ex17

a) $A(\alpha\vec{v}+\beta\vec{w}) = A\begin{pmatrix}\alpha v_1+\beta w_1\\ \alpha v_2+\beta w_2\end{pmatrix} = \begin{pmatrix}-(\alpha v_1+\beta w_1)\\ \alpha v_2+\beta w_2\end{pmatrix} = \begin{pmatrix}-\alpha v_1-\beta w_1\\ \alpha v_2+\beta w_2\end{pmatrix}$

$\alpha A(\vec{v})+\beta A(\vec{w}) = \alpha\begin{pmatrix}-v_1\\ v_2\end{pmatrix}+\beta\begin{pmatrix}-w_1\\ w_2\end{pmatrix} = \begin{pmatrix}-\alpha v_1\\ \alpha v_2\end{pmatrix}+\begin{pmatrix}-\beta w_1\\ \beta w_2\end{pmatrix} = \begin{pmatrix}-\alpha v_1-\beta w_1\\ \alpha v_2+\beta w_2\end{pmatrix}$

donc A est linéaire

b) $B\left(2\begin{pmatrix}1\\0\end{pmatrix}\right) \overset{?}{=} 2B\begin{pmatrix}1\\0\end{pmatrix} \Leftrightarrow B\begin{pmatrix}2\\0\end{pmatrix} \overset{?}{=} 2B\begin{pmatrix}1\\0\end{pmatrix}$

$\Leftrightarrow \begin{pmatrix}4\\-1\end{pmatrix} \overset{?}{=} 2\begin{pmatrix}3\\-1\end{pmatrix} \Leftrightarrow \begin{pmatrix}4\\-1\end{pmatrix} \overset{?}{=} \begin{pmatrix}6\\-2\end{pmatrix}$ non

donc B n'est pas linéaire

c) $C(\alpha\vec{v}+\beta\vec{w}) = C\begin{pmatrix}\alpha v_1+\beta w_1\\ \alpha v_2+\beta w_2\end{pmatrix} = \begin{pmatrix}\alpha v_2+\beta w_2\\ \alpha v_1+\beta w_1\end{pmatrix}$

$\alpha C(\vec{v})+\beta C(\vec{w}) = \alpha\begin{pmatrix}v_2\\ v_1\end{pmatrix}+\beta\begin{pmatrix}w_2\\ w_1\end{pmatrix} = \begin{pmatrix}\alpha v_2+\beta w_2\\ \alpha v_1+\beta w_1\end{pmatrix}$

donc C est linéaire

remarque

ou

2 façons différentes d'écrire la démo de linéarité... à vous de choisir celle qui vous convient le mieux

d) $D\left(2\begin{pmatrix}1\\1\end{pmatrix}\right) \overset{?}{=} 2D\begin{pmatrix}1\\1\end{pmatrix} \Leftrightarrow D\begin{pmatrix}2\\2\end{pmatrix} \overset{?}{=} 2D\begin{pmatrix}1\\1\end{pmatrix}$

$\Leftrightarrow \begin{pmatrix}4\\0\end{pmatrix} \overset{?}{=} 2\begin{pmatrix}1\\0\end{pmatrix} \Leftrightarrow \begin{pmatrix}4\\0\end{pmatrix} \overset{?}{=} \begin{pmatrix}2\\0\end{pmatrix}$ non

donc D n'est pas linéaire

e) $G\left(2\begin{pmatrix}0\\1\end{pmatrix}\right) \overset{?}{=} 2G\begin{pmatrix}0\\1\end{pmatrix} \Leftrightarrow G\begin{pmatrix}0\\2\end{pmatrix} \overset{?}{=} 2\cdot\begin{pmatrix}0\\3\end{pmatrix} \Leftrightarrow \begin{pmatrix}0\\3\end{pmatrix} \overset{?}{=} \begin{pmatrix}0\\6\end{pmatrix}$ non

donc G n'est pas linéaire

f) $F(\alpha\vec{v}+\beta\vec{w}) \overset{?}{=} \alpha F(\vec{v})+\beta F(\vec{w})$

$\Leftrightarrow F\begin{pmatrix}\alpha v_1+\beta w_1\\ \alpha v_2+\beta w_2\end{pmatrix} \overset{?}{=} \alpha\begin{pmatrix}v_2\\ v_1+v_2\end{pmatrix}+\beta\begin{pmatrix}w_2\\ w_1+w_2\end{pmatrix}$

$\Leftrightarrow \begin{pmatrix}\alpha v_2+\beta w_2\\ \alpha v_1+\beta w_1+\alpha v_2+\beta w_2\end{pmatrix} \overset{?}{=} \begin{pmatrix}\alpha v_2+\beta w_2\\ \alpha(v_1+v_2)+\beta(w_1+w_2)\end{pmatrix}$

$\Leftrightarrow \begin{pmatrix}\alpha v_2+\beta w_2\\ \alpha v_1+\beta w_1+\alpha v_2+\beta w_2\end{pmatrix} \overset{?}{=} \begin{pmatrix}\alpha v_2+\beta w_2\\ \alpha v_1+\alpha v_2+\beta w_1+\beta w_2\end{pmatrix}$ oui !

F est linéaire

g) $K(\alpha\vec{v}+\beta\vec{w}) \overset{?}{=} \alpha K(\vec{v})+\beta K(\vec{w})$

$\Leftrightarrow -3(\alpha\vec{v}+\beta\vec{w}) \overset{?}{=} \alpha(-3\vec{v})+\beta(-3\vec{w})$

$\Leftrightarrow -3\alpha\vec{v}-3\beta\vec{w} \overset{?}{=} \alpha(-3)\vec{v}+\beta(-3)\vec{w}$

$\Leftrightarrow -3\alpha\vec{v}-3\beta\vec{w} \overset{?}{=} -3\alpha\vec{v}-3\beta\vec{w}$ oui

K est linéaire

ex 18

T ? $T(\vec{0}) = \vec{t} \neq \vec{0}$ T pas linéaires

H ? cas 1 : Si $C \neq (0;0)$

$H(\vec{0}) \neq \vec{0}$ H pas linéaire

cas 2 Si $C = (0;0)$

$$H(\alpha\vec{v} + \beta\vec{w}) \overset{?}{=} \alpha \cdot H(\vec{v}) + \beta \cdot H(\vec{w})$$

$$\Leftrightarrow r(\alpha\vec{v} + \beta\vec{w}) \overset{?}{=} \alpha \cdot r\vec{v} + \beta \cdot r\vec{w}$$

$$\Leftrightarrow r \cdot \alpha\vec{v} + r \cdot \beta\vec{w} \overset{?}{=} \alpha r \vec{v} + \beta r \vec{w}$$ oui

H linéaire dans ce cas

R ? cas 1 : Si $C \neq (0;0)$

$R(\vec{0}) \neq \vec{0}$ R pas linéaire

cas 2 : rotation de centre $C(0;0)$ et d'angle θ

dém : $R(\alpha\vec{v} + \beta\vec{w}) \overset{?}{=} \alpha R(\vec{v}) + \beta R(\vec{w})$

$$R\left(\alpha\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} + \beta\begin{pmatrix} w_1 \\ w_2 \end{pmatrix}\right) \overset{?}{=} \alpha R\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} + \beta R\begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

$$R\begin{pmatrix} \alpha v_1 + \beta w_1 \\ \alpha v_2 + \beta w_2 \end{pmatrix} \overset{?}{=} \alpha R\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} + \beta R\begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

$$\begin{pmatrix} \cos(\theta)[\alpha v_1 + \beta w_1] - \sin(\theta)[\alpha v_2 + \beta w_2] \\ \sin(\theta)[\alpha v_1 + \beta w_2] + \cos(\theta)[\alpha v_2 + \beta w_2] \end{pmatrix}$$

$$\overset{?}{=} \alpha\begin{pmatrix} \cos(\theta)v_1 - \sin(\theta)v_2 \\ \sin(\theta)v_1 + \cos(\theta)v_2 \end{pmatrix} + \beta\begin{pmatrix} \cos(\theta)w_1 - \sin(\theta)w_2 \\ \sin(\theta)w_1 + \cos(\theta)w_2 \end{pmatrix}$$

OK

donc elles sont linéaires

S / Pr cas 1 : l'axe de symétrie ou la droite sur laquelle on projette ne contiennent pas le point $(0;0)$

$S(\vec{0}) \neq \vec{0}$; $Pr(\vec{0}) \neq \vec{0}$ donc pas linéaire

cas 2 : l'axe de symétrie contient $O(0;0)$

la droite de projection " "

Sans dém : les applications sont linéaires

ex 19

on écrit $\vec{i}\begin{pmatrix}1\\0\end{pmatrix}$ en fonction de $\begin{pmatrix}3\\1\end{pmatrix}$ et $\begin{pmatrix}-1\\0\end{pmatrix}$:

$$\begin{pmatrix}1\\0\end{pmatrix} = \alpha\begin{pmatrix}3\\1\end{pmatrix} + \beta\begin{pmatrix}-1\\0\end{pmatrix} \iff \begin{cases} ① \; 1 = 3\alpha - \beta \\ ② \; 0 = \alpha \end{cases}$$

dans ① : $1 = 3 \cdot 0 - \beta \iff \beta = -1$

d'où $F_1\begin{pmatrix}1\\0\end{pmatrix} = F_1\left(0\begin{pmatrix}3\\1\end{pmatrix} + (-1)\begin{pmatrix}-1\\0\end{pmatrix}\right) = F_1\left((-1)\cdot\begin{pmatrix}-1\\0\end{pmatrix}\right) = (-1)\cdot F_1\begin{pmatrix}-1\\0\end{pmatrix}$

$= (-1)\cdot\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}-1\\-1\end{pmatrix}$

idem pour $\begin{pmatrix}0\\1\end{pmatrix}$:

$$\begin{pmatrix}0\\1\end{pmatrix} = \gamma\begin{pmatrix}3\\1\end{pmatrix} + \delta\begin{pmatrix}-1\\0\end{pmatrix} \iff \begin{cases} ① \; 0 = 3\gamma - \delta \\ ② \; 1 = \gamma \end{cases}$$

dans ① : $0 = 3 \cdot 1 - \delta \iff \delta = 3$

d'où $F_1\begin{pmatrix}0\\1\end{pmatrix} = F_1\left(1\cdot\begin{pmatrix}3\\1\end{pmatrix} + 3\begin{pmatrix}-1\\0\end{pmatrix}\right) = 1\cdot F_1\begin{pmatrix}3\\1\end{pmatrix} + 3F_1\begin{pmatrix}-1\\0\end{pmatrix}$

$= 1\cdot\begin{pmatrix}1\\2\end{pmatrix} + 3\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}1\\2\end{pmatrix} + \begin{pmatrix}3\\3\end{pmatrix} = \begin{pmatrix}4\\5\end{pmatrix}$

et

$F_1\begin{pmatrix}x\\y\end{pmatrix} = F_1\left(x\begin{pmatrix}1\\0\end{pmatrix} + y\begin{pmatrix}0\\1\end{pmatrix}\right) = x\,F_1\begin{pmatrix}1\\0\end{pmatrix} + y\,F_1\begin{pmatrix}0\\1\end{pmatrix}$

$= x\begin{pmatrix}-1\\-1\end{pmatrix} + y\begin{pmatrix}4\\5\end{pmatrix} = \begin{pmatrix}-x+4y\\-x+5y\end{pmatrix}$

Pour F_2 : $\vec{i} = \alpha\begin{pmatrix}-1\\0\end{pmatrix} + \beta\begin{pmatrix}-1\\1\end{pmatrix} \iff \begin{cases} 1 = -\alpha - \beta \\ 0 = \beta \end{cases}$

d'où $\beta = 0$ et $\alpha = -1$

$\vec{j} = \delta\begin{pmatrix}-1\\0\end{pmatrix} + \gamma\begin{pmatrix}-1\\1\end{pmatrix} \iff \begin{cases} 0 = -\delta - \gamma \\ 1 = \gamma \end{cases}$

d'où $\gamma = 1$ et $\delta = -1$

on a donc : $\vec{i} = (-1)\cdot\begin{pmatrix}-1\\0\end{pmatrix}$

$\vec{j} = (-1)\begin{pmatrix}-1\\0\end{pmatrix} + 1\cdot\begin{pmatrix}-1\\1\end{pmatrix}$

puis $F_2(\vec{i}) = F_2\left((-1)\begin{pmatrix}-1\\0\end{pmatrix}\right) = (-1)F_2\begin{pmatrix}-1\\0\end{pmatrix} = (-1)\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}-1\\-1\end{pmatrix}$

$F_2(\vec{j}) = F_2\left((-1)\begin{pmatrix}-1\\0\end{pmatrix} + 1\begin{pmatrix}-1\\1\end{pmatrix}\right) = (-1)F_2\begin{pmatrix}-1\\0\end{pmatrix} + 1\cdot F_2\begin{pmatrix}-1\\1\end{pmatrix}$

$= (-1)\begin{pmatrix}1\\1\end{pmatrix} + 1\cdot\begin{pmatrix}6\\3\end{pmatrix} = \begin{pmatrix}5\\2\end{pmatrix}$

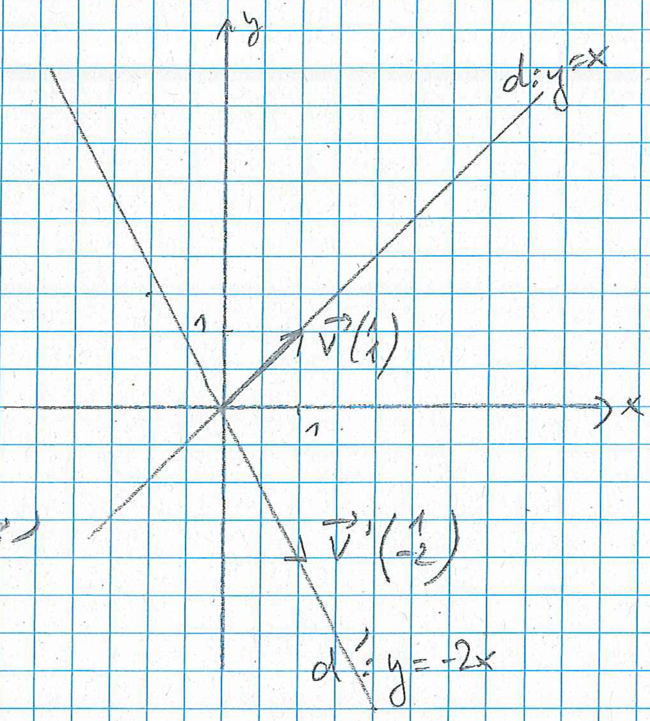
et

$F_2\begin{pmatrix}x\\y\end{pmatrix} = F_2\left(x\cdot\begin{pmatrix}1\\0\end{pmatrix} + y\begin{pmatrix}0\\1\end{pmatrix}\right) = F_2(x\vec{i} + y\vec{j})$

$= x\,F_2(\vec{i}) + y\,F_2(\vec{j}) = x\begin{pmatrix}-1\\-1\end{pmatrix} + y\begin{pmatrix}5\\2\end{pmatrix} = \begin{pmatrix}-x+5y\\-x+2y\end{pmatrix}$

ex 20* a) On considère un vecteur directeur de d : $\vec{v_d} = \begin{pmatrix}1\\1\end{pmatrix}$

on veut $F(\vec{v})$ soit un vecteur directeur de d', càd $F(\vec{v}) = \vec{v}' = \begin{pmatrix}1\\-2\end{pmatrix}$



ainsi, on aura :

$F(\alpha \vec{v}) = \alpha F(\vec{v}) = \alpha \vec{v}'$

càd F envoie d sur d'

Calculs : $F\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}1\\-2\end{pmatrix}$

$\Leftrightarrow F\left(1\cdot\begin{pmatrix}1\\0\end{pmatrix} + 1\cdot\begin{pmatrix}0\\1\end{pmatrix}\right) = \begin{pmatrix}1\\-2\end{pmatrix}$

$\Leftrightarrow 1\cdot F\begin{pmatrix}1\\0\end{pmatrix} + 1\cdot F\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\-2\end{pmatrix}$

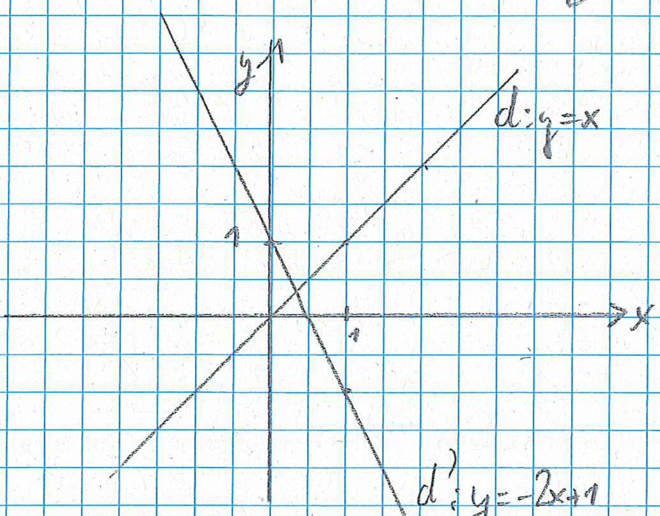
$\Leftrightarrow F\begin{pmatrix}1\\0\end{pmatrix} + F\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\-2\end{pmatrix}$

On peut - par exemple - choisir $F\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$ et $F\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\-2\end{pmatrix}$

d'où : $F\begin{pmatrix}x\\y\end{pmatrix} = x F\begin{pmatrix}1\\0\end{pmatrix} + y F\begin{pmatrix}0\\1\end{pmatrix}$

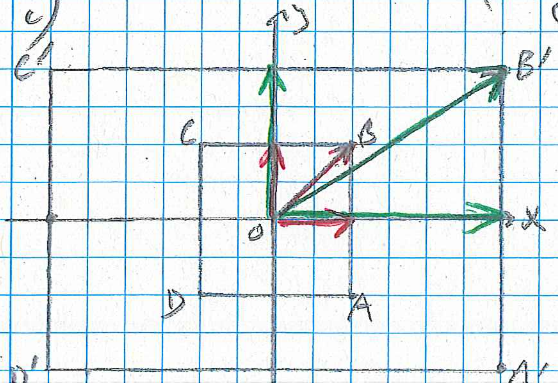
$= x\begin{pmatrix}1\\0\end{pmatrix} + y\begin{pmatrix}0\\-2\end{pmatrix} = \begin{pmatrix}x\\-2y\end{pmatrix}$

b)



On aurait $F(\vec{0}) \neq \vec{0}$, car $(0,0) \notin d'$ donc F ne peut pas être linéaire

c)



On veut, par ex, que $F(\vec{OB}) = \vec{OB'}$

càd $F\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}3\\2\end{pmatrix} \Leftrightarrow F\left(\begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix}\right) = \begin{pmatrix}3\\2\end{pmatrix}$

ou encore $F\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}3\\0\end{pmatrix}$ et $F\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\2\end{pmatrix}$

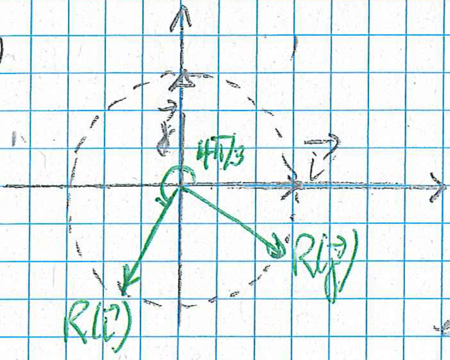
d'où $F\begin{pmatrix}x\\y\end{pmatrix} = x F\begin{pmatrix}1\\0\end{pmatrix} + y F\begin{pmatrix}0\\1\end{pmatrix} = x\begin{pmatrix}3\\0\end{pmatrix} + y\begin{pmatrix}0\\2\end{pmatrix} = \begin{pmatrix}3x\\2y\end{pmatrix}$

ex 22

a) $L\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix}$ et $L\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}0\\1\end{pmatrix}$ donc $M_L=\begin{pmatrix}1&0\\0&1\end{pmatrix}$

b) $L\begin{pmatrix}1\\0\end{pmatrix}=3\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}3\\0\end{pmatrix}$ et $L\begin{pmatrix}0\\1\end{pmatrix}=3\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}0\\3\end{pmatrix}$ donc $M=\begin{pmatrix}3&0\\0&3\end{pmatrix}$

c)



$R(\vec i)=\begin{pmatrix}\cos(\theta)\\ \sin(\theta)\end{pmatrix}$ et $R(\vec j)=\begin{pmatrix}-\sin(\theta)\\ \cos(\theta)\end{pmatrix}$

d'où $M_R=\begin{pmatrix}\cos(\theta) & -\sin(\theta)\\ \sin(\theta) & \cos(\theta)\end{pmatrix}$

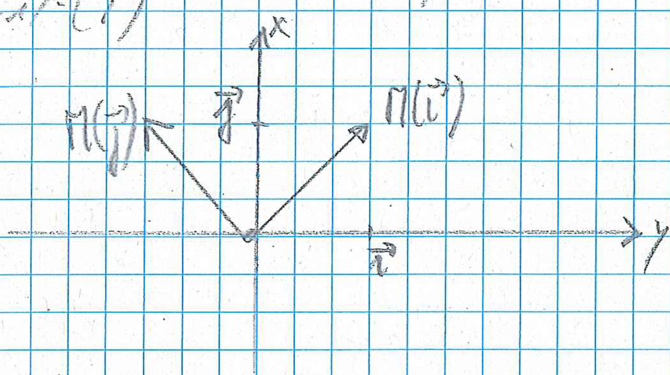
et $M_{R_{4\pi/3}}=\begin{pmatrix}\sqrt{3}/2 & -1/2\\ +1/2 & \sqrt{3}/2\end{pmatrix}$

ex 23

a) $F\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix}$

$F\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}-1\\1\end{pmatrix}$

$M_F=\begin{pmatrix}1&-1\\1&1\end{pmatrix}$

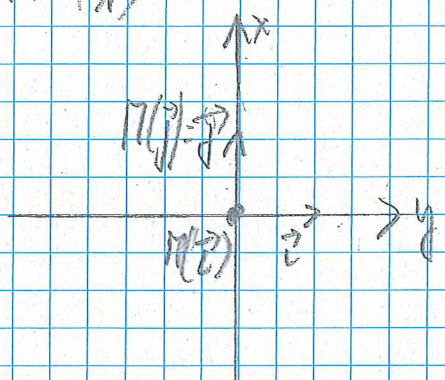


c'est une rotation d'angle $\pi/2$

b) $F\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$

$F\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}0\\1\end{pmatrix}$

$M_F=\begin{pmatrix}0&0\\0&1\end{pmatrix}$



c'est une projection sur l'axe Oy

ex 24

a) $F\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix}$, $F\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}0\\-1\end{pmatrix}$ $\quad M_F=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$

b) $F\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}-1\\0\end{pmatrix}$, $F\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}0\\1\end{pmatrix}$ $\quad M_F=\begin{pmatrix}-1&0\\0&1\end{pmatrix}$

c) $F\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}-1\\0\end{pmatrix}$, $F\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}0\\-1\end{pmatrix}$ $\quad M_F=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}$

d) $F\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}0\\-1\end{pmatrix}$, $F\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}-1\\0\end{pmatrix}$ $\quad M_F=\begin{pmatrix}0&-1\\-1&0\end{pmatrix}$

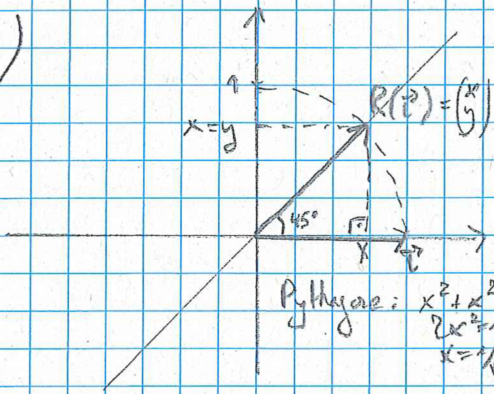
e) $F\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}0\\1\end{pmatrix}$, $F\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}-1\\0\end{pmatrix}$ $\quad M_F=\begin{pmatrix}0&-1\\1&0\end{pmatrix}$

f) $M\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}-1\\0\end{pmatrix}$, $M\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}0\\-1\end{pmatrix}$ $\quad M_F=\begin{pmatrix}-1&0\\0&-1\end{pmatrix}$

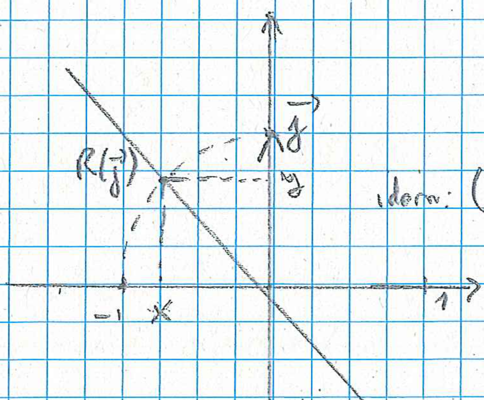
h) $M\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix}$, $M\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$ $\quad M_F=\begin{pmatrix}1&0\\0&0\end{pmatrix}$

i) $M\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$, $M\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}0\\1\end{pmatrix}$ $\quad M_F=\begin{pmatrix}0&0\\0&1\end{pmatrix}$

g)



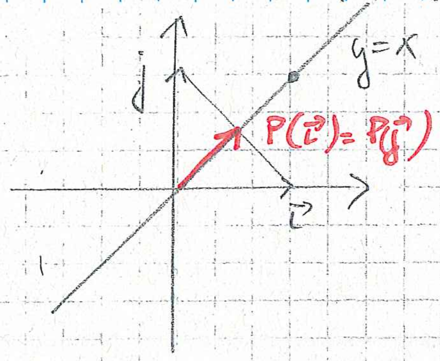
Pythagore : $x^2+x^2=1$
$2x^2=1$
$x=1/\sqrt{2}=\sqrt{2}/2=y$



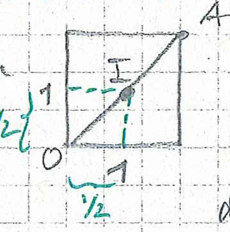
donc : $\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}-\sqrt{2}/2\\\sqrt{2}/2\end{pmatrix}$

donc $M_F=\begin{pmatrix}\sqrt{2}/2&-\sqrt{2}/2\\\sqrt{2}/2&\sqrt{2}/2\end{pmatrix}$

j)



On a

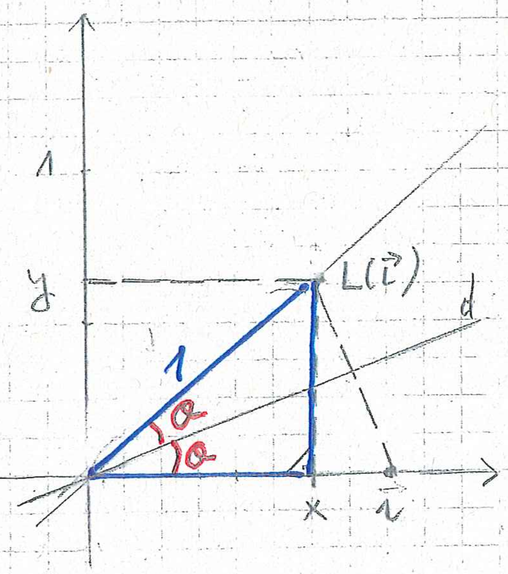


$\overline{OA}=\sqrt{2}$
$\overline{OI}=\sqrt{2}/2$

donc $P(\vec{i})=P(\vec{j})=\begin{pmatrix}1/2\\1/2\end{pmatrix}$

$\Rightarrow M=\begin{pmatrix}1/2&1/2\\1/2&1/2\end{pmatrix}$

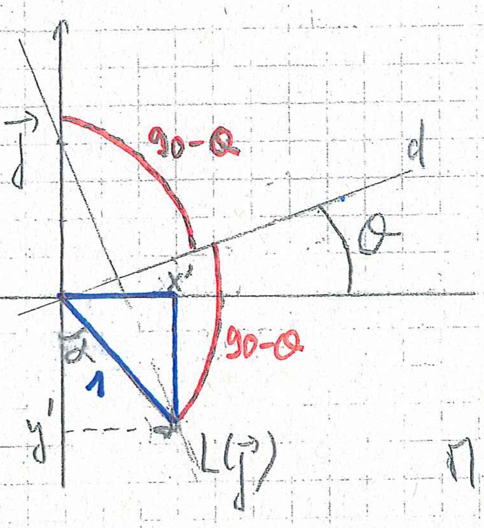
ex 25 :



On observe dans le triangle bleu :

$$\sin(2\theta) = \frac{y}{1} = y$$

$$\cos(2\theta) = \frac{x}{1} = x$$



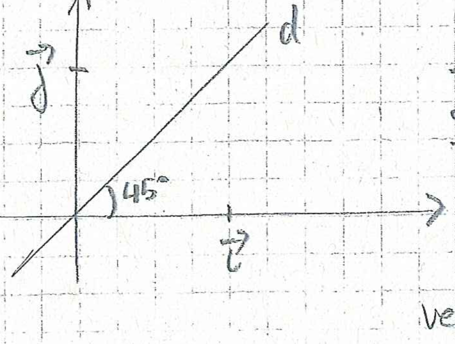
On a : $\alpha + (90-\theta) = 90+\theta$ d'où $\alpha = 2\theta$

$y' = -\cos(2\theta)$

$x' = \sin(2\theta)$

$$M = \begin{pmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{pmatrix}$$

Remarque : cas particulier où $\theta = 45°$



$$S_{45°}(\vec{i}) = \vec{j} \Leftrightarrow S\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$$

$$S_{45°}(\vec{j}) = \vec{i} \Leftrightarrow S\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$$

$$M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

vérif avec la formule générale :

$$M = \begin{pmatrix} \cos(2\cdot 45) & \sin(2\cdot 45) \\ \sin(2\cdot 45) & -\cos(2\cdot 45) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

ex 26

a) $M_F = \begin{pmatrix} 2 & -2 \\ 1 & 4 \end{pmatrix}$

b) $F\begin{pmatrix}3\\4\end{pmatrix} = M_F \cdot \begin{pmatrix}3\\4\end{pmatrix} = \begin{pmatrix}2 & -2\\1 & 4\end{pmatrix}\begin{pmatrix}3\\4\end{pmatrix} = \begin{pmatrix}2\cdot 3+(-2)\cdot 4\\1\cdot 3+4\cdot 4\end{pmatrix} = \begin{pmatrix}-2\\19\end{pmatrix}$

c) $F\begin{pmatrix}x\\y\end{pmatrix} = M_F\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}2 & -2\\1 & 4\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}2x-2y\\x+4y\end{pmatrix}$

ex 28

a) $\vec{a} = \begin{pmatrix}-3\\2\end{pmatrix} = -3\begin{pmatrix}1\\0\end{pmatrix} + 2\begin{pmatrix}0\\1\end{pmatrix} = -3\vec{i} + 2\vec{j}$

d'où $L(\vec{a}) = -3L(\vec{i}) + 2L(\vec{j}) = -3\begin{pmatrix}1\\1\end{pmatrix} + 2\begin{pmatrix}-1\\2\end{pmatrix} = \begin{pmatrix}-5\\1\end{pmatrix}$

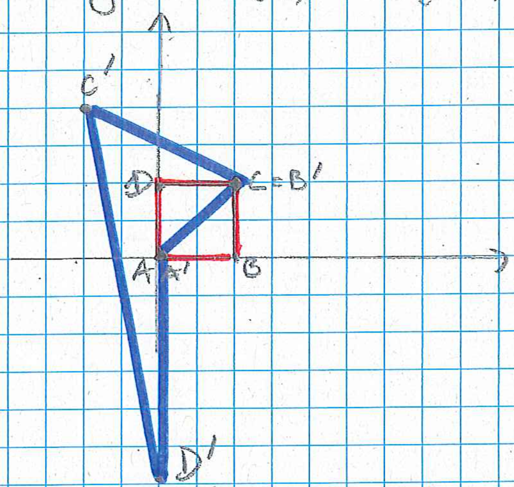
b) $L(\vec{v}) = v_1 L(\vec{i}) + v_2 L(\vec{j}) = v_1\begin{pmatrix}1\\1\end{pmatrix} + v_2\begin{pmatrix}-1\\2\end{pmatrix} = \begin{pmatrix}v_1 - v_2\\v_1 + 2v_2\end{pmatrix}$

c) $L\begin{pmatrix}0\\0\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$

$L\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix}$

$L\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}-1\\2\end{pmatrix}$

$L\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}0\\3\end{pmatrix}$



ex 29

a) $L(\vec{i}) = 2\vec{i}$, $L(\vec{j}) = 3\vec{j}$ $\quad M_L = \begin{pmatrix}2 & 0\\0 & 3\end{pmatrix} \rightsquigarrow L\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}2x\\3y\end{pmatrix}$

b) $L(\vec{i}) = \begin{pmatrix}0\\2\end{pmatrix}$, $L(\vec{j}) = \begin{pmatrix}-2\\0\end{pmatrix}$ $\quad M_L = \begin{pmatrix}0 & -2\\2 & 0\end{pmatrix} \rightsquigarrow L\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-2y\\2x\end{pmatrix}$

c) $L(\vec{i}) = -2\vec{i} + 3\vec{j} = \begin{pmatrix}-2\\3\end{pmatrix}$, $L(\vec{j}) = -2\vec{i} + \vec{j} = \begin{pmatrix}-2\\1\end{pmatrix}$ $\quad M_L = \begin{pmatrix}-2 & -2\\3 & 1\end{pmatrix} \rightsquigarrow L\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-2x+3y\\-2x+y\end{pmatrix}$

d) $L(\vec{i}) = 3\vec{i} + \vec{j} = \begin{pmatrix}3\\1\end{pmatrix}$, $L(\vec{j}) = 2\vec{j} = \begin{pmatrix}0\\2\end{pmatrix}$ $\quad M_L = \begin{pmatrix}3 & 0\\1 & 2\end{pmatrix} \rightsquigarrow L\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}3x\\x+2y\end{pmatrix}$

ex 30

a) $M = \begin{pmatrix}3 & -2\\-1 & 5\end{pmatrix}$ $\qquad$ b) $M = \begin{pmatrix}1 & 1\\0 & 0\end{pmatrix}$

c) $M = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$ $\qquad$ d) $M = \begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}$

ex31 : on écrit $\vec{i}$ et $\vec{j}$ comme comb. linéaire de $\begin{pmatrix}1\\1\end{pmatrix}$ et $\begin{pmatrix}1\\-1\end{pmatrix}$

$$\vec{i} = \begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix} \Rightarrow F(\vec{i}) = F\left(\frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}\right)$$

$$\underset{F \text{ linéaire}}{=} \frac{1}{2}F\begin{pmatrix}1\\1\end{pmatrix} + \frac{1}{2}F\begin{pmatrix}1\\-1\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}3\\-1\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}2\\1/2\end{pmatrix}$$

on peut poser l'équation $\begin{pmatrix}1\\0\end{pmatrix} = \alpha\begin{pmatrix}1\\1\end{pmatrix} + \beta\begin{pmatrix}1\\-1\end{pmatrix}$ ...

ou la trouver "au feeling"

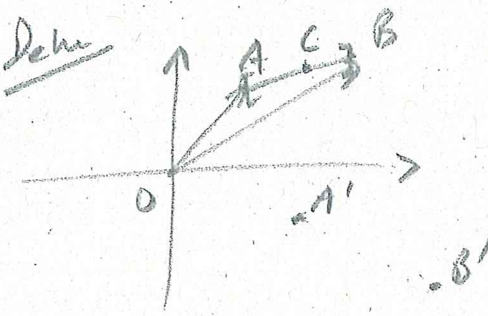
idem pour $\vec{j}$: $\vec{j} = \frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix} - \frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix} \Rightarrow F(\vec{j}) = \dots = \begin{pmatrix}1\\-3/2\end{pmatrix}$

donc $M = \begin{pmatrix}2 & 1\\ 1/2 & -3/2\end{pmatrix}$

ex27 :

Si L appl. lin.
$[A;B]$ segment , alors $L([A;B]) = [A';B']$

Dém



C tq $\vec{AC} = \lambda\vec{AB}$ avec $\lambda \in [0;1]$

Soient $A' = L(A)$ càd $L(\vec{OA}) = \vec{OA'}$
$B' = L(B)$ $\qquad L(\vec{OB}) = \vec{OB'}$

on a : $\vec{OC} = \vec{OA} + \vec{AC}$
$= \vec{OA} + \lambda\vec{AB}$
$= \vec{OA} + \lambda(\vec{OB} - \vec{OA})$
$= (1-\lambda)\vec{OA} + \lambda\vec{OB}$

d'où $L(\vec{OC}) = (1-\lambda)L(\vec{OA}) + \lambda L(\vec{OB})$
$= (1-\lambda)\vec{OA'} + \lambda\vec{OB'}$
$= (1-\lambda)\vec{OA'} + \lambda(\vec{OA'} + \vec{A'B'})$
$= \vec{OA'} + \lambda\vec{A'B'}$

donc L envoie le segment $[AB]$ sur le segment $[A'B']$

Ex 32

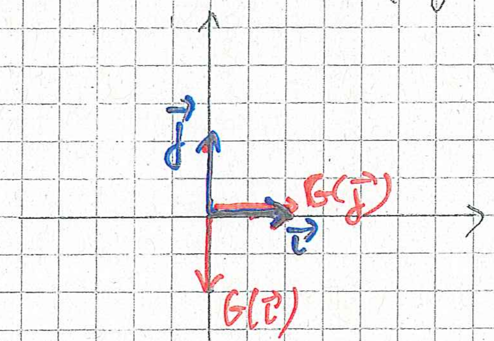
a) $F\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}0\\1\end{pmatrix}$

$F\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix}$

symétrie d'axe $y=x$ et de centre $(0;0)$
(déjà vue plusieurs fois)

b) $G\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}0\\-1\end{pmatrix}$

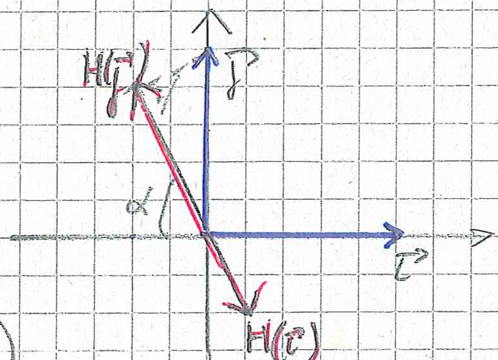
$G\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix}$



rotation $\theta=270°$
autour de $(0;0)$

c) $H\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}1/5\\-2/5\end{pmatrix}$

$H\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}-2/5\\4/5\end{pmatrix}$



projection sur la droite $y=-2x$

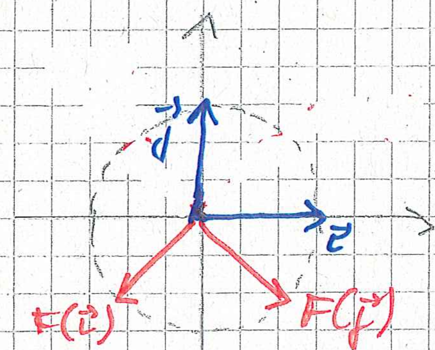
remarque

$H\begin{pmatrix}1\\0\end{pmatrix}=\alpha\cdot H\begin{pmatrix}0\\1\end{pmatrix}$
(vecteurs colinéaires)

d) $M\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}-\sqrt{2}/2\\-\sqrt{2}/2\end{pmatrix}$

$M\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}\sqrt{2}/2\\-\sqrt{2}/2\end{pmatrix}$

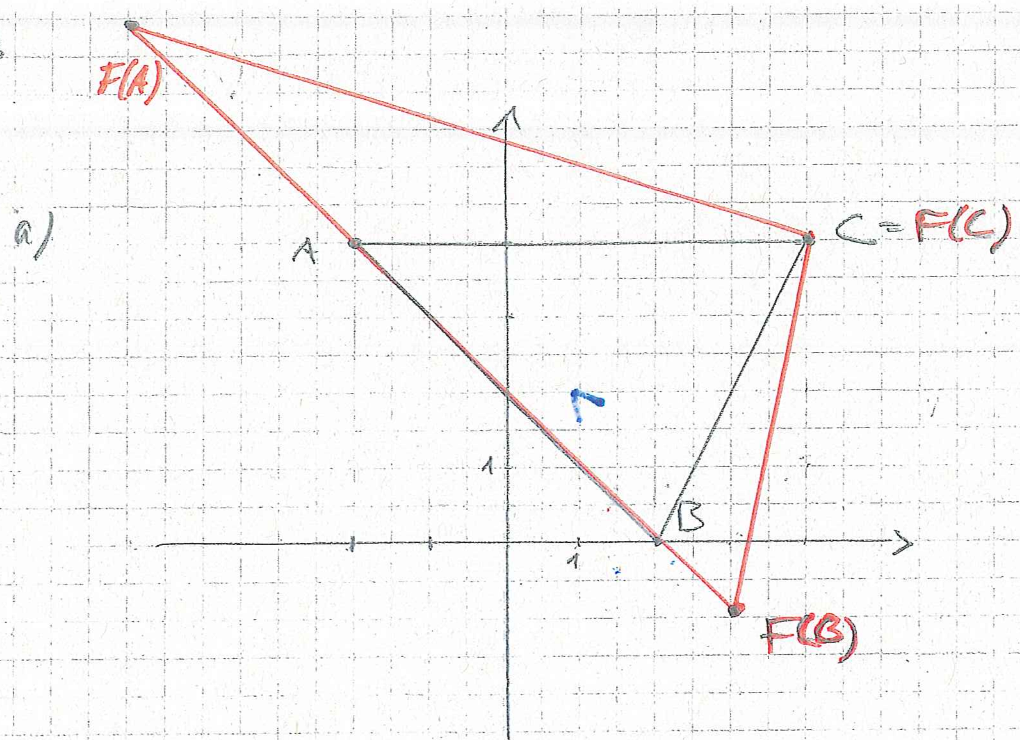
ces deux vecteurs pointent sur le cercle trigonométrique
(car $\left(\frac{\sqrt{2}}{2}\right)^2+\left(\frac{\sqrt{2}}{2}\right)^2=1$)



rotation de $-135°$
autour de $(0;0)$
(ou de $225°$)

ex 33

a)



$$F(A) = \begin{pmatrix} 3/2 & -1/2 \\ -1/2 & 3/2 \end{pmatrix} \begin{pmatrix} -2 \\ 4 \end{pmatrix} = \begin{pmatrix} -3-2 \\ 1+6 \end{pmatrix} = \begin{pmatrix} -5 \\ 7 \end{pmatrix}$$

$$F(B) = \begin{pmatrix} 3/2 & -1/2 \\ -1/2 & 3/2 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 3+0 \\ -1+0 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

$$F(C) = \begin{pmatrix} 3/2 & -1/2 \\ -1/2 & 3/2 \end{pmatrix} \begin{pmatrix} 4 \\ 4 \end{pmatrix} = \begin{pmatrix} 6-2 \\ -2+6 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

b) $F(\vec{v}) = \vec{v} \Leftrightarrow M \cdot \vec{v} = \vec{v} \Leftrightarrow \begin{pmatrix} 3/2 & -1/2 \\ -1/2 & 3/2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$$\Leftrightarrow \begin{cases} \frac{3}{2}v_1 - \frac{1}{2}v_2 = v_1 \\ -\frac{1}{2}v_1 + \frac{3}{2}v_2 = v_2 \end{cases} \Leftrightarrow \begin{cases} \frac{1}{2}v_1 - \frac{1}{2}v_2 = 0 \\ -\frac{1}{2}v_1 + \frac{1}{2}v_2 = 0 \end{cases} \Leftrightarrow \begin{cases} v_1 - v_2 = 0 \\ v_1 - v_2 = 0 \end{cases}$$

d'où $\{\vec{v} \in \mathbb{R}^2 \mid F(\vec{v}) = \vec{v}\} = \{\vec{v} \in \mathbb{R}^2 \mid v_1 = v_2\}$ = droite d'éq. $y = x$

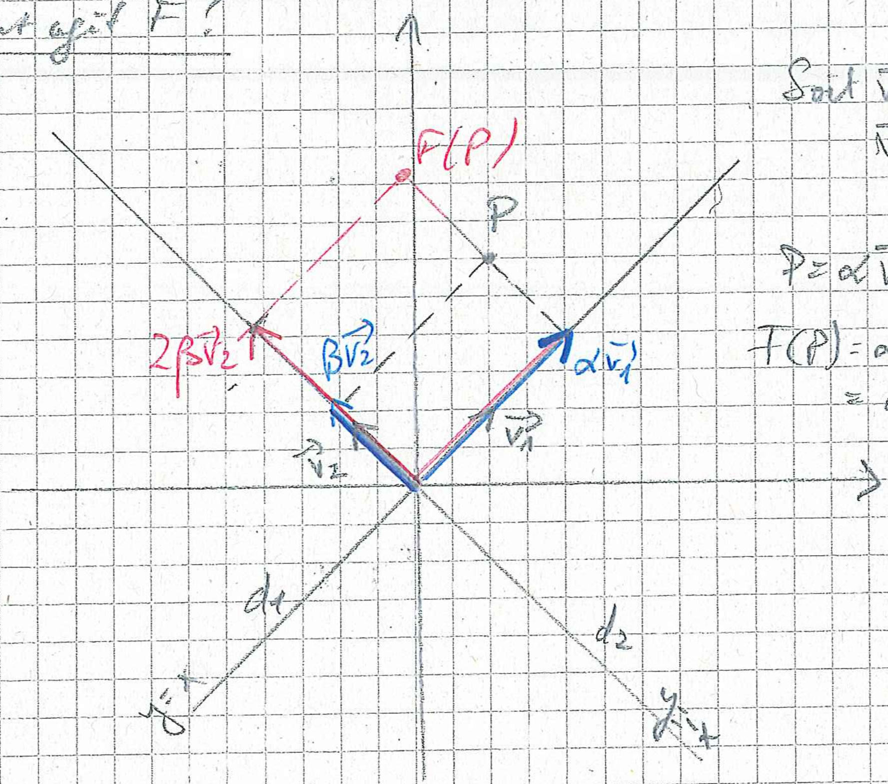
c) $F(\vec{w}) = 2\vec{w} \Leftrightarrow M\vec{w} = 2\vec{w} \Leftrightarrow \begin{pmatrix} 3/2 & -1/2 \\ -1/2 & 3/2 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} 2w_1 \\ 2w_2 \end{pmatrix}$

$$\Leftrightarrow \begin{cases} \frac{3}{2}w_1 - \frac{1}{2}w_2 = 2w_1 \\ -\frac{1}{2}w_1 + \frac{3}{2}w_2 = 2w_2 \end{cases} \Leftrightarrow \begin{cases} \frac{1}{2}w_1 + \frac{1}{2}w_2 = 0 \\ -\frac{1}{2}w_1 - \frac{1}{2}w_2 = 0 \end{cases} \Leftrightarrow \begin{cases} w_1 = -w_2 \\ w_1 = -w_2 \end{cases}$$

d'où $\{\vec{w} \in \mathbb{R}^2 \mid F(\vec{w}) = 2\vec{w}\} = \{\vec{w} \in \mathbb{R}^2 \mid w_1 = -w_2\}$ = droite d'éq. $w_1 = -w$

./.

Comment agit F ?



Soit $\vec{v_1}$ tq $F(\vec{v_1}) = \vec{v_1}$
$\vec{v_2}$ tq $F(\vec{v_2}) = 2\vec{v_2}$

$P = \alpha\vec{v_1} + \beta\vec{v_2}$

$F(P) = \alpha F(\vec{v_1}) + \beta F(\vec{v_2})$
$= \alpha\vec{v_1} + 2\beta\vec{v_2}$

Pour tout $P \in \mathbb{R}^2$, F agit ainsi : on double la composante de P le long de d_2, celle de d_1 ne change pas.