ex 5 a) $10-8i$ b) $362-315i$ c) $112+40i$ d) $19+30i$

e) -3 f) 20 g) -4 h) $-\frac{5}{8}-\frac{7}{8}i$

i) $\frac{9}{5}-\frac{12}{5}i$ j) 109

ex 6 $-64 = z^2 \Leftrightarrow z^2 = 64(-1) = 64i^2 = 8^2 i^2 = (8i)^2 \Leftrightarrow z = \pm 8i$

$-11 = z^2 \Leftrightarrow \dots \Leftrightarrow z = \pm\sqrt{11}\,i$

ex 7 $x^2+2 = x \Leftrightarrow x^2 - x + 2 = 0$

$\Delta = 1-8 = -7 \qquad z_{1,2} = \frac{1 \pm \sqrt{-7}}{2} = \frac{1 \pm \sqrt{7}\,i}{2}$

$x^4 = 1 \Leftrightarrow x^4 - 1 = 0 \Leftrightarrow \underbrace{(x^2-1)}_{x=\pm 1}\underbrace{(x^2+1)}_{\substack{x^2=-1 \\ x=\pm i}} = 0 \qquad S = \{\pm 1; \pm i\}$

ex 8

$z+w = i$

$(z+w)^{100} = i^{100} = (i^2)^{50} = (-1)^{50} = 1$

$z \cdot w = \frac{3}{2} + \frac{3}{2}\sqrt{2}\,i$

$z^2 = -\frac{1}{2} - \sqrt{2}\,i$

ex 9 a) $-45+28i$ b) $-12-16i$ c) 13 d) $-i$ e) $-2-2i$ f) $\frac{1}{4}-\frac{\sqrt{3}}{4}i$

g) $\frac{11}{5}-\frac{7}{5}i$ h) $-\frac{2}{65}+\frac{29}{65}i$ i) $-i$

ex 10 a) $1+i+i^2+\dots+i^{1000} = [\underset{0^e : i^0}{1}+i+(-1)+(-i)] + [\underset{4^e : i^4}{1}+i+(-1)+(-i)] + \dots + [\underset{996^e : i^{996}}{1}+i+(-1)+(-i)] + 1$

$= 250\,[\underbrace{1+i+(-i)+-1}_{0}] + 1 = 1$

ou utiliser la formule connue $1+r+r^2+\dots+r^n = \frac{1-r^{n+1}}{1-r} \Leftrightarrow 1+i+\dots+i^{1000} = \frac{1-i^{1001}}{1-i}$

$= \frac{1-i}{1-i} = 0$

b) $(1+i)^2 = 1+2i+i^2 = 2i \rightarrow (1+i)^4 = (2i)^2 = 4i^2 = -4$

c) $(1+i)^{100} = [(1+i)^2]^{50} = (2i)^{50} = 2^{50} \cdot i^{50} = 2^{50} \cdot (i^2)^{25} = 2^{50} \cdot (-1)^{25}$

$= -2^{50}$

b) $(a+bi)^2 = 3+4i \Leftrightarrow \begin{cases} a^2-b^2=3 \\ 2ab=4 \end{cases} \Leftrightarrow \begin{cases} a^2-b^2=3 \\ b=2/a \end{cases}$

$a^2-\left(\frac{2}{a}\right)^2 = 3 \Leftrightarrow a^4-3a^2-4=0 \quad (a\neq 0)$

$\Leftrightarrow (a^2+4)(a^2-1)=0$

$a^2=-4$ ou $a^2=1$

$a=\pm 2i$ $\quad a=\pm 1$ $\quad S=\{\pm 1; \pm 2i\}$

ex12 a) $\sqrt[3]{8}=z \Leftrightarrow z^3=8 \Leftrightarrow z^3-8=0 \Leftrightarrow (z-2)(z^2+2z+4)=0$

$z=2$ ou $z_{1,2}=\dfrac{-2\pm\sqrt{-12}}{2}=-1\pm\sqrt{-3}$

$=-1\pm\sqrt{3}i$

b) $\sqrt[4]{1+i}=z \Leftrightarrow z^2=1+i \Leftrightarrow (a+b)^2=1+i \Leftrightarrow a^2-b^2+2abi=1+i$

$\Leftrightarrow \begin{cases} a^2-b^2=1 \\ 2ab=1 \end{cases} \Leftrightarrow \begin{cases} a^2-b^2=1 \\ b=\frac{1}{2a} \end{cases}$

d'où $a^2-\left(\frac{1}{2a}\right)^2=1 \Leftrightarrow 4a^4-4a^2-1=0$

$\Delta = 16+16=32$

$a_{1,2}=\dfrac{4\pm\sqrt{32}}{8}=\frac{1}{2}\pm\frac{1}{2}\sqrt{2}$

$a_1=\frac{1}{2}+\frac{1}{2}\sqrt{2} \Rightarrow b_1=\dfrac{1}{1-\sqrt{2}}\dfrac{(1+\sqrt{2})}{(1+\sqrt{2})}=\dfrac{1+\sqrt{2}}{3}$

$a_2=\frac{1}{2}+\frac{1}{2}\sqrt{2} \Rightarrow b_2=\dfrac{1}{1+\sqrt{2}}\dfrac{(1-\sqrt{2})}{(1-\sqrt{2})}=\dfrac{1-\sqrt{2}}{3}$

$S=\left\{\frac{1}{2}-\frac{\sqrt{2}}{2}+\left(\frac{1+\sqrt{2}}{3}\right)i ; \frac{1}{2}-\frac{\sqrt{2}}{2}+\left(\frac{1-\sqrt{2}}{3}\right)i\right\}$

ex13 a) $z=a+bi \Rightarrow \bar{z}=a+(-b)i \Rightarrow \bar{\bar{z}}=a+(-(-b))i=a+bi=z$

c'est vrai

b) $z=a+bi$
$\bar{z}=a-bi$: $z=\bar{z} \Leftrightarrow a+bi=a-bi \Leftrightarrow 2bi=0 \Leftrightarrow b=0$

c'est vrai : tous les nbres complexes de la forme $z=a+0i$ sont égaux à leur conjugué

c) $z=a+bi$
$\bar{z}=a-bi$: $z=-\bar{z} \Leftrightarrow a+bi=-(a-bi) \Leftrightarrow 2a=0 \Leftrightarrow a=0$

c'est vrai, ce sont les imaginaires purs ex17

a) $(1-i)\,\overline{z} - 2 + i = 0$:

$z = a + bi$: $(1-i)(a-bi) - 2 + i = 0$

$(a-b) - (a+b) = 2 - i$

d'où $\begin{cases} a-b=2 \\ a+b=1 \end{cases}$

$2a = 3$

$a = 3/2 \Rightarrow b = -1/2$

$S = \left\{\frac{3}{2} - \frac{1}{2}i\right\}$

ou $\overline{z} = \frac{2-i}{1-i} \cdot \frac{(1+i)}{(1+i)}$

$= \frac{(2+1)+i(2-1)}{1^2+1^2}$

$= \frac{3+i}{2} = \frac{3}{2} + \frac{1}{2}i$

d'où $z = \frac{3}{2} - \frac{1}{2}i$

b) $z^2 + 2z + 5 = 0$:

$z_{1,2} = \frac{-2 \pm \sqrt{4-20}}{2} = \frac{-2 \pm \sqrt{-16}}{2} = -1 \pm 2\sqrt{-1} = 1 \pm 2i$

c) $z^2 + 2iz + 3 = 0$

$z_{1,2} = \frac{-2i \pm \sqrt{4(-1)-12}}{2} = \frac{-2i \pm \sqrt{-16}}{2} = \frac{-2i \pm 4i}{2} \rightarrow z_1 = \frac{-6i}{2} =$

$\rightarrow z_2 = \frac{2i}{2} =$

d) $z^2 + z + 1 = 0$

$z_{1,2} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}i}{2}$

e) $z = a + bi$: $(3+2i)(a+bi) + (1-i)(a-bi) = 1 + 0i$

$\Leftrightarrow (3a-2b) + (3b+2a)i + (a-b) + (-a-b)i = 1 + 0i$

$\Leftrightarrow \begin{cases} 3a-2b+a-b=1 \\ 3b+2a-a-b=0 \end{cases} \Leftrightarrow \begin{cases} 4a-3b=1 \\ a+2b=0 \quad |{-4}| \end{cases}$

$-11b = 1$

$b = -1/11$

puis $a = -2b = 2/11$

$S = \left\{\frac{2}{11} - \frac{1}{11}i\right\}$

f) $2iz + 4 = -3z + i$

on pose $z = a + bi$: $2i(a+bi) + 4 = -3(a+bi) + i$

$2ia - 2b + 4 = -3a - 3bi + i$

$(-2b+4) + 2ai = -3a + (-3b+1)i$

$\Leftrightarrow \begin{cases} -2b+4 = -3a \end{cases} \Leftrightarrow \begin{cases} 3a-2b = -4 \quad |3| \end{cases}$

ex 18

$$\begin{array}{l} (1) \\ (2) \end{array} \left\{ \begin{array}{l} 3z+2w = 7+i \\ 5z-3w = -1+8i \end{array} \right| \begin{array}{l} 3 \\ 2 \end{array}$$

$$19z = 21+3i-2+16i$$

$$\Leftrightarrow 19z = 19+19i$$

$$\Leftrightarrow z = 1+i$$

dans (1) $2w = 7+i-3(1+i) = 7+i-3-3i = 4-2i$

$$\Leftrightarrow w = 2-i$$

$$S = \{(1+i ; 2-i)\}$$

ex. 14 $\overline{\frac{(2-3i)(1+i)}{(3+2i)^2}} = \frac{(2+3i)(1-i)}{(3-2i)^2}\,\frac{(3+2i)^2}{(3+2i)^2} = \frac{(2+3i)(1-i)(3+2i)^2}{(9+4)^2}$

$= \frac{((2+3)+(3-2)i)(9-4+6i)}{169} = \frac{(5+i)(5+6i)}{169} = \frac{19+35i}{169}$

$= \frac{2+5i}{13} = \frac{2}{13}+\frac{5}{13}i$

ex 15 Soit $z \in \mathbb{C}$ solution de $az^2+bz+c=0$

on a : $a\bar{z}^2+b\bar{z}+c = \overline{az^2}+\overline{bz}+\bar{c}$ (1)

$= \overline{az^2+bz+c}$

$= \bar{0}$

$= 0$ cqfd

ex 16 $z = a+bi \rightarrow \bar{z} = a-bi$: $z-\bar{z} = 2bi$

$z\cdot\bar{z} = a^2+b^2$

$\frac{z-\bar{z}}{z\bar{z}} = \frac{2b}{a^2+b^2}i$

$\Rightarrow Re\left(\frac{z-\bar{z}}{z\bar{z}}\right) = 0$