[illegible] 2001 4N Corrigé

Q1: a) $\int x\cos(2x)\,dx$

$g(x) = x \Rightarrow g'(x) = 1$
$f'(x) = \cos(2x) \Rightarrow f(x) = \frac{1}{2}\sin(2x)$

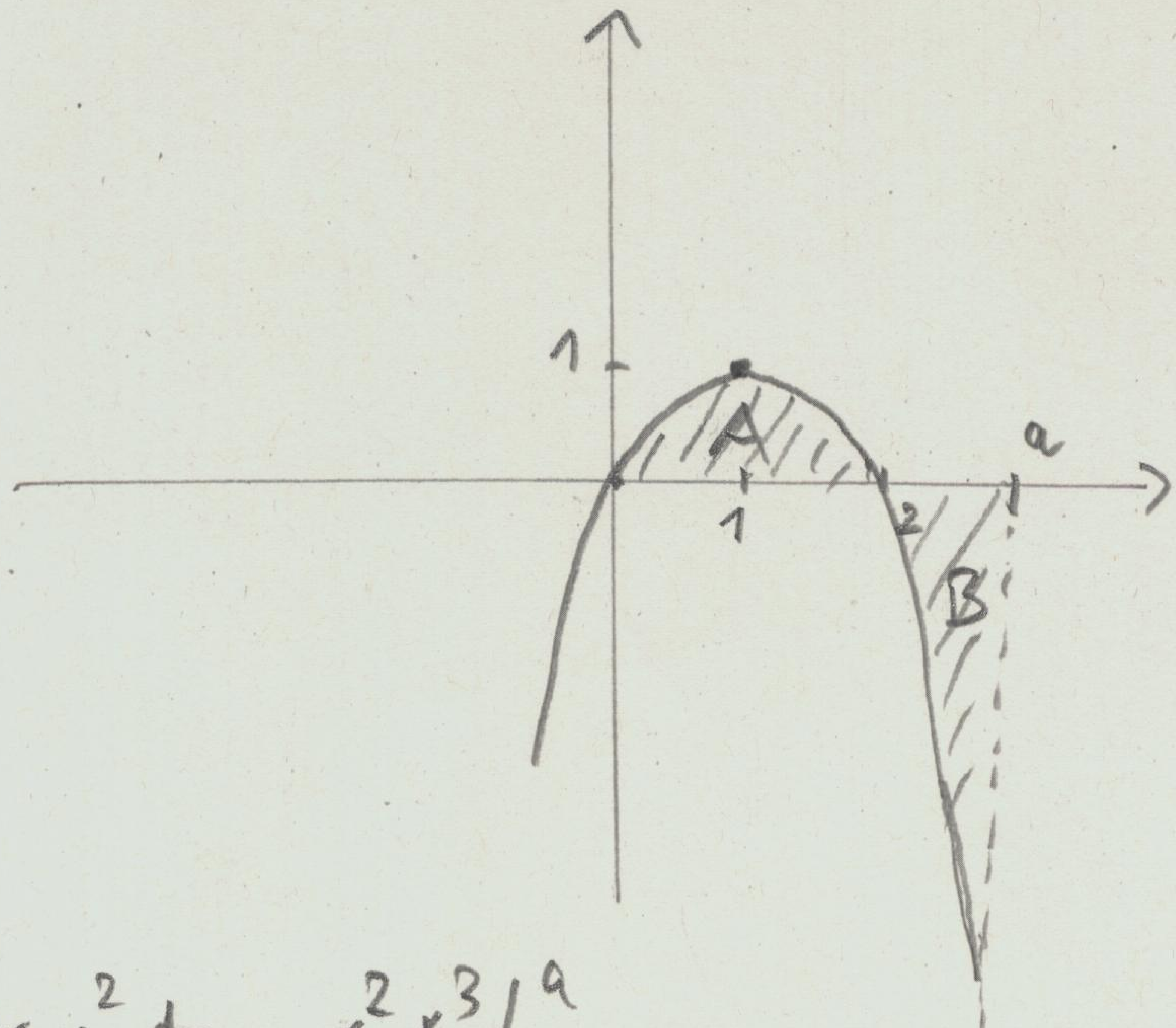
$$= x\cdot\frac{1}{2}\sin(2x) - \int 1\cdot\frac{1}{2}\sin(2x)\,dx + C$$
$$= \frac{x}{2}\sin(2x) - \frac{1}{2}\cdot\left(-\frac{1}{2}\cos(2x)\right) + C$$
$$= \frac{x}{2}\sin(2x) + \frac{1}{4}\cos(2x) + C$$

b) $\int \frac{e^x}{2+e^x}\,dx = \int \frac{(e^x+2)'}{e^x+2}\,dx = \ln|e^x+2| + C$

c) $\int_{-e}^{-1} \frac{\ln(x)}{x}\,dx = \int_{-e}^{-1} \ln(x)\cdot\frac{1}{x}\,dx = \int_{-e}^{-1} \ln(x)\cdot[\ln(x)]'\,dx$

$$= \frac{1}{2}\ln^2|x|\Big|_{-e}^{-1}$$
$$= \frac{1}{2}\ln^2|-1| - \frac{1}{2}\ln^2|-e|$$
$$= 0 - \frac{1}{2}\ln^2(e) = -\frac{1}{2}\cdot 1 = -\frac{1}{2}$$

Q2:



$f(x) = x(2-x)$
$f(1) = 1$

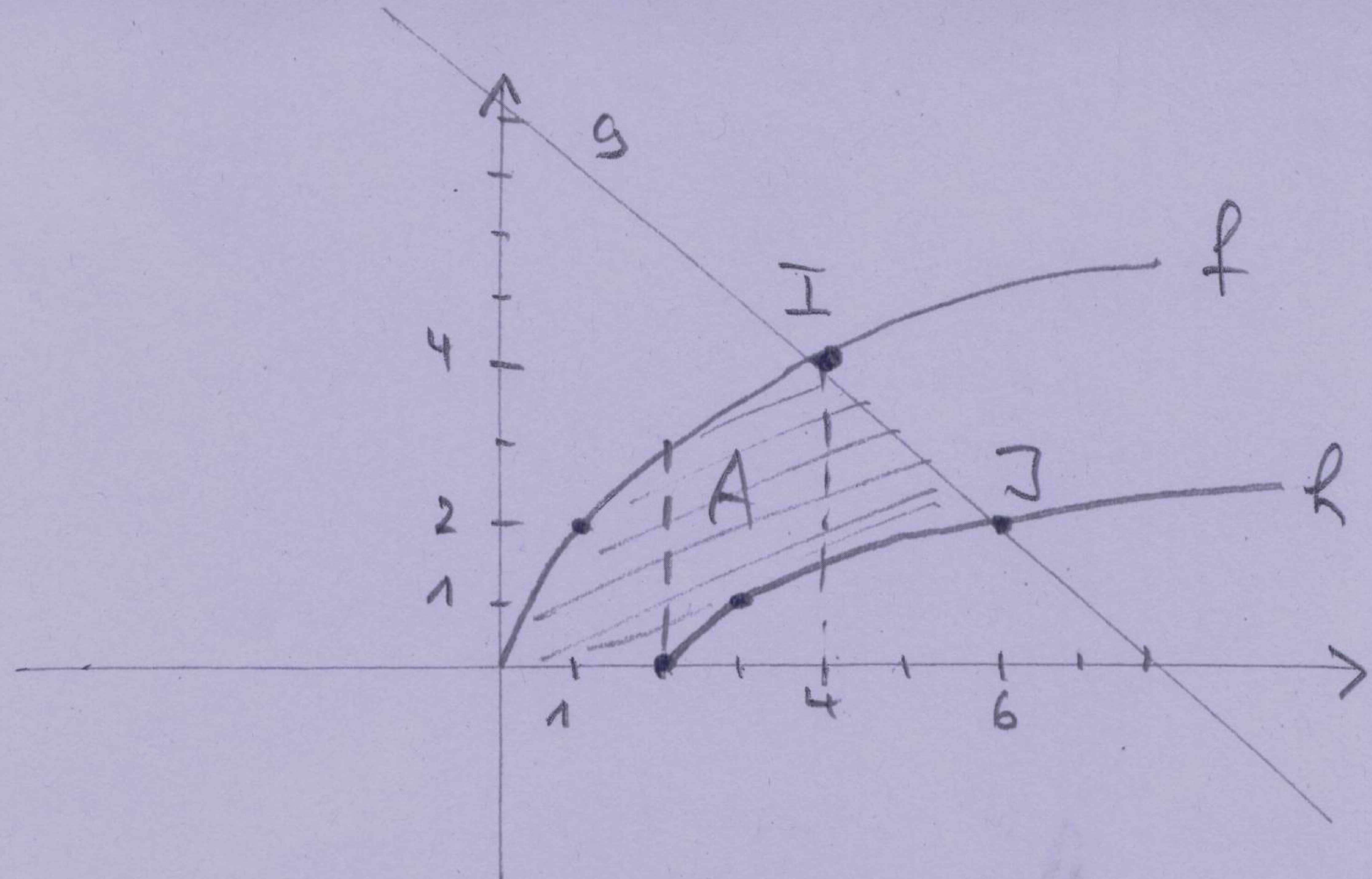
$$0 = \int_0^a 2x - x^2\,dx = x^2 - \frac{x^3}{3}\Big|_0^a$$
$$= a^2 - \frac{a^3}{3}$$

$\Leftrightarrow a^2\left(1-\frac{a}{3}\right) = 0 \Leftrightarrow a = 0$ ou $a = 3$

l'aire géométrique A est égale à l'aire géométrique B

Q3:

a)



b) $f(4) = g(4) = 4 \Rightarrow I = (4;4)$

$g(6) = h(6) = 2 \Rightarrow J = (6;2)$

$$A = \int_0^2 f(x)\,dx + \int_2^4 f(x) - h(x)\,dx + \int_4^6 g(x) - h(x)\,dx$$

$$= \int_0^4 f(x)\,dx + \int_4^6 g(x)\,dx - \int_2^6 h(x)\,dx$$

$$= 2\cdot\frac{x^{3/2}}{3/2}\Big|_0^4 + 8x - x^2\Big|_4^6 - \frac{(x-2)^{3/2}}{3/2}\Big|_2^6$$

$$= \frac{4}{3}\cdot 4^{3/2} + (48-36) - (32-16) - \frac{2}{3}4^{3/2}$$

$$= \frac{4}{3}\cdot 8 + (12-16) - \frac{2}{3}\cdot 8 = \frac{32}{3} - \frac{16}{3} - 4 = \frac{16}{3} - 4 = \frac{4}{3}$$

c) $$V = \pi\int_0^4 f^2(x)\,dx - \pi\int_2^4 h^2(x)\,dx + \pi\int_4^6 g^2(x)\,dx - \pi\int_4^6 h^2(x)\,dx$$

$$= \pi\left[\int_0^4 f^2(x)\,dx + \int_4^6 g^2(x)\,dx - \int_2^6 h^2(x)\,dx\right]$$

$$= \pi\left[\int_0^4 4x\,dx + \int_4^6 (8-x)^2\,dx + \int_2^6 (x-2)\,dx\right]$$

$$= \pi\left[2x^2\Big|_0^4 + \frac{1}{3}(8-x)^3\Big|_4^6 - \frac{(x-2)^2}{2}\Big|_2^6\right]$$

$$= \pi\left[32 - \frac{1}{3}(8-64) - \left(\frac{16}{2} - 0\right)\right] = \pi\left[32 + \frac{56}{3} - 8\right]$$

$$= \pi\left[24 + \frac{56}{3}\right] = \pi\,\frac{128}{3}$$

Q 4 :

$\pi_1 : x + y + 2z = 6$

$\pi_2 : x + z = 2$

a) $\vec{n}\begin{pmatrix}1\\1\\2\end{pmatrix} \perp \pi_1$, donc $\vec{n} \perp \pi$

éq. de π : $1x + 1y + 2z + d = 0$

$A(2;3;1) \in \pi \Leftrightarrow 1\cdot 2 + 1\cdot 3 + 2\cdot 1 + d = 0$

$\Leftrightarrow d = -7$

éq. de π : $\boxed{x + y + 2z - 7 = 0}$

b) $\begin{cases} x + y + 2z = 6 \\ x + z = 2 \end{cases}$

$-$

$y + z = 4$

posons $y = 4 \Rightarrow z = 0 \Rightarrow x = 2 \Rightarrow B'(2;4;0) \in d$

$y = 3 \Rightarrow z = 1 \Rightarrow x = 1 \Rightarrow C'(1;3;1) \in d$

Soit $P(x;y;z) \in d$: $\overrightarrow{B'P} = \lambda \overrightarrow{B'C'}$

$\Leftrightarrow \begin{pmatrix}x-2\\y-4\\z-0\end{pmatrix} = \lambda\begin{pmatrix}-1\\-1\\1\end{pmatrix}$ éq. param.

$\left(\Leftrightarrow \begin{cases} x - 2 = -\lambda \\ y - 4 = -\lambda \\ z = \lambda \end{cases} \right)$

c) $\left.\begin{array}{l} d' \parallel \pi_1 \\ d' \parallel \pi_2 \end{array}\right\} \Rightarrow d' \parallel d \Rightarrow \overrightarrow{B'C'}\begin{pmatrix}-1\\-1\\1\end{pmatrix}$ vect dir de d'

$Q(x;y;z) \in d' \Leftrightarrow \overrightarrow{BQ} = \mu \overrightarrow{B'C'}$

$\Leftrightarrow \begin{pmatrix}x-1\\y-3\\z-2\end{pmatrix} = \mu\begin{pmatrix}-1\\-1\\1\end{pmatrix} \Leftrightarrow \begin{cases} x - 1 = -\mu \\ y - 3 = -\mu \\ z - 2 = \mu \end{cases}$

$\Leftrightarrow -(x-1) = -(y-3) = z - 2$

[éq. cart]

d) $d(C;\pi_1) = \dfrac{|2+4+2\cdot 3-6|}{\sqrt{1^2+1^2+2^2}}$

$= \dfrac{6}{\sqrt{6}} = \sqrt{6}$

Q5. b) $\begin{pmatrix}1\\0\\1\end{pmatrix} = \alpha\begin{pmatrix}2\\1\\3\end{pmatrix} + \beta\begin{pmatrix}1\\3\\1\end{pmatrix} + \gamma\begin{pmatrix}-1\\1\\3\end{pmatrix}$

$$\Leftrightarrow \begin{cases} 1 = 2\alpha + \beta - \gamma \\ 0 = \alpha + 3\beta + \gamma \\ 1 = 3\alpha + \beta + 3\gamma \end{cases}$$

$$\Leftrightarrow \underbrace{\begin{pmatrix}2 & 1 & -1\\1 & 3 & 1\\3 & 1 & 3\end{pmatrix}}_{M}\begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix} = \begin{pmatrix}1\\0\\1\end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix} = M^{-1}\begin{pmatrix}1\\0\\1\end{pmatrix}$$

$$\det M = 2\begin{vmatrix}3 & 1\\1 & 3\end{vmatrix} - 1\cdot\begin{vmatrix}1 & -1\\1 & 3\end{vmatrix} + 3\begin{vmatrix}1 & -1\\3 & 1\end{vmatrix}$$
$$= 2\cdot 8 - 1\cdot 4 + 3\cdot 4 = 24$$

$$M^{-1} = \frac{1}{24}\begin{pmatrix}8 & -0 & -8\\-4 & 8 & +1\\+4 & -3 & +5\end{pmatrix}^t = \frac{1}{24}\begin{pmatrix}8 & -4 & +4\\0 & 8 & -3\\-8 & 1 & +5\end{pmatrix}$$

donc $\begin{pmatrix}\alpha\\\beta\\\gamma\end{pmatrix} = \frac{1}{24}\begin{pmatrix}8 & -4 & +4\\0 & 8 & -3\\-8 & 1 & +5\end{pmatrix}\begin{pmatrix}1\\0\\1\end{pmatrix} = \frac{1}{24}\begin{pmatrix}12\\3\\-3\end{pmatrix} = \begin{pmatrix}1/2\\-1/8\\-1/8\end{pmatrix}$

vérif math : $\frac{1}{2}\begin{pmatrix}2\\1\\3\end{pmatrix} - \frac{1}{8}\begin{pmatrix}1\\3\\1\end{pmatrix} - \frac{1}{8}\begin{pmatrix}-1\\1\\3\end{pmatrix} = \begin{pmatrix}1\\0\\1\end{pmatrix}$ ✓

Q6:

a) A a 3 rois ; dans son jeu reste 29 cartes, dont 1 roi

$P(\text{A ait un carré de rois}) = \frac{1}{29}$

b) B a 2 as ; dans son jeu restent 30 cartes, dont 2 rois

$P(\text{B ait un carré d'as}) = \frac{1}{C^{2}_{30}} = \frac{1}{\frac{30\cdot 29}{2}} = \frac{2}{29\cdot 30}$ $\left(\text{ou } \underbrace{\frac{2}{30}}_{\text{tirer un as parmi les 2 restants}} \cdot \underbrace{\frac{1}{29}}_{\text{retirer un as}} \right)$

$= \frac{1}{15\cdot 29}$

$P(\text{B pas carré d'as}) = 1 - \frac{1}{15\cdot 29}$

c) $P(A|B) = P(A)$
$P(B|A) = P(B)$ $\Rightarrow$ les 2 événements sont indép.

$P(A\cap B) = P(A)\cdot P(B)$
$= \frac{1}{29}\cdot\frac{1}{15\cdot 29}$

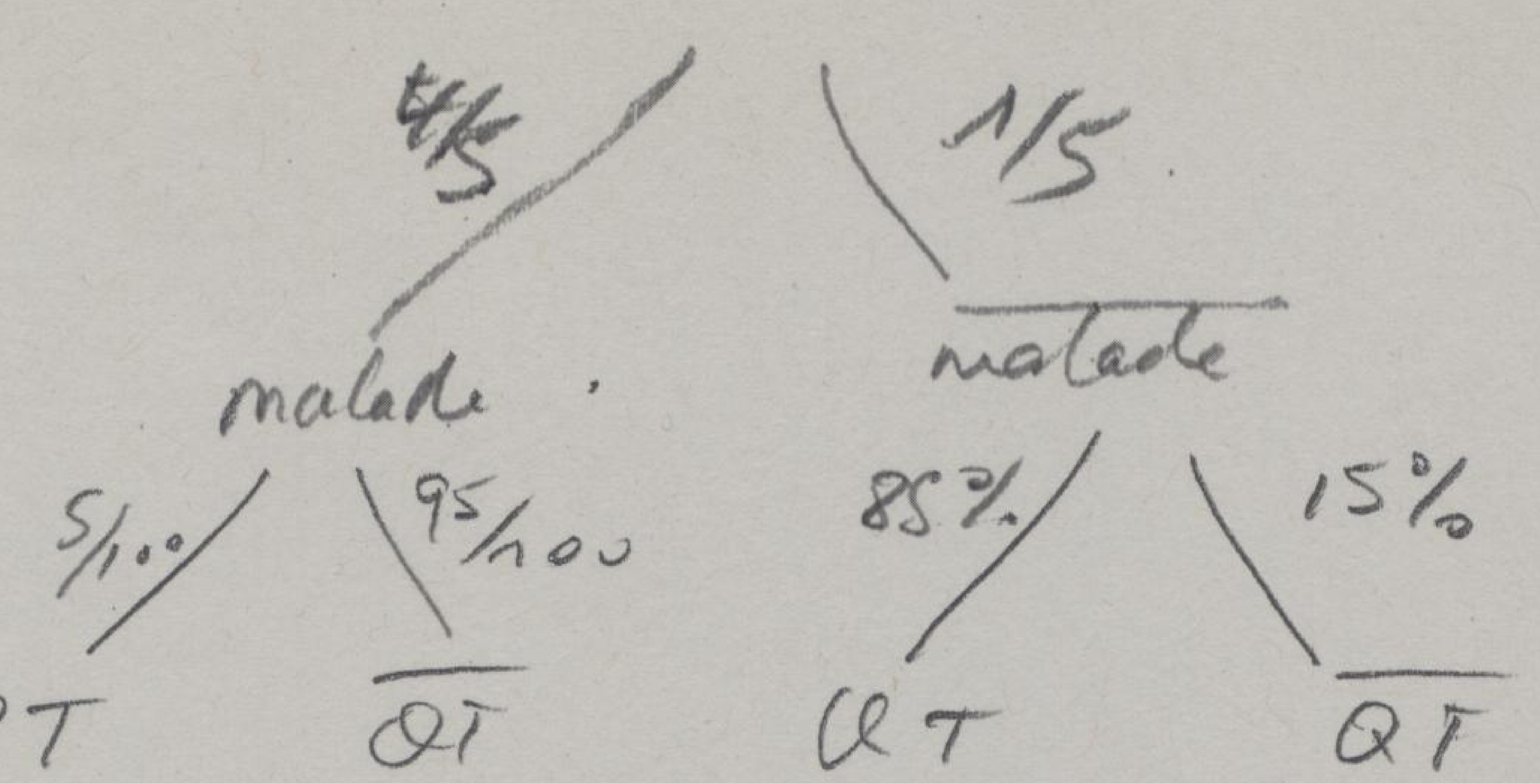
d) A gagne $\Leftrightarrow$ A a un carré

$P(\text{A gagne}) = \frac{1}{29}$

e) B gagne $\Leftrightarrow$ B a un carré et A n'a pas de carré (indép)

$P(\text{B gagne}) = \frac{1}{29\cdot 15}\cdot\frac{28}{29}$

Q7:



$P(M \mid \overline{QT}) = \frac{P(\overline{M}\cap\overline{QT})}{P(\overline{QT})} = \frac{\frac{4}{5}\cdot\frac{95}{100}}{\frac{4}{5}\cdot\frac{95}{100} + \frac{1}{5}\cdot\frac{15}{100}} = \frac{\frac{19}{25}}{\frac{79}{100}}$

$= \frac{76}{79} \simeq 96.2\%$