/ 1 IATU 2011 corrigé /

Ex 1

[/6]

a) $\int \cos(2x)\,dx = \frac{1}{2}\sin(2x) + C$

on veut : $\frac{1}{2}\sin\left(2\cdot\frac{\pi}{4}\right) + C = 0$

$\Leftrightarrow \frac{1}{2}\sin\left(\frac{\pi}{2}\right) + C = 0$

$\Leftrightarrow \frac{1}{2} + C = 0$

$\Leftrightarrow C = -\frac{1}{2}$

Sol : $F(x) = \frac{1}{2}\sin(2x) - \frac{1}{2}$ 3 pts

b) $(x^3 e^{2x})' = 3x^2 e^{2x} + x^3 e^{2x}(2x)'$

$= 3x^2 e^{2x} + 2x^3 e^{2x}$

$= x^2 e^{2x}(3 + 2x)$ 3 pts

Q2

[/6]

a) $\int \frac{3x^2+1}{3x^3+3x-1}\,dx = \frac{1}{3}\int \frac{9x^2+3}{3x^3+3x-1}\,dx$

$= \frac{1}{3}\ln|3x^3+3x+1| + C$ 3pts

b) $\int x^7 \ln(x)\,dx$ $\quad f(x)=\ln(x) \Rightarrow f'(x)=\frac{1}{x}$

$g'(x)=x^7 \Rightarrow g(x)=\frac{x^8}{8}$

$\int x^7 \ln(x)\,dx = \frac{x^8}{8}\ln|x| - \int \frac{1}{x}\cdot\frac{x^8}{8}\,dx$

$= \frac{x^8}{8}\ln|x| - \frac{1}{8}\int x^7\,dx$

$= \frac{x^8}{8}\ln|x| - \frac{1}{8}\cdot\frac{x^8}{8} + C$

$= \frac{x^8}{8}\ln|x| - \frac{1}{64}x^8 + C$ 3pts

Remarque:

Q3 a) $f(x) = g(x) \Leftrightarrow \frac{1}{4}x^2 - 2x + 6 = -\frac{1}{4}x^2 + 2x$

[/9]

$\Leftrightarrow \frac{1}{2}x^2 - 4x + 6 = 0$

$\Leftrightarrow x^2 - 8x + 12 = 0$

$\Leftrightarrow (x-6)(x-2) = 0$

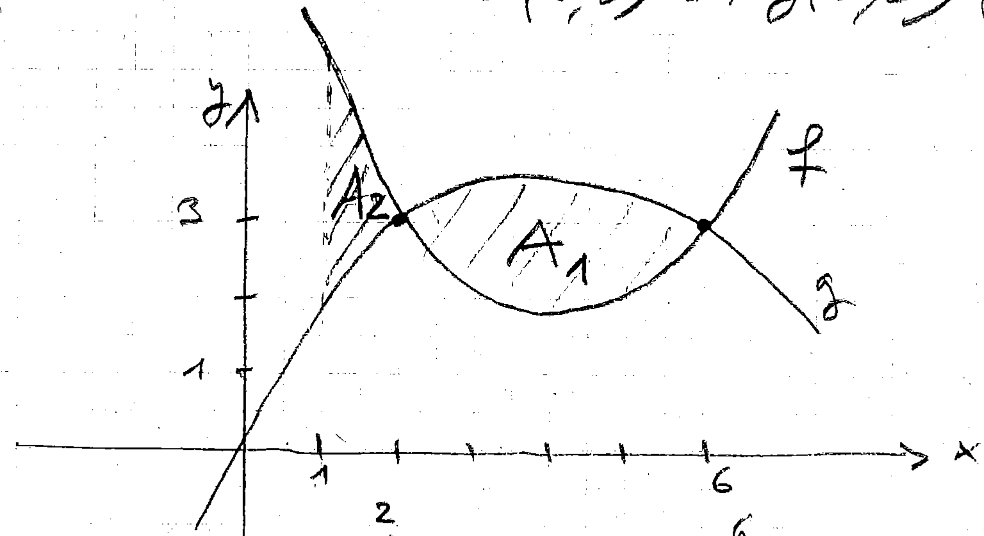
$x = 6$ ou $x = 2$

$x=6$: $g(6) = -\frac{1}{4} \cdot 36 + 12 = -9 + 12 = 3$

$x=2$: $g(2) = -\frac{1}{4} \cdot 4 + 4 = -1 + 4 = 3$

Pts d'intersection : $I(6;3)$ et $J(2;3)$ 3 pts

b)



$A = A_2 + A_1 = \int_1^2 f(x) - g(x)\,dx + \int_2^6 g(x) - f(x)\,dx$

$A_1 = \int_2^6 g(x) - f(x)\,dx = \int_2^6 \left(-\frac{1}{4}x^2 + 2x\right) - \left(\frac{1}{4}x^2 - 2x + 6\right) dx$

$= \int_2^6 \left(-\frac{1}{2}x^2 + 4x - 6\right) dx = -\frac{x^3}{6} + 2x^2 - 6x \Big|_2^6$

$= (-36 + 72 - 36) - \left(-\frac{4}{3} + 8 - 12\right)$

$= \frac{4}{3} + 4$

$= \frac{16}{3}$

6 pts

Q3 (suite)

$$A_2 = \int_1^2 f(x) - g(x)\,dx = \int_1^2 \left(\tfrac{1}{4}x^2 - 2x + 6\right) - \left(-\tfrac{1}{4}x^2 + 2x\right) dx$$

$$= \int_1^2 \left(\frac{x^2}{2} - 4x + 6\right) dx = \frac{x^3}{6} - 2x^2 + 6x \Big|_1^2$$

$$= \left(\frac{8}{6} - 8 + 12\right) - \left(\frac{1}{6} - 2 + 6\right) = \frac{7}{6}$$

donc $A = A_1 + A_2 = \dfrac{16}{3} + \dfrac{7}{6} = \dfrac{32+7}{6} = \dfrac{39}{6} = 6{,}5$

ou mathématiquement

$$A_1 = \int_2^6 \left(-\tfrac{1}{4}x^2 + 2x\right) dx - \int_2^6 \left(\tfrac{1}{4}x^2 - 2x + 6\right) dx$$

$$= \left(-\frac{x^3}{12} + x^2\right)\Big|_2^6 - \left(\frac{x^3}{12} - x^2 + 6x\right)\Big|_2^6$$

$$= \left(-\frac{216}{12} + 36\right) - \left(-\frac{8}{12} + 4\right) - \left[\left(\frac{216}{12} - 36 + 36\right) - \left(\frac{8}{12} - 4 + 12\right)\right]$$

$$= \left(-\frac{208}{12} + 32\right) - \left(\frac{208}{12} - 8\right)$$

$$= -\frac{208}{6} + 40 = 40 - \frac{104}{3} = \frac{16}{3}$$

$$A_2 = \int_1^2 \left(\tfrac{1}{4}x^2 - 2x + 6\right) dx - \int_1^2 \left(-\tfrac{1}{4}x^2 + 2x\right) dx$$

$$= \left(\frac{x^3}{12} - x^2 + 6x\right)\Big|_1^2 - \left(-\frac{x^3}{12} + x^2\right)\Big|_1^2$$

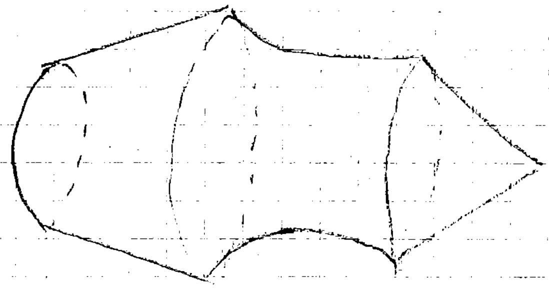
$$= \left(\frac{8}{12} - 4 + 12\right) - \left(\frac{1}{12} - 1 + 6\right) - \left[\left(-\frac{8}{12} + 4\right) - \left(-\frac{1}{12} + 1\right)\right]$$

$$= \left(\frac{7}{12} + 3\right) - \left(-\frac{7}{12} + 3\right)$$

$$= \frac{14}{12} = \frac{7}{6}$$

Q4 a)

[/8]



2pt

b) $V = \int_0^2 \pi \cdot (\tfrac{1}{2}x+1)^2\,dx + \int_2^4 \pi\left(\tfrac{4}{x}\right)^2 dx + \int_4^5 (-x+5)^2\,dx$

$= \pi\left[\tfrac{2}{3}(\tfrac{1}{2}x+1)^3\Big|_0^2 + 16\left(-\tfrac{1}{x}\right)\Big|_2^4 - \tfrac{1}{3}(-x+5)^3\Big|_4^5\right]$

$= \pi\left[\tfrac{2}{3}(8-1) + 16\left(-\tfrac{1}{4}+\tfrac{1}{2}\right) - \tfrac{1}{3}(0-1)\right]$

$= \pi\left[\tfrac{14}{3} + 4 + \tfrac{1}{3}\right]$

$= \pi\left[\tfrac{15}{3} + 4\right]$

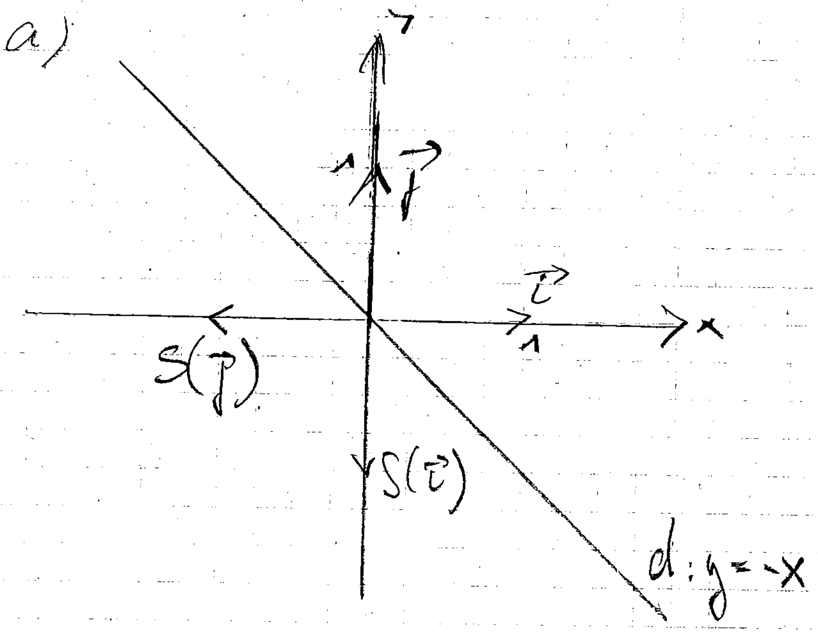
$= \pi \cdot 9$

$= 9\pi$

6pt

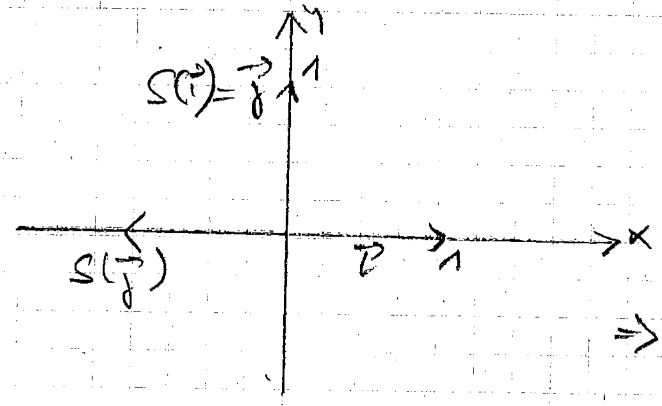
Q5 a)

[/12]



$S(\vec{i}) = -\vec{j} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$
$S(\vec{j}) = -\vec{i} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ $\Rightarrow M_S = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ 2 pts

b)



$S(\vec{i}) = \vec{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$S(\vec{j}) = -\vec{i} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$
$\Rightarrow M_R = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ 2 pts

c) $M_{R\circ S} = M_R \cdot M_S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$M_{S\circ R} = M_S \cdot M_R = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ (3)

d'où : $R\circ S(\vec{i}) = \vec{i}$ et $R\circ S(\vec{j}) = -\vec{j}$

$R\circ S$ est la symétrie d'axe Ox

et $S\circ R(\vec{i}) = -\vec{i}$ et $S\circ R(\vec{j}) = \vec{j}$

$S\circ R$ est la symétrie d'axe Oy (3) 6 pts

d) $X\circ S = R \Leftrightarrow X\circ S\circ S^{-1} = R\circ S^{-1}$

$\Leftrightarrow X\circ I = R\circ S^{-1} \Leftrightarrow X = R\circ S^{-1} = R\circ S$

$M_X = M_{R\circ S} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ 2 pts

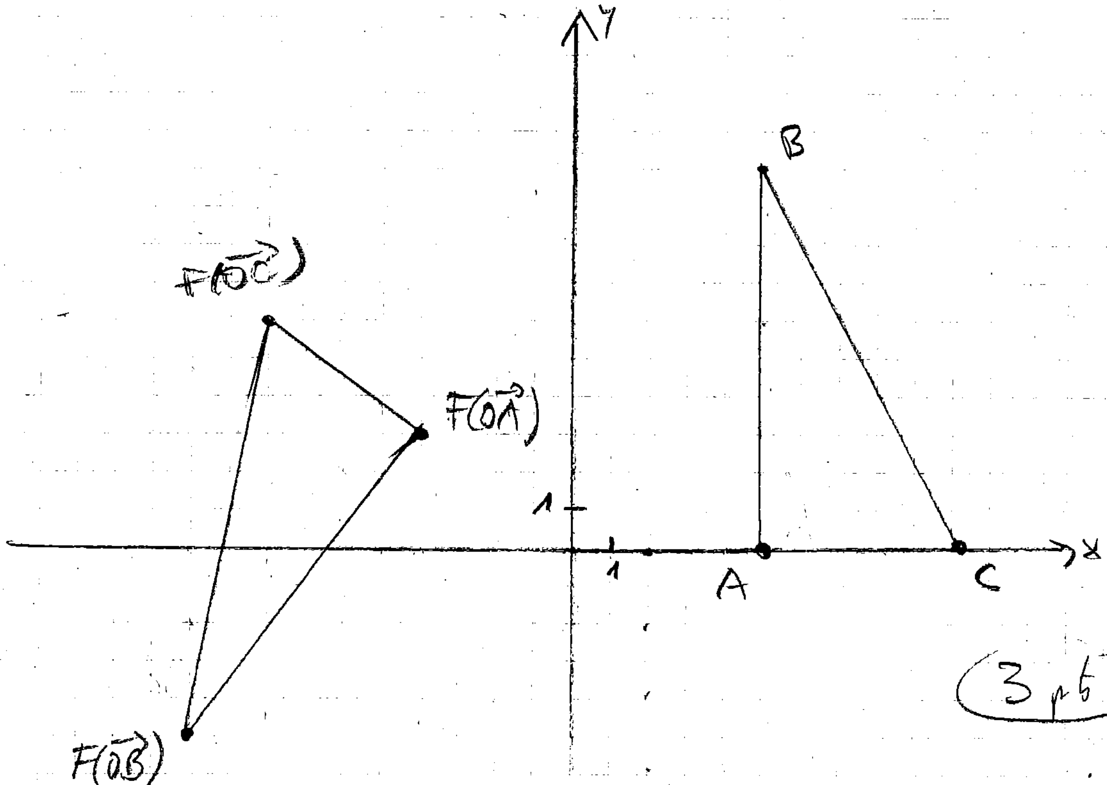
Q6

[/10]

a) $F(\vec{OA}) = F\begin{pmatrix}5\\0\end{pmatrix} = \begin{pmatrix}-4\\3\end{pmatrix}$

$F(\vec{OB}) = F\begin{pmatrix}5\\10\end{pmatrix} = \begin{pmatrix}-10\\-5\end{pmatrix}$

$F(\vec{OC}) = F\begin{pmatrix}10\\0\end{pmatrix} = \begin{pmatrix}-8\\6\end{pmatrix}$



(3 pt)

b) $\left.\begin{array}{l} F(\vec{i}) = F\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}-4/5\\3/5\end{pmatrix} \\ F(\vec{j}) = F\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}-3/5\\-4/5\end{pmatrix}\end{array}\right\} \Rightarrow M_F = \begin{pmatrix}-4/5 & -3/5\\ 3/5 & -4/5\end{pmatrix}$

D'où :

$\det M_F = \frac{16}{25} + \frac{9}{25} = \frac{25}{25} = 1$

$M_F^{-1} = \frac{1}{1}\begin{pmatrix}-4/5 & 3/5\\ -3/5 & -4/5\end{pmatrix}$

d'où $F^{-1}(\vec{OB}) = M_F^{-1} \cdot \begin{pmatrix}5\\10\end{pmatrix} = \frac{1}{5}\begin{pmatrix}-4 & 3\\ -3 & -4\end{pmatrix}\begin{pmatrix}5\\10\end{pmatrix}$

$= \frac{1}{5}\begin{pmatrix}10\\-55\end{pmatrix} = \begin{pmatrix}2\\-11\end{pmatrix}$

(4 pt)

c) $M_{R\alpha} = \begin{pmatrix}\cos(\alpha) & -\sin(\alpha)\\ \sin(\alpha) & \cos(\alpha)\end{pmatrix} = \begin{pmatrix}-4/5 & -3/5\\ 3/5 & -4/5\end{pmatrix}$

$\cos(\alpha) = -\frac{4}{5} \Leftrightarrow \alpha = \cos^{-1}\left(-\frac{4}{5}\right) \simeq 143°$

(3 pt)

Q7 [/8]

a) $\frac{2}{8} \cdot \frac{1}{7} = \frac{2}{56} = \frac{1}{28}$

choisir une botte de d'Artagnan — choisir l'autre botte de D'A.

ou $\frac{1}{C_2^8} = \frac{1}{\frac{8\cdot 7}{2}} = \frac{1}{28}$

2pts

b) $\frac{8}{8} \cdot \frac{4}{7} = \frac{4}{7}$

choisir une botte — choisir une botte de l'autre pied

2pts

c) $\frac{4}{8} \cdot \frac{3}{7} = \frac{3}{14}$

choisir un pied droit — choisir un autre pied droit

2pts

d) $\frac{8}{8} \cdot \frac{6}{7} = \frac{6}{7}$

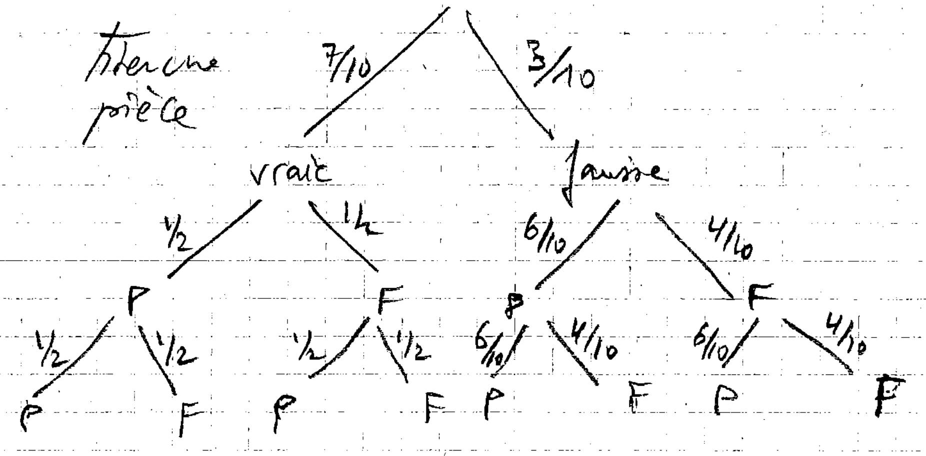
choisir une botte — choisir une botte d'une autre personne

2pts

Q8

[/8]

tirer une pièce



1°/ a) $p = \frac{3}{10} \cdot \frac{6}{10} \cdot \frac{6}{10} = \frac{108}{1000} = 10{,}8\,\%$ (2 pts)

b) $p = \frac{7}{10} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{3}{10} \cdot \frac{6}{10} \cdot \frac{6}{10} = \frac{7}{40} + \frac{108}{1000} = \frac{283}{1000} \simeq 28{,}3\,\%$ (2 pts)

2°/ A : tirer fausse pièce $p(A) = 0.3$

B : obtenir 2x pile

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{108}{1000}}{\frac{283}{1000}} = \frac{108}{283} \simeq 38{,}2\,\%$$ (4 pts)

Q9

[/8]

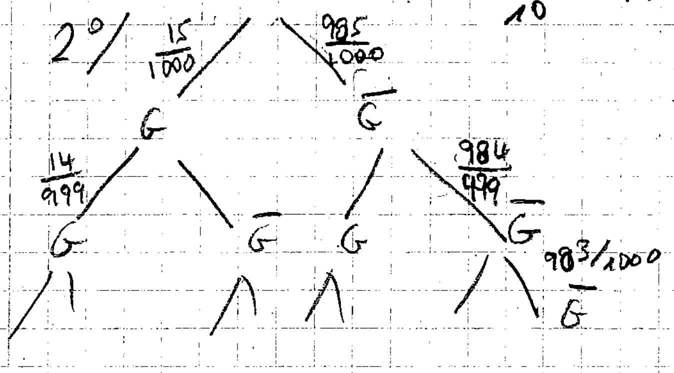
1°/ a) 1000 billets, dont 15 gagnants : $p(\overline{G}) = \frac{985}{1000}$ (2 pts)

b) X : gain

x	0	50	100	200	500
P	$\frac{985}{1000}$	$\frac{8}{1000}$	$\frac{4}{1000}$	$\frac{2}{1000}$	$\frac{1}{1000}$

$$E(X) = 0 \cdot \frac{985}{1000} + 50 \cdot \frac{8}{1000} + 100 \cdot \frac{4}{1000} + 200 \cdot \frac{2}{1000} + 500 \cdot \frac{1}{1000}$$
$$= \frac{17}{10} = 1{,}7$$ (4 pts)

2°/



$\left(\text{ou } \frac{C^{985}_{3}}{C^{1000}_{3}} \right)$

$$p = \frac{985}{1000} \cdot \frac{984}{999} \cdot \frac{983}{998} \simeq 95{,}6\,\%$$ (2 pts)