Ma4A - chAr2

ex1 a) $L(\alpha\vec{a}) = \alpha L(\vec{a}) = \alpha L\begin{pmatrix}2\\-1\end{pmatrix} = \alpha\begin{pmatrix}2\cdot2+4(-1)\\2+2(-1)\end{pmatrix} = \alpha\begin{pmatrix}0\\0\end{pmatrix} = \vec{0}$

donc $\{\alpha\vec{a} \mid \alpha\in\mathbb{R}\} \subseteq Ker(L)$

b) $L\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix} \iff \begin{cases}2x+4y=0\\x+2y=0\end{cases} \iff \begin{cases}x+2y=0\\x+2y=0\end{cases}$

donc $Ker(L) = \left\{\begin{pmatrix}x\\y\end{pmatrix}\in\mathbb{R}^2 \mid x=-2y\right\} = \left\{\begin{pmatrix}-2y\\y\end{pmatrix} \mid y\in\mathbb{R}\right\} = \left\{y\begin{pmatrix}-2\\1\end{pmatrix} \mid y\in\mathbb{R}\right\}$

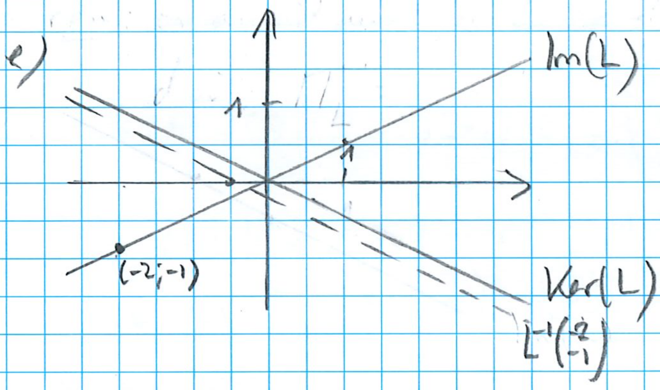
direct : $L\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}2(x+2y)\\x+2y\end{pmatrix} = (x+2y)\begin{pmatrix}2\\1\end{pmatrix}$ donc $Im(L) = \left\{\alpha\begin{pmatrix}2\\1\end{pmatrix} \mid \alpha\in\mathbb{R}\right\}$

ou $L\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x'\\y'\end{pmatrix} \iff \begin{cases}2(x+2y)=x'\\x+2y=y' \quad |2|\end{cases}$; $0 = x'-2y'$ donc $Im(L) = \left\{\begin{pmatrix}x'\\y'\end{pmatrix} \mid x'=2y'\right\} = \left\{y'\begin{pmatrix}2\\1\end{pmatrix} \mid y'\in\mathbb{R}\right\}$

d) $L\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-2\\-1\end{pmatrix} \iff \begin{cases}2x+4y=-2\\x+2y=-1\end{cases} \iff \begin{cases}x+2y=-1\\x+2y=-1\end{cases}$

donc $L^{-1}\begin{pmatrix}-2\\-1\end{pmatrix} = \left\{\begin{pmatrix}x\\y\end{pmatrix}\in\mathbb{R}^2 \mid x+2y=-1\right\}$

e)



f) $\dim(Ker L) + \dim(Im(L)) = 1+1 = 2 = \dim\mathbb{R}^2$

ex2 a) $L\begin{pmatrix}x\\y\end{pmatrix} = \vec{0} \iff L(x\vec{\imath}+y\vec{\jmath}) = \vec{0} \iff xL(\vec{\imath})+yL(\vec{\jmath}) = \vec{0}$

$\iff x\begin{pmatrix}1\\1\end{pmatrix} + y\begin{pmatrix}-1\\2\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$

$\iff \begin{cases}x-y=0\\x+2y=0\end{cases}$

$-3y=0$

$y=0$, donc $x=0$, càd $Ker(L)=\{\vec{0}\}$

$L\begin{pmatrix}x\\y\end{pmatrix} = x\begin{pmatrix}1\\1\end{pmatrix} + y\begin{pmatrix}-1\\2\end{pmatrix} = \begin{pmatrix}x-y\\x+2y\end{pmatrix} = \begin{pmatrix}a\\b\end{pmatrix}$

$\iff \begin{cases}a=x-y\\b=x+2y\end{cases}$ càd que $\forall a,b\in\mathbb{R}$, on a $x,y\in\mathbb{R}$ tq $L\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}a\\b\end{pmatrix}$

càd $Im(L) = \mathbb{R}^2$

b) $\dim(Ker(L)) + \dim(Im(L)) = 0+2 = 2 = \dim\mathbb{R}^2$