi}{4}\right)\right| - \ln|\cos(0)|\right)$

$= -\left(\ln\left(\frac{\sqrt{2}}{2}\right) - \ln(1)\right) = -\ln(2^{-1/2}) = \frac{1}{2}\ln(2)$

ex 14

a) $$\int_0^{\pi} \sin(x)\, e^{\cos(x)}\, dx = -\int_0^{\pi} e^{\cos(x)} \cdot (-\sin(x))\, dx = -\left[e^{\cos(x)} \Big|_0^{\pi} \right]$$

$$= -\left[e^{\cos(\pi)} - e^{\cos(0)} \right] = -\left[e^{-1} - e^{1} \right] = e - \frac{1}{e}$$

b) $$\int_1^4 \frac{1}{\sqrt{x}\, e^{\sqrt{x}}}\, dx = -2\int_1^4 e^{-\sqrt{x}} \left(-\frac{1}{2\sqrt{x}} \right) dx = -2 \cdot e^{-\sqrt{x}} \Big|_1^4 = -2\left(e^{-2} - e^{-1}\right) = 2\left(\frac{1}{e} - \frac{1}{e^2}\right)$$

c) $$I = \int_0^{\pi/2} e^x \cdot \cos(x)\, dx \qquad \begin{matrix} f(x) = e^x & g'(x) = \cos(x) \\ f'(x) = e^x & g(x) = \sin(x) \end{matrix}$$

$$I = \underbrace{e^x \sin(x) \Big|_0^{\pi/2}}_{e^{\pi/2}} - \underbrace{\int_0^{\pi/2} e^x \sin(x)\, dx}_{J}$$

$$\begin{matrix} f(x) = e^x & g'(x) = \sin(x) \\ f'(x) = e^x & g(x) = -\cos(x) \end{matrix}$$

$$J = -e^x \cos(x) \Big|_0^{\pi/2} + \int_0^{\pi/2} e^x \cos(x)\, dx$$

$$= e^{\pi/2} - \left[(0 - (-1)) + I \right] = e^{\pi/2} - 1 - I$$

$$\Leftrightarrow \quad 2I = e^{\pi/2} - 1$$

$$\Leftrightarrow \quad I = \frac{e^{\pi/2} - 1}{2}$$

d) $$\int_1^e \ln(x)\, dx = \int_1^e 1 \cdot \ln(x)\, dx \qquad \begin{matrix} f(x) = \ln(x) & g'(x) = 1 \\ f'(x) = \frac{1}{x} & g(x) = x \end{matrix}$$

$$= x \ln(x) \Big|_1^e - \int_1^e \frac{1}{x} \cdot x\, dx$$

$$= (e \cdot 1 - 1 \cdot 0) - x \Big|_1^e$$

$$= e - (e - 1)$$

$$= 1$$

ex 14 : $\displaystyle\int_0^1 \frac{2x}{x^2+1}\,dx = \ln(x^2+1)\Big|_0^1 = \ln(2) - \ln(1) = \ln(2)$

ex 15 : $\displaystyle\int 5e^{2x} - 3x\,dx = 5\left(\frac{1}{2}e^{2x}\right) - 3\frac{x^2}{2} + C = \frac{5}{2}e^{2x} - \frac{3}{2}x^2 + C$

Celle dont la représentation graphique passe par $(0;2)$:

$$\frac{5}{2}e^{2\cdot 0} - \frac{3}{2}\cdot 0^2 + c = 2 \iff \frac{5}{2}\,1 + C = 0 \iff C = -\frac{5}{2}$$

d'où $F(x) = \frac{5}{2}e^{2x} - \frac{3}{2}x^2 - \frac{5}{2}$

ex 16 : a) $F(x) = \displaystyle\int_0^x \frac{1}{(t^2+1)^{100}}\,dt$ bien définie sur $\mathbb{R}$ car la fonction f définie par $f(x) = \frac{1}{(x^2+1)^{100}}$ est continue sur $\mathbb{R}$

donc, par thm fond I, on a : $F'(x) = f(x)$

Comme $f(x) \geq 0$ sur $\mathbb{R}$, F est croissante sur $\mathbb{R}$ [par lien AF]

C'est donc faux.

(⚠ la fonction f est-elle bien décroissante !)

b) $\left[\ln\left((x+1)^3\right)\right]' = \dfrac{\left[(x+1)^3\right]'}{(x+1)^3} = \dfrac{3(x+1)^2 \cdot (x+1)'}{(x+1)^3} = \dfrac{3}{x+1}$

$\left[3\ln(5x+5)\right]' = 3\left[\ln(5x+5)\right]' = 3\cdot\dfrac{(5x+5)'}{5x+5} = 3\cdot\dfrac{5}{5(x+1)} = \dfrac{3}{x+1}$

les dérivées sont égales, donc f et g sont deux primitives de la même fonction définie par $h(x) = \frac{3}{x+1}$.

Remarque : elles diffèrent donc d'une même constante $C \iff f(x) - g(x) = C$

On peut grâce aux propriétés du ln déterminer cette constante : $\ln\left[(x+1)^3\right] - 3\ln(5x+5)$

$= 3\ln(x+1) - 3\ln\left[5(x+1)\right]$

$= 3\ln(x+1) - 3\left[\ln 5 + \ln(x+1)\right]$

$= 3\cancel{\ln(x+1)} - 3\ln 5 - \cancel{3\ln(x+1)}$

$= -3\ln 5$

d'où $C = -3\ln 5$