

ex 36

$$P_n(z) = a_n z^n + \dots + a_1 z + a_0 = 0$$

$$\Rightarrow P_n(\bar{z}) = a_n \bar{z}^n + \dots + a_1 \bar{z} + a_0$$

$$= \overline{a_n z^n + \dots + a_1 z + a_0}$$

$$= \overline{a_n z^n + \dots + a_1 z + a_0}$$

$$= \overline{a_n z^n + \dots + a_1 z + a_0}$$

$$= \overline{0}$$

$$= 0$$

on utilise les propriétés du conjugué:

$$\overline{z \cdot w} = \bar{z} \cdot \bar{w}$$

$$\overline{z + w} = \bar{z} + \bar{w}$$

ex 37

$$a) \bullet i^5 = 3i^4 + 7i^3 - 7i^2 + 6i - 4 = (-1) - 3 \cdot 1 + 7(-1) - 7(1) + 6i - 4 = 0 \checkmark$$

$$\bullet (2e^{i\pi/3})^5 - 3(2e^{i\pi/3})^4 + 7(2e^{i\pi/3})^3 - 7(2e^{i\pi/3})^2 + 6(2e^{i\pi/3}) - 4$$

$$= 32e^{i5\pi/3} - 48e^{i4\pi/3} + 56e^{i\pi} - 28e^{i2\pi/3} + 12e^{i\pi/3} - 4$$

$$= 32\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right) - \dots$$

$$= 0 \checkmark$$

$$b) z_1 = i \Rightarrow \bar{z}_1 = -i \text{ aussi solution}$$

$$z_2 = 2e^{i\pi/3} \Rightarrow \bar{z}_2 = 2e^{-i\pi/3} \text{ aussi sol.}$$

c) non, car son conjugué serait aussi sol., on en aurait 6 différentes possible par la suite de la factorisation dans \mathbb{C}

ex 38

$$z^3 + 1 = (z - 1)(z^2 + z + 1)$$

$$\Delta = 1 - 4 = -3$$

$$z_{1,2} = \frac{-1 \pm \sqrt{3}i}{2}$$

$$\text{donc } P(z) = (z - 1)\left(z - \left(\frac{-1 - \sqrt{3}i}{2}\right)\right)\left(z - \left(\frac{-1 + \sqrt{3}i}{2}\right)\right) \quad (\text{forme cart})$$

$$\text{ou } z^3 = -1 = e^{i\pi} \Leftrightarrow \begin{cases} \rho^3 = 1 \\ \theta = \frac{\pi + k2\pi}{3} \end{cases} \quad z_0 = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3} = \frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$z_1 = \cos(\pi) + i\sin(\pi) = -1$$

$$z_2 = \cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3} = \frac{1}{2} - i\frac{\sqrt{3}}{2}$$

$$P(z) = (z - z_0)(z - z_1)(z - z_2) \quad \text{forme (Froj)}$$

ex 30

$$z^3 + i = 0$$

$$a) z^3 = -i \Leftrightarrow (pe^{i\theta})^3 = e^{i3\pi/2} \Leftrightarrow \begin{cases} p=1 \\ 3\theta = \frac{3\pi}{2} + k2\pi \end{cases} \Leftrightarrow \begin{cases} p=1 \\ \theta = \frac{\pi}{2} + k\frac{2\pi}{3} \end{cases}$$

$$z_0 = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i$$

$$z_1 = \cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6} = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$$

$$z_2 = \cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6} = \frac{\sqrt{3}}{2} - \frac{1}{2}i$$

$$c) z_0 + z_1 + z_2$$

$$= \left(-\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) + \left(1 - \frac{1}{2} - \frac{1}{2}\right)i = 0$$

