ex 27 a) $\mu = 2{,}8 = \bar{x}$ cf calculatrice

$\sigma \simeq 2{,}27$

$V(X) \simeq 5{,}16$

b) $\mu = 100.2 = \bar{x}$

$\sigma \simeq 0{,}9798$

$V(X) = 0{,}96$

c) $\mu = \sum_{i=1}^{n} i \frac{1}{n} = \frac{1}{n} \sum_{i=1}^{n} i = \frac{1}{n} \frac{n(n+1)}{2} = \frac{n+1}{2}$

$V(X) = E(X^2) - \mu^2 = \sum_{i=1}^{n} i^2 \frac{1}{n} - \left(\frac{n+1}{2}\right)^2 = \frac{1}{\cancel{n}} \frac{\cancel{n}(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$

$= \frac{(n+1)}{6}\left[2n+1 - \frac{3}{2}(n+1)\right] = \frac{n+1}{6} \cdot \left[\frac{n}{2} - \frac{1}{2}\right] = \frac{(n+1)(n-1)}{12}$

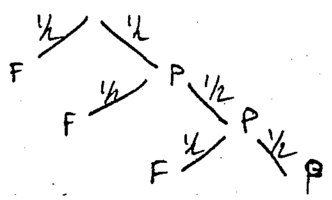
$= \frac{n^2-1}{12}$

ex 28

$E(X) = E(Y) = 3{,}5$ $\quad E(X+Y) = E(2X) = 7$

$V(X) = V(Y) = 5{,}16$ $\quad V(X+Y) = 5{,}8\overline{3}$ $\quad V(2X) = 11{,}\overline{6}$

x 29



X	1	2	3
$f_X(x)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}+\frac{1}{8}$

$E(X) = 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} + 3 \cdot \frac{1}{4} = \frac{7}{4}$

Posons $Y = X^2$:

x	1	4	9
$f_Y(x)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

$E(Y) = 1 \cdot \frac{1}{2} + 4 \cdot \frac{1}{4} + 9 \cdot \frac{1}{4} = \frac{15}{4}$

D'où $V(X) = E(X^2) - [E(X)]^2 = \frac{15}{4} - \left(\frac{7}{4}\right)^2 = \frac{60-49}{16} = \frac{11}{16}$

ex30

Si $X = \text{cte}$:

x	cte
$f_X(x)$	1

$\Rightarrow E(X) = \mu = \text{cte}$

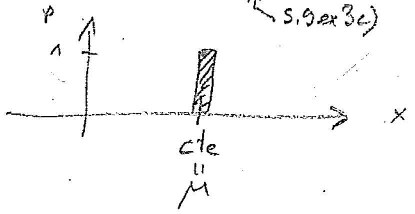
Posons $Y = X^2$

x	$(\text{cte})^2$
$f_Y(x)$	1

$\rightarrow E(Y) = (\text{cte})^2$

d'où $V(X) \underset{\text{s.9 ex3c)}}{=} E(X^2) - \mu^2 = E(Y) - (\text{cte})^2 = (\text{cte})^2 - (\text{cte})^2 = 0$

int. géom :



pas de dispersion autour de μ

ex31

$V(X) = 0 \underset{\text{car...}}{\Leftrightarrow} E((X - E(X))^2) = 0 \underset{\text{car...}}{\Leftrightarrow} X - E(X) = 0 \Leftrightarrow X = E(X)$

$\underset{\text{car...}}{\Leftrightarrow} X = \text{cte}$

ex32

$E(\alpha X)$

$$V(\alpha + X) = E((\alpha + X)^2) - [E(\alpha + X)]^2 = E(\alpha^2 + 2\alpha X + X^2) - [E(\alpha) + E(X)]^2$$

$$= E(\alpha^2) + 2\alpha E(X) + E(X^2) - [\alpha + E(X)]^2$$

$$= \cancel{\alpha^2} + \cancel{2\alpha E(X)} + E(X^2) - \cancel{\alpha^2} - \cancel{2\alpha E(X)} - [E(X)]^2$$

$$= E(X^2) - [E(X)]^2 = V(X)$$

$$V(\alpha X) = E((\alpha X)^2) - [E(\alpha X)]^2 = E(\alpha^2 X^2) - [\alpha E(X)]^2$$

$$= \alpha^2 E(X^2) - \alpha^2 [E(X)]^2 = \alpha^2 (E(X^2) - [E(X)]^2) = \alpha^2 V(X)$$

ex33

<u>Théo</u> : X, Y v.a. indép. $\Rightarrow V(X+Y) = V(X) + V(Y)$

<u>dém</u> : $V(X+Y) = E((X+Y)^2) - [E(X+Y)]^2$ car...

$$= E(X^2 + 2XY + Y^2) - [E(X) + E(Y)]^2$$ car...

$$= E(X^2) + 2E(XY) + E(Y^2) - (E(X))^2 - 2E(X)E(Y) - (E(Y))^2$$ ...

$$= E(X^2) - (E(X))^2 + E(Y^2) - (E(Y))^2 + \cancel{2E(X)E(Y)} - \cancel{2E(X)E(Y)}$$ ...

$$= V(X) + V(Y)$$

ex34

1. Déterminer la loi de probabilité de la v.a. X :
On a

$$\mathbb{P}(X=1)=\frac{1}{4}.$$

$$\mathbb{P}(X=2)=\frac{3}{4}\times\frac{1}{3}=\frac{1}{4}.$$

$$\mathbb{P}(X=3)=\frac{3}{4}\times\frac{2}{3}\times\frac{1}{2}=\frac{1}{4}.$$

$$\mathbb{P}(X=4)=\frac{3}{4}\times\frac{2}{3}\times\frac{1}{2}\times 1=\frac{1}{4}.$$

La loi de probabilité de la v.a. X :

x_i	1	2	3	4
$\mathbb{P}(X=x_i)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Puisque $\mathbb{P}(X=1)=\mathbb{P}(X=2)=\mathbb{P}(X=3)=\mathbb{P}(X=4)$, on déduit que la v.a. X suit une loi uniforme, on écrit $X \hookrightarrow \mathcal{U}_{\{1,2,3,4\}}$.

3. Calculer $E(X)$ et $V(X)$:

2. loi uniforme

$$E(X)=\frac{n+1}{2}=\frac{5}{2}.$$

$$V(X)=\frac{n^2-1}{12}=\frac{5}{4}.$$