ex 26 a) $\mu = 2{,}8 = \bar{x}$ cf calculatrice

$\sigma \cong 2{,}27$

$V(X) \cong 5{,}16$

b) $\mu = 100.2 = \bar{x}$

$\sigma \cong 0{,}9798$

$V(X) = 0{,}96$

c) $\mu = \sum_{i=1}^{n} i \frac{1}{n} = \frac{1}{n} \sum_{i=1}^{n} i = \frac{1}{n} \frac{n(n+1)}{2} = \frac{n+1}{2}$

$$V(X) = E(X^2) - \mu^2 = \sum_{i=1}^{n} i^2 \frac{1}{n} - \left(\frac{n+1}{2}\right)^2 = \frac{1}{\cancel{n}} \frac{\cancel{n}(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

$$= \frac{(n+1)}{6}\left[2n+1 - \frac{3}{2}(n+1)\right] = \frac{n+1}{6} \cdot \left[\frac{n}{2} - \frac{1}{2}\right] = \frac{(n+1)(n-1)}{12} = \frac{n^2-1}{12}$$

ex 27

Si $X = cte$:

x	cte
$f_X(x)$	1

$\Rightarrow E(X) = \mu = cte$

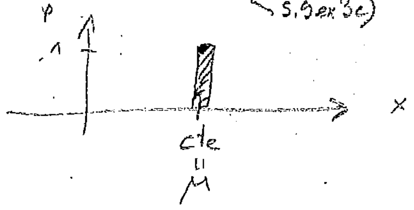
Posons $Y = X^2$:

x	$(cte)^2$
$f_Y(x)$	1

$\rightarrow E(Y) = (cte)^2$

d'où $V(X) \underset{\text{s.9 ex 3c)}}{=} E(X^2) - \mu^2 = E(Y) - (cte)^2 = (cte)^2 - (cte)^2 = 0$

Int. géom :



pas de dispersion autour de μ

b) $$V(\alpha + X) \overset{\text{s.9 ex 3c)}}{=} E((\alpha+X)^2) - [E(\alpha+X)]^2 = E(\alpha^2 + 2\alpha X + X^2) - [E(\alpha) + E(X)]^2$$

$$= E(\alpha^2) + 2\alpha E(X) + E(X^2) - [\alpha + E(X)]^2$$

$$= \cancel{\alpha^2} + \cancel{2\alpha E(X)} + E(X^2) - \cancel{\alpha^2} - \cancel{2\alpha E(X)} - [E(X)]^2$$

$$= E(X^2) - [E(X)]^2 = V(X)$$

c) $$V(\alpha X) = E((\alpha X)^2) - [E(\alpha X)]^2 = E(\alpha^2 X^2) - [\alpha E(X)]^2$$

$$= \alpha^2 E(X^2) - \alpha^2 [E(X)]^2 = \alpha^2 \left(E(X^2) - [E(X)]^2\right) = \alpha^2 V(X)$$

ex 28 a) cf. p14 ex 2

x	0	100
$f_X(x)$	$1/2$	$1/2$

$\Rightarrow E(X) = 50$

x	0	10000
$f_{X^2}(x)$	$1/2$	$1/2$

$\Rightarrow E(X^2) = 5000$

$\hookrightarrow V(X) = E(X^2) - (E(X))^2 = 5000 - 50^2 = 2500$

y	0	10
$f_Y(y)$	$1/2$	$1/2$

$\Rightarrow E(Y) = 5$

x	0	100
$f_{Y^2}(x)$	$1/2$	$1/2$

$\Rightarrow E(Y^2) = 50$

$\hookrightarrow V(Y) = E(Y^2) - (E(Y))^2 = 50 - 5^2 = 25$

x	10	100
$f_{X+Y}(x)$	$1/2$	$1/2$

$\Rightarrow E(X+Y) = \dots = 55$

x	100	10000
$f_{(X+Y)^2}(x)$	$1/2$	$1/2$

$\Rightarrow E((X+Y)^2) = \dots = 5050$

$\hookrightarrow V(X+Y) = \dots = 5050 - 55^2 = 2025$

donc $V(X+Y) \neq V(X) + V(Y)$

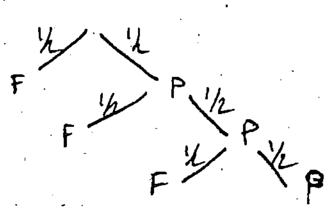
b) Thm: X, Y v.a. indép. $\Rightarrow V(X+Y) = V(X) + V(Y)$

dém:
$$\begin{aligned} V(X+Y) &= E((X+Y)^2) - [E(X+Y)]^2 \quad \downarrow \text{car} \dots \\ &= E(X^2 + 2XY + Y^2) - [E(X) + E(Y)]^2 \quad \downarrow \text{car} \dots \\ &= E(X^2) + 2E(XY) + E(Y^2) - (E(X))^2 - 2E(X)E(Y) - (E(Y))^2 \\ &= E(X^2) - (E(X))^2 + E(Y^2) - (E(Y))^2 + \cancel{2E(X)E(Y) - 2E(X)E(Y)} \\ &= V(X) + V(Y) \end{aligned}$$

ex 30 $V(X) = 0 \underset{\text{car}\dots}{\Leftrightarrow} E((X - E(X))^2) = 0 \underset{\text{car}\dots}{\Leftrightarrow} X - E(X) = 0 \Leftrightarrow X = E(X)$
$\underset{\text{car}\dots}{\Leftrightarrow} X = \text{cte}$

ex 29 $E(X) = E(Y) = 3{,}5 \qquad E(X+Y) = E(2X) = 7$
$V(X) = V(Y) = 5{,}16 \qquad V(X+Y) = 5{,}8\overline{3} \qquad V(2X) = 11{,}\overline{6}$

ex 31



X	1	2	3
$f_X(x)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}+\frac{1}{8}$

$$E(X) = 1\cdot\frac{1}{2} + 2\cdot\frac{1}{4} + 3\cdot\frac{1}{4} = \frac{7}{4}$$

Posons $Y = X^2$:

x	1	4	9
$f_Y(x)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

$$E(Y) = 1\cdot\frac{1}{2} + 4\cdot\frac{1}{4} + 9\cdot\frac{1}{4} = \frac{15}{4}$$

D'où $V(X) = E(X^2) - [E(X)]^2 = \frac{15}{4} - \left(\frac{7}{4}\right)^2 = \frac{60-49}{16} = \frac{11}{16}$

---

ex 32

a) X = nbre de 5, $n = 15$, $p = 1/6$, $k = 7$ $\Rightarrow$ $p(X=7) = C_7^{15}\left(\frac{1}{6}\right)^7\left(1-\frac{1}{6}\right)^{15-7} = C_7^{15}\left(\frac{1}{6}\right)^7\left(\frac{5}{6}\right)^8$

$\approx 0{,}58\ \%$

b) X = nbre de $\overline{2}$, $n = 15$, $p = 5/6$, $k = 9$ $\Rightarrow$ $p(X=9) = C_9^{15}\left(\frac{5}{6}\right)^9\left(1-\frac{5}{6}\right)^{15-9} = C_9^{15}\left(\frac{5}{6}\right)^9\left(\frac{1}{6}\right)^6$

$\approx 2{,}08\ \%$

c) idem b) avec $k = 13$: $P(X=13) = C_{13}^{15}\left(\frac{5}{6}\right)^{13}\left(\frac{1}{6}\right)^2 \approx 27{,}[illegible]\%$

d) X = multiple de 3, càd 3 ou 6, $n = 15$, $p = \frac{1}{3}$, $k = 4$ $\Rightarrow$ $p(X=4) = C_4^{15}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^{11} \approx 19{,}48\%$

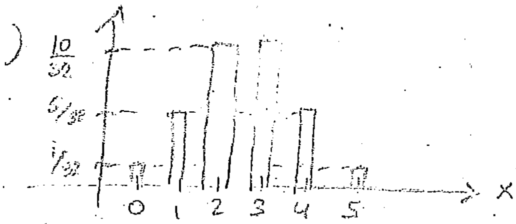
e) X = nbre de 6, $n = 15$, $p = \frac{1}{6}$, $k \geq 2$ $\Rightarrow$ $p(X \geq 2) = 1 - p(X < 2) = 1 - [p(X=0) + p(X=1)]$

$= 1 - \left[C_0^{15}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{15} + C_1^{15}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{14}\right]$

$\approx$ [illegible] %

ex 33

a)



$p(X=0) = C_0^5 \left(\frac{1}{2}\right)^5 = \frac{1}{32}$

$p(X=1) = C_1^5 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^4 = 5 \cdot \frac{1}{32}$

$p(X=2) = C_2^5 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^3 = 10 \cdot \frac{1}{32}$

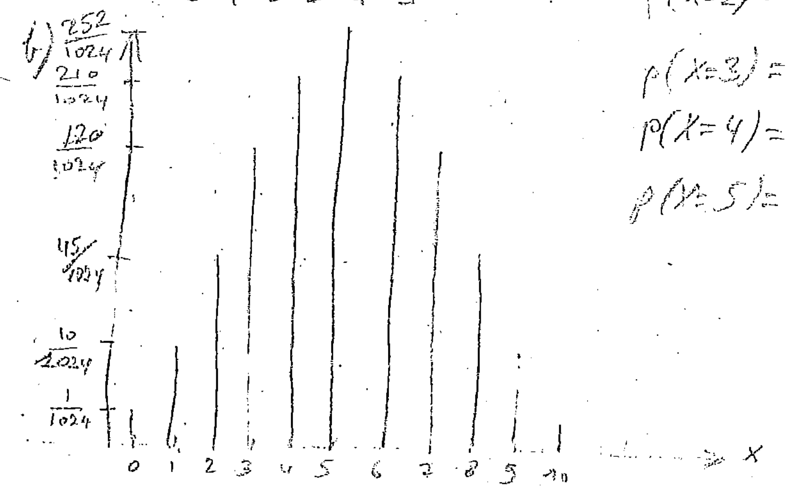
$p(X=3) = \ldots = 10 \cdot \frac{1}{32}$

$p(X=4) = \ldots = 5 \cdot \frac{1}{32}$

$p(X=5) = \ldots = \frac{1}{32}$

Remarque : $P(X=0) + \ldots + P(X=5) = 1$!!! (ok)

b)



c) et d) semblable...

ex 34

a) X = nbre de P

$n = 100$

$p = \frac{1}{2}$

$X \sim B(100 ; \frac{1}{2})$

$k = 48$

$\Rightarrow p(X=48) = C_{48}^{100} \left(\frac{1}{2}\right)^{48} \left(1-\frac{1}{2}\right)^{100-48}$

$= C_{48}^{100} \left(\frac{1}{2}\right)^{48} \left(\frac{1}{2}\right)^{52}$

$= C_{48}^{100} \frac{1}{2^{100}} \cong 7,35\,\%$

b) X = nbre de F

$n = 100$

$p = \frac{1}{2}$

$X \sim B(100 ; \frac{1}{2})$

$k = 17$

$\Rightarrow p(X=17) = C_{17}^{100} \left(\frac{1}{2}\right)^{17} \left(\frac{1}{2}\right)^{83}$

$\cong 5,25 \cdot 10^{-10}\ \%$

c) X = nbre de P

$n = 100$

$p = \frac{1}{2}$

$X \sim B(100 ; \frac{1}{2})$

$k \geq 2$

$\Rightarrow p(X \geq 2) = 1 - p(X < 2)$

$= 1 - \left[P(X=0) + P(X=1)\right]$

$= 1 - \left[C_0^{100} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{100} + C_1^{100} \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{99}\right]$

$\cong 0,999...$

ex 35

a) $X \sim B(10; \frac{95}{100})$ $p(X \geq 9) = p(X=9) + p(X=10)$

$= C_9^{10}\, 0{,}95^9\, 0{,}05^1 + C_{10}^{10}\, 0{,}95^{10}\, 0{,}05^0 \simeq 91{,}4\%$

b) $X \sim B(10; \frac{95}{100})$ $p(X \geq 9) = \ldots = C_9^{10}\, 0{,}95^9\, 0{,}05 + C_{10}^{10}\, 0{,}95^{10}\, 0{,}05^0 \simeq 91{,}4\%$

ex 36

$X \sim B(5; \frac{1}{5})$

a) $p(X=1) = C_1^5 \left(\frac{1}{5}\right)^1 \left(\frac{4}{5}\right)^{5-1} \simeq 40{,}96\%$

b) $p(X \geq 1) = 1 - p(X < 1) = 1 - p(X=0)$

$= 1 - C_0^5 \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^5 \simeq 67{,}23\%$

$X \sim B(20; \frac{1}{5})$

c) $p(X=4) = C_4^{20} \left(\frac{1}{5}\right)^4 \left(\frac{4}{5}\right)^{16} \simeq 22\%$

$p(X \geq 4) = 1 - [p(X=0) + p(X=1) + p(X=2) + p(X=3)]$

$= 1 - \left[C_0^{20} \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^{20} + \cdots + C_3^{20} \left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right)^{17}\right] = \ldots \simeq 59\%$

ex 37

$X_n \sim B(n; 0{,}01)$

$p(X_n \geq 1) = 1 - p(X_n = 0) = 1 - C_0^n (0{,}01)^0 (0{,}99)^n$

$= 1 - 1 \cdot 0{,}99^n$

On veut n tel que $0{,}99 \geq 1 - 0{,}99^n$

$\Leftrightarrow 0{,}99^n \geq 0{,}01$

$\Leftrightarrow \ln(0{,}99^n) \geq \ln(0{,}01)$

$\Leftrightarrow n \ln(0{,}99) \geq \ln(0{,}01)$

$\Leftrightarrow n \geq \frac{\ln(0{,}01)}{\ln(0{,}99)} \simeq 458{,}21$

il faut au moins 459 personnes !