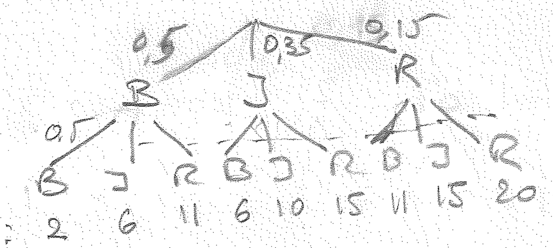
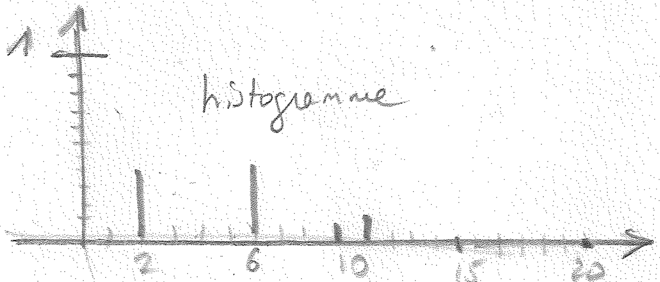
Act 11 [1] a) X : "pts après 2 tirs"



X :

x	2	6	10	11	15	20
p	0,25	0,35	0,1225	0,15	0,105	0,0225



$p(2) = 0,5^2 = 0,25$

$p(6) = 0,5 \cdot 0,35 + 0,35 \cdot 0,5 = 0,35$

$p(10) = 0,35^2 = 0,1225$

$p(11) = 0,5 \cdot 0,15 \cdot 2 = 0,15$

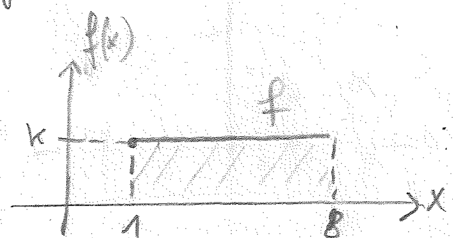
$p(15) = 0,35 \cdot 0,15 \cdot 2 = 0,105$

$p(20) = 0,0225$

vérif. $\Sigma = 1$ ✓

b) cf théorie 9 avec 2 exemples

[2] a)



on veut : • $\int_1^8 f(x)\,dx = 1 \Leftrightarrow (8-1)\cdot k = 1 \Leftrightarrow k = 1/7$

• f positive, continue sur $[1;8]$

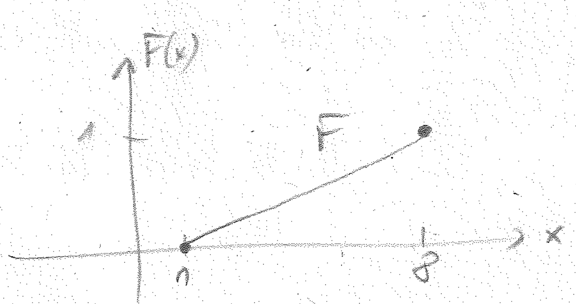
$\Rightarrow f : \mathbb{R} \to \mathbb{R}$, $x \mapsto \begin{cases} \frac{1}{7} & \text{si } 1 \leq x \leq 8 \\ 0 & \text{sinon} \end{cases}$

b) $P(3 < X < 5) = \int_3^5 f(x)\,dx = \int_3^5 \frac{1}{7}\,dx = \frac{1}{7} x \Big|_3^5 = \frac{1}{7}(5-3) = \frac{2}{7} = P(3 \leq X < 5)$

$P(3 < X) = P(3 < X < 8) = (8-3)\cdot\frac{1}{7} = 5/7$

$P(X = 2) = 0$

c) $F(x) = \int_{-\infty}^{x} f(t)\,dt = \int_1^x \frac{1}{7}\,dt = \frac{1}{7} t \Big|_1^x = \frac{x}{7} - \frac{1}{7}$ $\forall x \in [1;8]$

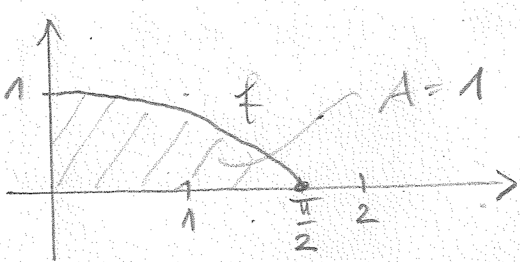


$F : \mathbb{R} \to \mathbb{R}$

$x \mapsto \begin{cases} \frac{x-1}{7} & \text{si } x \in [1;8] \\ 0 & \text{si } x \leq 1 \\ 1 & \text{si } x \geq 8 \end{cases}$

Act 11.3

$$f(x) = \begin{cases} \cos(x) & \text{si } 0 \le x \le \pi/2 \\ 0 & \text{sinon} \end{cases}$$

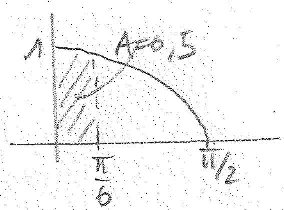
a) 

f est continue et positive sur $[0; \pi/2]$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^{\pi/2} \cos(x)\,dx = \sin(x)\Big|_0^{\pi/2} = \sin(\pi/2) - \sin(0) = 1$$

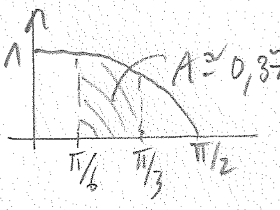
donc f est une fct de densité

b) $P(0 \le Y \le \frac{\pi}{6}) = \int_0^{\pi/6} \cos(x)\,dx = \sin(x)\Big|_0^{\pi/6} = \sin(\pi/6) = 0{,}5 = 50\%$

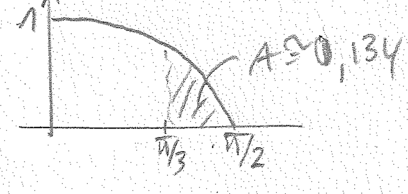


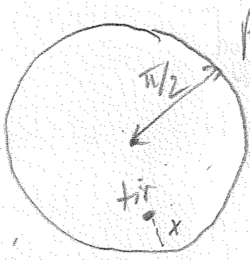
rem: $P(0 \le Y \le \pi/6) = P(0 < Y < \pi/6)$
$= P(0 < Y \le \pi/6) \ldots$

$P(\frac{\pi}{6} \le Y \le \frac{\pi}{3}) = \int_{\pi/6}^{\pi/3} \cos(x)\,dx = \sin(x)\Big|_{\pi/6}^{\pi/3} = \frac{\sqrt{3}}{2} - \frac{1}{2} \cong 36{,}6\%$



$P(\frac{\pi}{3} \le Y \le 0) = \ldots = 1 - \frac{\sqrt{3}}{2} \cong 13{,}4\%$



c) 

Posons que le rayon = π/2 et X = distance entre tir et bord

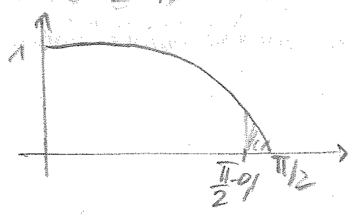
On a: $X \in [0; \pi/2]$

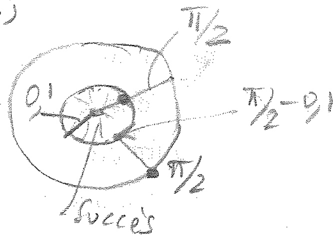
Le modèle indique qu'on a plus de chances de tirer proche du bord !

Art 12 HYP : rayon de la cible = $\pi/2$ [m]

$X = \#$ touchés à $\leq 10\,cm = 0,1\,m$ du centre en n essais

<u>un tir</u> : $p = \int_{\pi/2 - 0,1}^{\pi/2} \cos x \, dx = \sin(\pi/2) - \sin(\pi/2 - 0,1) \cong 5\%$





<u>n tirs</u> : $X \sim B(n;p)$:

$$P(X \geq 1) \geq 0,9$$

$$1 - P(X < 1) \geq 0,9$$

$$P(X = 0) \leq 0,1$$

$$C_0^n \, p^0 (1-p)^n \leq 0,1$$

$$(1-p)^n \leq 0,1$$

$$n \ln(1-p) \leq \ln 0,1$$

$$n \geq \frac{\ln 0,1}{\ln(1-p)} \cong 44,89$$

Il faut donc 45 essais.

Act 13 $\boxed{1}$ 1) $E(X) = \int_1^8 x \cdot \frac{1}{7}\, dx = \frac{1}{7}\frac{x^2}{2}\Big|_1^8 = \frac{1}{7}\left(\frac{64}{2} - \frac{1}{2}\right) = \frac{63}{14} = 4{,}5$

$$V(X) = \int_1^8 \left(x - \frac{9}{2}\right)^2 \cdot \frac{1}{7}\, dx = \frac{1}{7}\left(x - \frac{9}{2}\right)^3 \cdot \frac{1}{3}\Big|_1^8 = \frac{1}{21}\left[\left(\frac{7}{2}\right)^3 - \left(-\frac{7}{2}\right)^3\right]$$

$$= \frac{1}{21} \cdot 2 \cdot \frac{7^3}{2^3} = \frac{49}{12}$$

Act 14 4) $E(X) = \int_0^{\pi/2} x \cos(x)\, dx = x\sin x + \cos x \Big|_0^{\pi/2} = \frac{\pi}{2} - 1 = \mu$

↑ table (ou par parties)

$$V(X) = E(X^2) - \mu^2 = \int_0^{\pi/2} x^2 \cos(x)\, dx - \mu^2$$

$$= \ldots = x^2 \sin x - 2\sin x + 2x\cos x \Big|_0^{\pi/2} - \mu^2$$

↑ (2x int. par parties)

$$= \frac{\pi^2}{4} - 2 - \mu^2 = \frac{\pi^2}{4} - 2 - \left(\frac{\pi}{2} - 1\right)^2 = \pi - 3$$