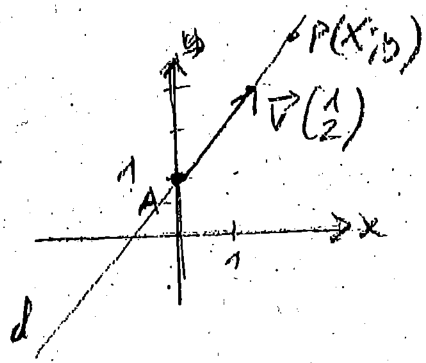
ex1



Soit $P(x;y) \in d$:

$P \in d \iff \overrightarrow{AP} = \lambda \vec{v}$ avec un $\lambda \in \mathbb{R}$

$\iff \begin{pmatrix}x-0\\y-1\end{pmatrix} = \lambda \begin{pmatrix}1\\2\end{pmatrix}$ éq. vectorielle

$\iff \begin{cases} x = \lambda \\ y-1 = 2\lambda \end{cases}$ syst. éq. paramétriques

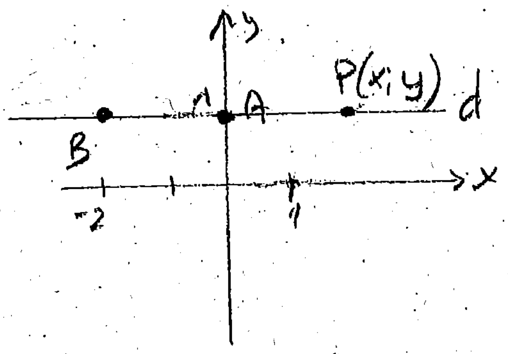
Résoudre le système :

$\begin{matrix} ① \\ ② \end{matrix} \begin{cases} x = \lambda \\ y-1 = 2\lambda \end{cases} \begin{vmatrix} -2 \\ 1 \end{vmatrix}$

$\iff \begin{matrix} ① \\ ② \end{matrix} \begin{cases} -2x = -2\lambda \\ y-1 = 2\lambda \end{cases}$

$+$: $-2x + y - 1 = 0 \iff -2x + y = -1$ éq. cartésienne

ex2



un vecteur directeur de d : $\overrightarrow{AB} = \begin{pmatrix}-2-0\\1-1\end{pmatrix} = \begin{pmatrix}-2\\0\end{pmatrix}$

$P \in d \iff \overrightarrow{AP} = \lambda \overrightarrow{AB}$

$\iff \begin{pmatrix}x-0\\y-1\end{pmatrix} = \lambda \begin{pmatrix}-2\\0\end{pmatrix}$ éq. vectorielle

$\iff \begin{pmatrix}x\\y-1\end{pmatrix} = \begin{pmatrix}-2\lambda\\0\end{pmatrix}$

Résoudre le système : $\begin{matrix} ① \\ ② \end{matrix} \begin{cases} x = -2\lambda \\ y-1 = 0 \end{cases}$ syst. d'éq. paramétriques

② donne immédiatement $y - 1 = 0$

$y = 1$ éq. cart

Remarque : en voyant la repr. graphique, on voyait directement l'éq. cartésienne de d : $y = 1$!

ex3

a) $\vec{AB}\begin{pmatrix}3-12\\2-(-5)\\1/2-(-3)\end{pmatrix}=\begin{pmatrix}-9\\7\\3,5\end{pmatrix}$

$\vec{AC}\begin{pmatrix}0-12\\0-(-5)\\-7-(-3)\end{pmatrix}=\begin{pmatrix}-12\\5\\-4\end{pmatrix}$

$\vec{AB} \neq k\vec{AC}$ ✓

ég. vect : soit $P(x;y;z)\in\pi$: $\vec{AP}=\lambda\vec{AB}+\mu\vec{AC}$

$$\begin{pmatrix}x-12\\y-(-5)\\z-(-3)\end{pmatrix}=\lambda\begin{pmatrix}-9\\7\\3,5\end{pmatrix}+\mu\begin{pmatrix}-12\\5\\-4\end{pmatrix}$$

ég. cat :
$$\begin{cases}① & x-12=-9\lambda-12\mu\\ ② & y+5=7\lambda+5\mu\\ ③ & z+3=3,5\lambda-4\mu\end{cases}$$

$$\begin{array}{l}7\cdot①\\18\cdot③\end{array}\begin{cases}7x-84=-63\lambda-84\mu\\18z+54=63\lambda-72\mu\end{cases}$$
④ $7x+18z-30=-156\mu$

$$\begin{array}{l}7\cdot①\\9\cdot②\end{array}\begin{cases}7x-84=-63\lambda-84\mu\\9y+45=63\lambda+45\mu\end{cases}$$
⑤ $7x+9y-39=-39\mu$

$$\begin{array}{l}④\\4\cdot⑤\end{array}\begin{cases}7x+18z-30=-156\mu\\-28x-36y+156=-156\mu\end{cases}$$
$[\,-21x+36y+18z+126=0\,]$

Vérif : $A\overset{?}{\in}\pi$: $-21\cdot12-36(-5)+18(-3)+126\overset{?}{=}0$ ✓

$B\overset{?}{\in}\pi$: $-21\cdot3-36\cdot2+18\left(\frac{1}{2}\right)+126\overset{?}{=}0$ ✓

$C\overset{?}{\in}\pi$: $0+0-7\cdot18+126\overset{?}{=}0$ ✓

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b) $\vec{AB}\begin{pmatrix}3-2\\4-0\\1-(-2)\end{pmatrix}=\begin{pmatrix}1\\4\\3\end{pmatrix}$

$\vec{AC}\begin{pmatrix}-1-2\\-1-0\\0-(-2)\end{pmatrix}=\begin{pmatrix}-3\\-1\\2\end{pmatrix}$

$\vec{AB}\neq k\vec{AC}$ ✓

ég. vect : $P\in\pi \Leftrightarrow \begin{pmatrix}x-2\\y-0\\z-(-2)\end{pmatrix}=\lambda\begin{pmatrix}1\\4\\3\end{pmatrix}+\mu\begin{pmatrix}-3\\-1\\2\end{pmatrix}$

/.

éq. cart: $\begin{cases} ① & x-2 = \lambda - 3\mu \\ ② & y = 4\lambda - \mu \\ ③ & z+2 = 3\lambda + 2\mu \end{cases}$

$$\begin{array}{rl} ① & x-2 = \lambda - 3\mu \\ -3\cdot② & -3y = -12\lambda + 3\mu \\ \hline ④ & x-3y-2 = -11\lambda \end{array}$$

$$\begin{array}{rl} 2\cdot② & 2y = 8\lambda - 2\mu \\ ③ & z+2 = 3\lambda + 2\mu \\ \hline & 2y+z+2 = 11\lambda \end{array}$$

$$\begin{array}{rl} ④ & x-3y-2 = -11\lambda \\ ⑤ & 2y+z+2 = 11\lambda \\ \hline & [\ x-y+z = 0\ ] \end{array}$$

Vérif: $A \overset{?}{\in} \pi$: $2-0+(-2) \overset{?}{=} 0$ ✓

$B \overset{?}{\in} \pi$: $3-4+1 \overset{?}{=} 0$ ✓

$C \overset{?}{\in} \pi$: $\cancel{2}-(-1)+0 \overset{?}{=} 0$ ✓

c) $\left.\begin{array}{l} \overrightarrow{AB}\begin{pmatrix} 3-4 \\ 5-5 \\ 7-6 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \\ \overrightarrow{AC}\begin{pmatrix} 10-4 \\ 5-5 \\ 1-6 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \\ -5 \end{pmatrix} \end{array}\right\}$ $\overrightarrow{AB} \neq k\overrightarrow{AC}$ ✓

ép. vect: $P(x;y;z) \in \pi \Leftrightarrow \overrightarrow{AP} = \lambda\overrightarrow{AB} + \mu\overrightarrow{AC}$

$\Leftrightarrow \begin{pmatrix} x-4 \\ y-5 \\ z-6 \end{pmatrix} = \lambda\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + \mu\begin{pmatrix} 6 \\ 0 \\ -5 \end{pmatrix}$

ép. cart: $\begin{cases} ① & x-4 = -\lambda + 6\mu \\ ② & y-5 = 0 \\ ③ & z-6 = \lambda - 5\mu \end{cases}$

l'équation est déjà là ! ② $y-5=0$

càd $[\ 0\cdot x + y + 0\cdot z - 5 = 0\ ]$

vérif immédiate : $A, B, C \in \pi$ ✓

d) $\vec{AB}\begin{pmatrix} 3 - (-3) \\ -4 - 2 \\ 20 - 5 \end{pmatrix} = \begin{pmatrix} 6 \\ -6 \\ 15 \end{pmatrix}$

$\vec{AC}\begin{pmatrix} 0 - (-3) \\ 0 - 2 \\ 10 - 5 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix}$

$\left.\right\} \vec{AB} \neq k\vec{AC}$ ✓

<u>éq. vect :</u> $P(x;y;z) \in \Pi \Leftrightarrow \vec{AP} = \lambda\vec{AB} + \mu\vec{AC}$

$\Leftrightarrow \begin{pmatrix} x - (-3) \\ y - 2 \\ z - 5 \end{pmatrix} = \lambda\begin{pmatrix} 6 \\ -6 \\ 15 \end{pmatrix} + \mu\begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix}$

<u>éq. cart :</u> $\begin{cases} x+3 = 6\lambda + 3\mu & (1) \\ y-2 = -6\lambda - 2\mu & (2) \\ z-5 = 15\lambda + 5\mu & (3) \end{cases}$

(1) $x+3 = 6\lambda + 3\mu$
(2) $y-2 = -6\lambda - 2\mu$

$x+y+1 = \mu$

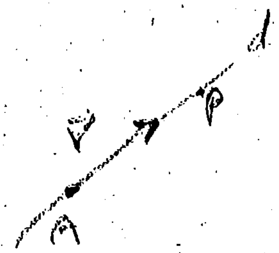
$5\cdot(2)$ $5y - 10 = -30\lambda - 10\mu$
$2\cdot(3)$ $2z - 10 = 30\lambda + 10\mu$

$5y + 2z - 20 = 0$

on obtient l'équation de Π

<u>vérif:</u>
$A \in \Pi$: $5(2) + 2(5) - 20 = 0$ ✓
$B \in \Pi$: $5(-4) + 2\cdot 20 - 20 = 0$ ✓
$C \in \Pi$: $5(0) + 2\cdot 10 - 20 = 0$ ✓

ex4



$P(x;y;z) \in d \Leftrightarrow \overrightarrow{AP} = \lambda \vec{v}$

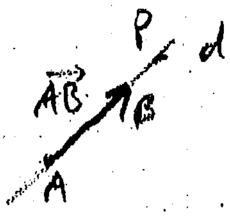
$\Leftrightarrow \begin{pmatrix} x+2 \\ y-1 \\ z-3 \end{pmatrix} = \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ éq. vectorielle de d

$\Leftrightarrow \begin{cases} ① \ x+2 = 2\lambda \\ ② \ y-1 = \lambda \\ ③ \ z-3 = -\lambda \end{cases}$ Syst éq paramétriques de d

$\begin{cases} ① \ \frac{x+2}{2} = \lambda \\ ② \ y-1 = \lambda \\ ③ \ \frac{z-3}{-1} = \lambda \end{cases}$ d'où $\lambda = \left[\frac{x+2}{2} = y-1 = -z+3\right]$

$\boxed{x+2 = 2y \text{ et } y+z = 4}$ éq. cart de d

ex5



$P(x;y;z) \in d \Leftrightarrow \overrightarrow{AP} = \lambda \overrightarrow{AB}$

$\Leftrightarrow \begin{pmatrix} x-0 \\ y-1 \\ z-0 \end{pmatrix} = \lambda \begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix}$ éq. vect de d

$\Leftrightarrow \begin{cases} ① \ x = -2\lambda \\ ② \ y-1 = 0 \\ ③ \ z = 3\lambda \end{cases}$

$\Leftrightarrow \begin{cases} ① \ -\frac{x}{2} = \lambda \\ ② \ y-1 = 0 \\ ③ \ \frac{z}{3} = \lambda \end{cases}$ d'où $\lambda = \left[-\frac{x}{2} = \frac{z}{3}\right]$ et $y = 1$

$\boxed{-3x = 2z \text{ et } y = 1}$ éq. cart de d