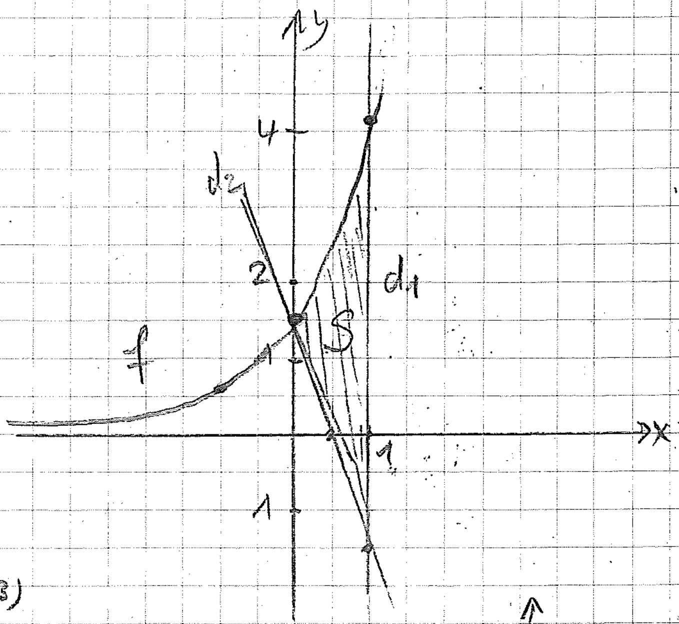
ex 17

$d_1 : x = 1$

$d_2 : 6x + 2y = 3$

$y = -3x + \frac{3}{2}$

$f(x) = \frac{3}{2} e^x$

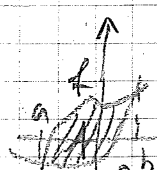


zéro de d_2 :

$-3x + \frac{3}{2} = 0 \quad \Big| -\frac{3}{2}$

$\Leftrightarrow -3x = -\frac{3}{2} \quad \Big| \div(-3)$

$\Leftrightarrow x = +\frac{1}{2}$

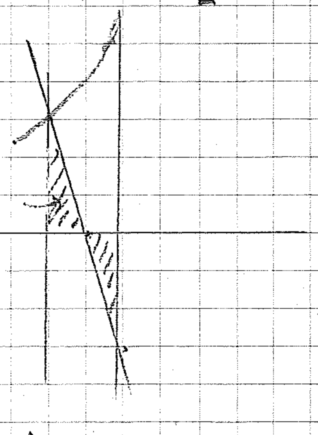
d'où : $A = \int_0^1 f(x)\,dx - \int_0^1 \left(-3x + \frac{3}{2}\right) dx$ (cas  $A = \int_a^b f(x) - g(x)\,dx$)

$= \int_0^1 \frac{3}{2} e^x\,dx - \int_0^1 \left(-3x + \frac{3}{2}\right) dx$

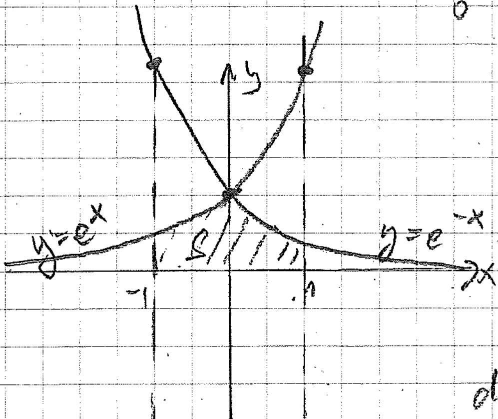
$= \frac{3}{2} e^x \Big|_0^1 - \left(-\frac{3x^2}{2} + \frac{3}{2}x\right)\Big|_0^1 = \frac{3}{2}(e - 1) - \left[\left(-\frac{3}{2} + \frac{3}{2}\right) - (0)\right]$

$= \frac{3}{2}(e - 1)$

Remarque : on peut aussi remarquer que les 2 aires hachurées se compensent et donc que $A = \int_0^1 f(x)\,dx$



ex 18



$A = \int_{-1}^{0} e^x\,dx + \int_0^1 e^{-x}\,dx$

ou, par symétrie, $A = 2 \int_{-1}^{0} e^x\,dx$

d'où

$A = 2\, e^x \Big|_{-1}^{0} = 2\left(e^0 - e^{-1}\right)$

$= 2\left(1 - \frac{1}{e}\right)$

$= 2\left(\frac{e - 1}{e}\right)$

ex19 $\quad f(x) = 2x \cdot \ln^3(x)$

(a) $f'(x) = (2x)' \ln^3(x) + 2x \left([\ln(x)]^3 \right)'$

$= 2\ln^3(x) + 2x \cdot 3\ln^2(x) \cdot [\ln(x)]'$

$= 2\ln^3(x) + 6\cancel{x} \cdot \ln^2(x) \cdot \frac{1}{\cancel{x}}$

$= 2\ln^3(x) + 6\ln^2(x)$

(b) Points critiques = zéros de la dérivée :

$f'(x) = 0 \iff 2\ln^2(x) \cdot [\ln(x) + 3] = 0$

$\ln(x) = 0$ ou $\ln(x) + 3 = 0$

$x = 1$ $\qquad \ln(x) = -3$

$x = e^{-3} = \frac{1}{e^3}$

x	0		$1/e^3$		1		$+\infty$
$2\ln^2(x)$		$+$	$+$	$+$	0	$+$	
$\ln(x)+3$		$-$	0	$+$	$+$	$+$	
$f'(x)$		$-$	0	$+$	0	$+$	
$f(x)$		$\searrow$	min	$\nearrow$	pt inflexion	$\nearrow$	

tangente horizontale en $\left(\frac{1}{e^3}; f\left(\frac{1}{e^3}\right)\right) = \left(\frac{1}{e^3}; 2 \cdot \frac{1}{e^3} \ln^3(e^{-3})\right)$

$= \left(\frac{1}{e^3}; 2 \cdot \frac{1}{e^3} [\ln(e^{-3})]^3\right)$

$= \left(\frac{1}{e^3}; \frac{2}{e^3} \cdot [-3 \underbrace{\ln(e)}_{=1}]^3\right)$

$= \left(\frac{1}{e^3}; \frac{2}{e^3} \cdot (-27)\right) = \left(\frac{1}{e^3}; \frac{-54}{e^3}\right)$

et en $(1; f(1)) = (1; 2 \cdot 1 \cdot \underbrace{\ln^3(1)}_{=0}) = (1; 0)$

(c) éq. de la tg. (table) : $y = f'(a)(x-a) + f(a)$

en $a = e$: $y = \left(2[\underbrace{\ln(e)}_{=1}]^3 + 6[\underbrace{\ln(e)}_{=1}]^2\right) \cdot (x-e) + 2e[\underbrace{\ln(e)}_{=1}]^3$

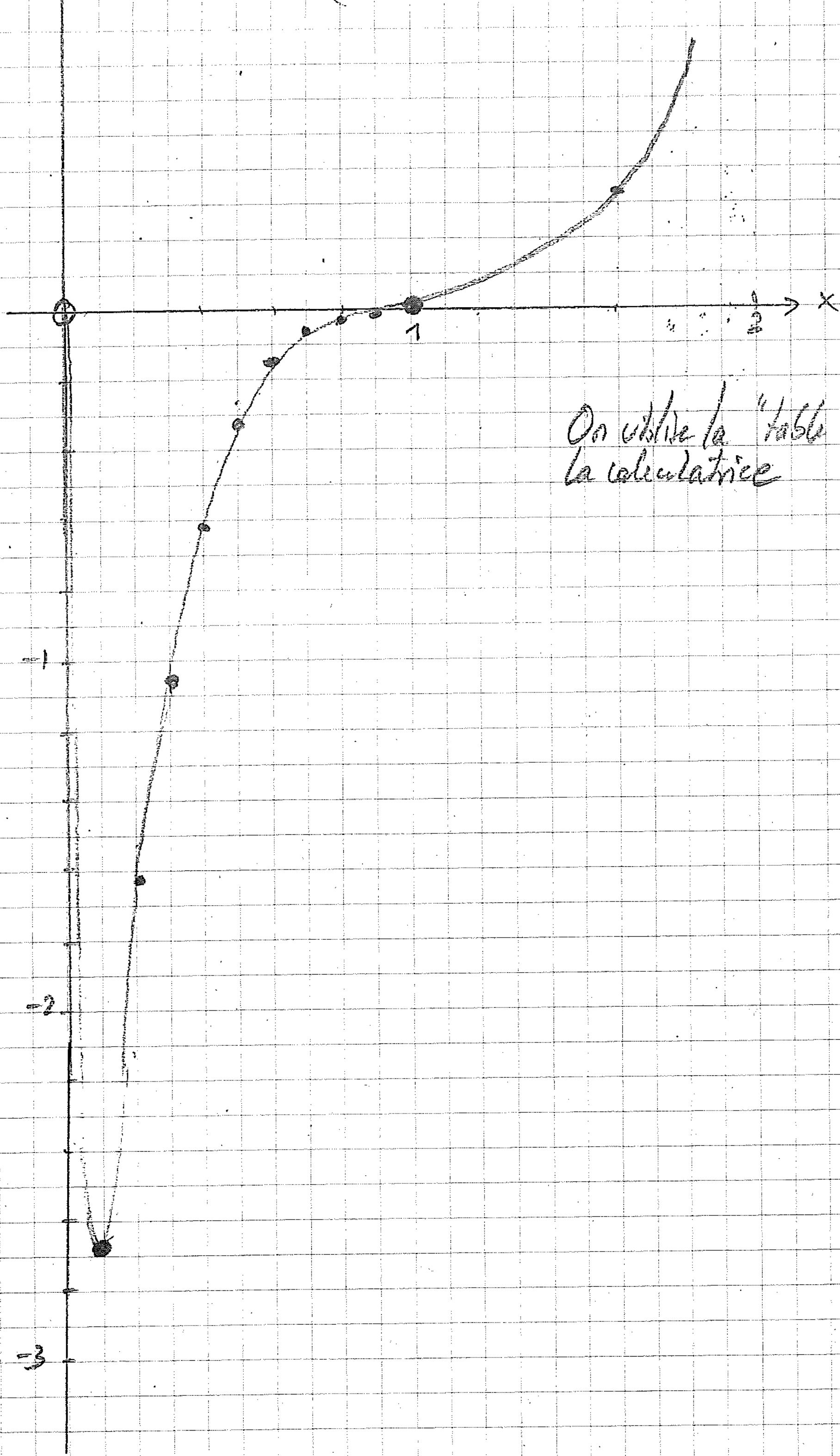
$= 8(x-e) + 2e$

$= 8x + 2e - 8e$

$[\, y = 8x - 6e \,]$

%

$\left(\frac{1}{e^3} ; -\frac{54}{e^3}\right) \simeq (0{,}05 ; -2{,}69)$



On utilise la "table" de la calculatrice ex 20

(a) $h(0) = \frac{e^{-1}+e}{10} \approx 0,3086$ hm $= 30,86$ m

(b) On cherche le minimum :

$$h'(x) = \frac{1}{10}\left[(e^{x-1})' + (e^{-(x-1)})'\right] = \frac{1}{10}\left[e^{x-1} + e^{-(x-1)} \cdot (-1)\right]$$

$$= \frac{1}{10}\left[e^{x-1} - e^{-(x-1)}\right] = \frac{1}{10}\left[e^{x-1} - \frac{1}{e^{x-1}}\right] = \frac{1}{10}\left[\frac{(e^{x-1})^2 - 1}{e^{x-1}}\right]$$

$$= \frac{1}{10}\left[\frac{e^{2x-2} - 1}{e^{x-1}}\right]$$

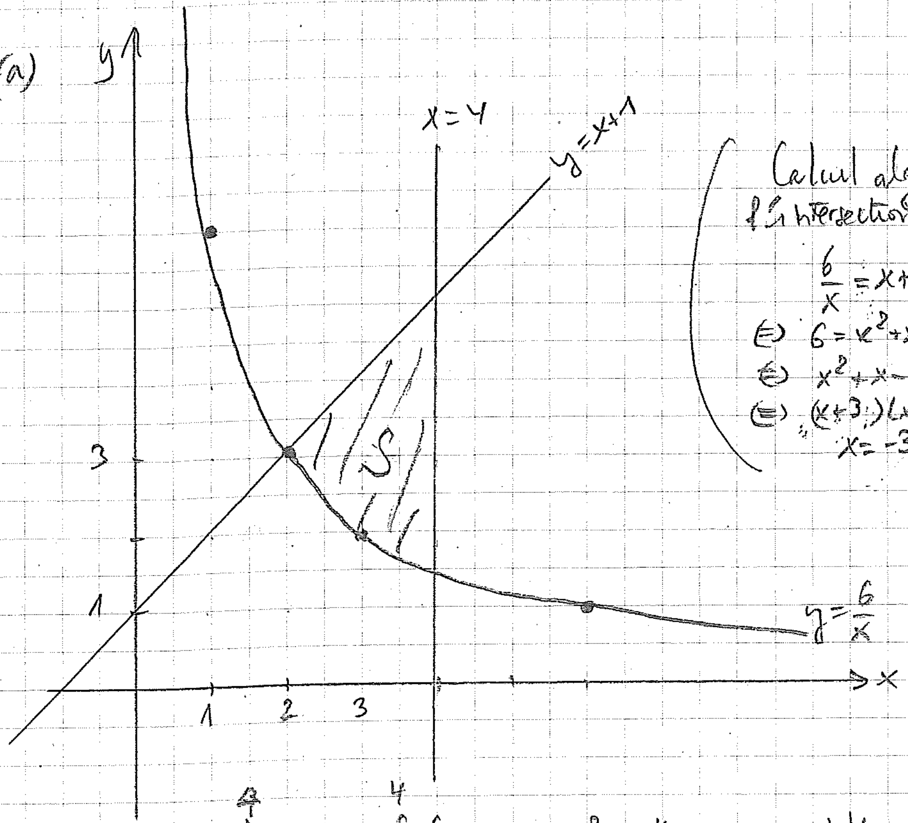
zéro de h' : $e^{2x-2} - 1 = 0 \Leftrightarrow e^{2x-2} = 1$

$\Leftrightarrow 2x-2 = 0 \Leftrightarrow x = 1$ hm ou $x = 100$ m

x		1	
$f'(x)$	$-$	0	$+$
$f(x)$	$\searrow$	min	$\nearrow$

c'est bien un minimum

ex 21 (a)



(Calcul algébrique de l'intersection :

$\frac{6}{x} = x+1 \quad \big| \cdot x \qquad D = \mathbb{R}^*$

$\Leftrightarrow 6 = x^2 + x$

$\Leftrightarrow x^2 + x - 6 = 0$

$\Leftrightarrow (x+3)(x-2) = 0$

$x = -3$ ou $x = 2$)

(b) $A = \int_2^4 (x+1)\,dx - \int_2^4 \frac{6}{x}\,dx = \frac{x^2}{2} + x \Big|_2^4 - 6 \ln|x| \Big|_2^4$

$= \left[(8+4) - (2+2)\right] - \left[6(\ln 4 - \ln 2)\right]$

$= 8 - 6 \ln \frac{4}{2} = 8 - 6 \ln 2 \doteq 3{,}84$

(rem : résultat plausible avec le graphe)

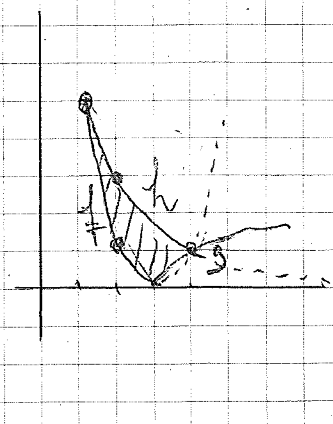
(c) $V = \pi \int_2^4 (x+1)^2\,dx - \pi \int_2^4 \left(\frac{6}{x}\right)^2 dx = \pi \frac{(x+1)^3}{3} \Big|_2^4 - \pi \cdot 36 \int_2^4 \frac{1}{x^2}\,dx$

$= \pi \frac{(x+1)^3}{3} \Big|_2^4 - 36\pi \left(-\frac{1}{x}\right) \Big|_2^4 = \pi \left[\frac{125}{3} - \frac{27}{3}\right] + 36\pi \left[\frac{1}{4} - \frac{1}{2}\right]$

$= \pi \cdot \frac{98}{3} + 36\pi \left(-\frac{1}{4}\right) = \frac{98\pi}{3} - 9\pi = \frac{71\pi}{3} \doteq 74{,}4$

ex 22

$A = \int_1^3 \frac{4}{x}\,dx - \int_1^3 (x-3)^2\,dx + \int_3^4 \frac{4}{x}\,dx - \int_3^4 \sqrt{x-3}\,dx$



$= \int_1^4 \frac{4}{x}\,dx - \int_1^3 (x-3)^2\,dx - \int_3^4 \sqrt{x-3}\,dx$

$= 4 \ln|x| \Big|_1^4 - \frac{(x-3)^3}{3} \Big|_1^3 - \frac{(x-3)^{3/2}}{3/2} \Big|_3^4$

$= 4[\ln 4 - \underbrace{\ln 1}_{=0}] - \left(0 - \frac{(-2)^3}{3}\right) - \frac{2}{3}(1 - 0)$

$= 4 \ln 4 - \frac{8}{3} - \frac{2}{3} = 4 \ln 4 - \frac{10}{3} \left(= 4 \ln(2^2) - \frac{10}{3} = 4 \cdot 2 \ln 2 - \frac{10}{3} = 8 \ln 2 - \frac{10}{3} \right)$

$\cong 2{,}21$

23 $M' = \alpha M$ $\quad$ M = masse restante à t. (M en grammes, t en années)

$\alpha < 0$

$$\frac{M'}{M} = \alpha \Leftrightarrow \int \frac{M'}{M} dt = \int \alpha \, dt \Leftrightarrow \ln M = \alpha t + C$$
$$\Leftrightarrow M(t) = Ce^{\alpha t}$$

a) • $M(0) = 1000 \Leftrightarrow 1000 = Ce^{\alpha \cdot 0}$
$\Leftrightarrow C = 1000$

$M(10) = 950 \Leftrightarrow 950 = 1000 e^{\alpha \cdot 10}$
$\Leftrightarrow e^{\alpha 10} = \frac{950}{1000}$
$\Leftrightarrow \alpha = \frac{1}{10} \ln 0,95 = \ln(0,95^{0,1}) \simeq 5,13 \cdot 10^{-3}$

• $M(250) = 1000 \, e^{[\ln(0,95^{0,1}) \cdot 250]}$
$\simeq 277,39$ gr.

• $500 = 1000 \, e^{\ln(0,95^{0,1}) \cdot t}$

$\frac{\ln 0,5}{\ln(0,95^{0,1})} = t$

$t \simeq 135,13$ ans $\simeq 135$ ans, 48j ; 22h, $28'$ $57''$ (½ vie)

b) $M(t) = Ce^{\alpha t}$

• $M(0) = 100 = Ce^{\alpha \cdot 0} = C$

$M(5500) = 50 = Ce^{\alpha 5500} \Leftrightarrow e^{\alpha 5500} = \frac{50}{100} = 0,5$
$\Leftrightarrow \alpha 5500 = \ln(0,5)$
$\alpha = \frac{1}{5500} \ln(0,5)$

• $M(1000) = 100 \, e^{\frac{1}{5500} \ln(0,5) \cdot 1000} \simeq$

• $M(10000) = 100 \, e^{\frac{1}{5500} \ln(0,5) \cdot 10000} \simeq$

ex 24 $[T(t)-M]' = \alpha (T(t)-M)$

$\Leftrightarrow \dfrac{(T(t)-M)'}{T(t)-M} = \alpha \Leftrightarrow \displaystyle\int \frac{(T(t)-M)'}{T(t)-M}\,dt = \int \alpha\, dt$

$\Leftrightarrow \ln|T(t)-M| = \alpha t + C$

$\Leftrightarrow |T(t)-M| = e^{\alpha t + C}$

$\Leftrightarrow T(t)-M = \pm e^{\alpha t}\cdot e^{C} = \underbrace{\pm e^{C}}_{T_0} e^{\alpha t} = T_0 \cdot e^{\alpha t}$

$T_0 = 100$ [°C]

$M = 18$ [°C]

$T(5) = 70$ °C (t en minutes)

a) $T(t) - 18 = 100 e^{\alpha t}$

en $t=5$: $70 - 18 = 100 e^{\alpha \cdot 5} \Leftrightarrow 52 = 100 e^{5\alpha} \Leftrightarrow e^{5\alpha} = \frac{52}{100}$

$\Leftrightarrow 5\alpha = \ln\frac{52}{100} \Leftrightarrow \alpha = \frac{1}{5}\ln\frac{52}{100} \overset{\text{"C"}}{\cong} -0,130785$

b) $19 - 18 = 100 e^{\alpha t} \Leftrightarrow \frac{1}{100} = e^{\alpha t} \Leftrightarrow \alpha t = \ln\left(\frac{1}{100}\right)$

$\Leftrightarrow t = \frac{1}{\alpha}\ln\left(\frac{1}{100}\right) \overset{\text{"C"}}{\cong} 35,21$ min

c) $[T(0) - M]' = \alpha (T(0) - M))$

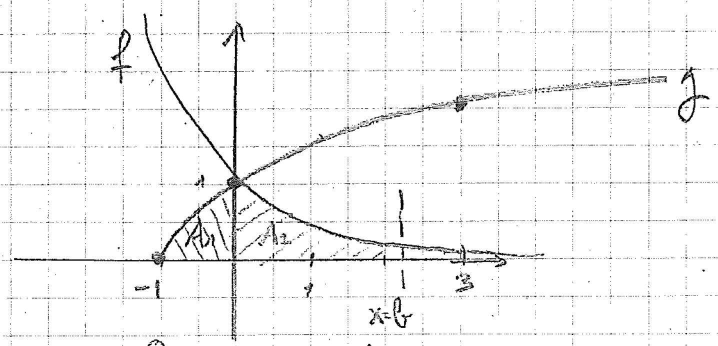
$\Leftrightarrow T'(0) = \alpha \cdot (100 - 18)$

$\Leftrightarrow T'(0) = \alpha \cdot 82 \overset{\text{"C"}}{\cong} -10,72$ °C/min

et $[T(\text{fin}) - M]' = \alpha (T(\text{fin}) - M)$

$\Leftrightarrow T'(\text{fin}) = \alpha (19 - 18) = \alpha \cong -0,13$ °C/min ex 25 $f(x) = e^{-x}$ $g(x) = \sqrt{x+1}$

a)



b) $A_1 = \int_{-1}^{0} \sqrt{x+1}\,dx = \int_{-1}^{0} (x+1)^{1/2}dx = \frac{2}{3}(x+1)^{3/2}\Big|_{-1}^{0} = \frac{2}{3}[1-0] = \frac{2}{3}$

$A_2 = \int_{0}^{b} e^{-x}dx = -e^{-x}\Big|_0^b = -[e^{-b}-1] = 1-e^{-b}$

$A_1 = A_2 \Leftrightarrow \frac{2}{3} = 1-e^{-b} \Leftrightarrow e^{-b} = \frac{1}{3} \Leftrightarrow -b = \ln(\frac{1}{3})$

$\Leftrightarrow b = -\ln(\frac{1}{3}) = -[\ln(1)-\ln(3)] = -[0-\ln(3)] = \ln(3)$

c) $V = \pi\left(\int_{-1}^{0} (\sqrt{x+1})^2 dx + \int_{0}^{b} (e^{-x})^2 dx\right)$

$= \pi\left(\int_{-1}^{0} x+1\,dx + \int_{0}^{b} e^{-2x}dx\right)$

$= \pi\left(\frac{(x+1)^2}{2}\Big|_{-1}^{0} + -\frac{1}{2}e^{-2x}\Big|_0^b\right)$

$= \pi\left(\left(\frac{1}{2}-0\right) + \left(-\frac{1}{2}\right)\cdot\left(e^{-2b}-1\right)\right)$

$= \pi\left(\frac{1}{2}+\frac{1}{2}-\frac{e^{-2b}}{2}\right)$

$= \pi\left(1-\frac{e^{-2b}}{2}\right) = \pi\left(1-\frac{1}{2e^{2b}}\right)$