

Ma4A - ch A2

ex 1 a) $L(\alpha \vec{a}) = \alpha L(\vec{a}) = \alpha L\left(\begin{pmatrix} 2 \\ -1 \end{pmatrix}\right) = \alpha \begin{pmatrix} 2 \cdot 2 + 4(-1) \\ 2 + 2(-1) \end{pmatrix} = \alpha \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \vec{0}$
 donc $\{\alpha \vec{a} \mid \alpha \in \mathbb{R}\} \subseteq \text{Ker}(L)$

b) $L\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Leftrightarrow \begin{cases} 2x + 4y = 0 \\ x + 2y = 0 \end{cases} \Leftrightarrow \begin{cases} x + 2y = 0 \\ x + 2y = 0 \end{cases}$

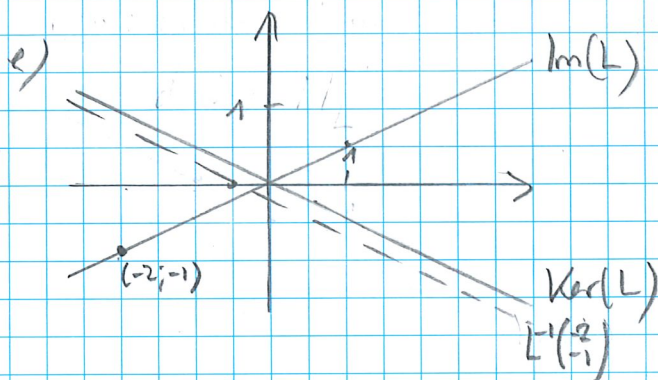
donc $\text{Ker}(L) = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \mid x = -2y \right\} = \left\{ \begin{pmatrix} -2y \\ y \end{pmatrix} \mid y \in \mathbb{R} \right\} = \left\{ y \begin{pmatrix} -2 \\ 1 \end{pmatrix} \mid y \in \mathbb{R} \right\}$

direct: $L\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} 2(x+2y) \\ x+2y \end{pmatrix} = (x+2y) \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ donc $\text{Im}(L) = \left\{ \alpha \begin{pmatrix} 2 \\ 1 \end{pmatrix} \mid \alpha \in \mathbb{R} \right\}$

ou $L\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} x' \\ y' \end{pmatrix} \Leftrightarrow \begin{cases} 2(x+2y) = x' \\ x+2y = y' \\ 0 = x'-2y' \end{cases} \mid \times 2 \text{ } \Leftrightarrow \begin{cases} x+2y = y' \\ x+2y = 2y' \end{cases} \Leftrightarrow \begin{cases} x+2y = y' \\ x+2y = 2y' \end{cases}$ donc $\text{Im}(L) = \left\{ \begin{pmatrix} x' \\ y' \end{pmatrix} \mid x' = 2y' \right\} = \left\{ y' \begin{pmatrix} 2 \\ 1 \end{pmatrix} \mid y' \in \mathbb{R} \right\}$

d) $L\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \Leftrightarrow \begin{cases} 2x + 4y = -2 \\ x + 2y = -1 \end{cases} \Leftrightarrow \begin{cases} x + 2y = -1 \\ x + 2y = -1 \end{cases}$

donc $L^{-1}\left(\begin{pmatrix} -2 \\ -1 \end{pmatrix}\right) = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \mid x + 2y = -1 \right\}$



f) $\dim(\text{Ker}(L)) + \dim(\text{Im}(L)) = 1 + 1 = 2 = \dim \mathbb{R}^2$

ex 2 a) $L\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \vec{0} \Leftrightarrow L(x\vec{e}_1 + y\vec{e}_2) = \vec{0} \Leftrightarrow xL(\vec{e}_1) + yL(\vec{e}_2) = \vec{0}$

$\Leftrightarrow x \begin{pmatrix} 1 \\ 1 \end{pmatrix} + y \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\Leftrightarrow \begin{cases} x - y = 0 \\ x + 2y = 0 \end{cases}$

$\begin{matrix} -3y = 0 \\ y = 0 \end{matrix}$, donc $x = 0$, car $\text{Ker}(L) = \{\vec{0}\}$

$L\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = x \begin{pmatrix} 1 \\ 1 \end{pmatrix} + y \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} x-y \\ x+2y \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$

$\Leftrightarrow \begin{cases} a = x - y \\ b = x + 2y \end{cases}$ car pour $\forall a, b \in \mathbb{R}$, on a $x, y \in \mathbb{R}$
 tq $L\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} a \\ b \end{pmatrix}$

car $\text{Im}(L) = \mathbb{R}^2$

b) $\dim(\text{Ker}(L)) + \dim(\text{Im}(L)) = 0 + 2 = 2 = \dim \mathbb{R}^2$