

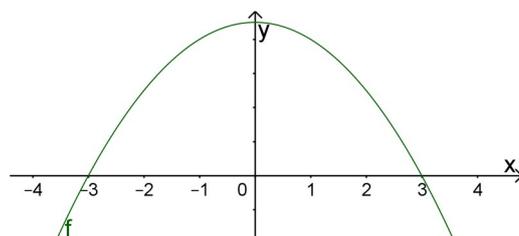
Exercice 1

a) $(f+g)(x) = f(x) + g(x) = x^3 + 2x^2 + 3x^2 - 1 = x^3 + 5x^2 - 1$ et $D_{f+g} = D_f \cap D_g = \mathbb{R}$
 $(f-g)(x) = f(x) - g(x) = x^3 + 2x^2 - (3x^2 - 1) = x^3 - x^2 + 1$ et $D_{f-g} = D_f \cap D_g = \mathbb{R}$
 $(f \cdot g)(x) = f(x) \cdot g(x) = (x^3 + 2x^2)(3x^2 - 1) = 3x^5 + 6x^4 - x^3 - 2x^2$ et $D_{f \cdot g} = D_f \cap D_g = \mathbb{R}$
 $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^3 + 2x^2}{3x^2 - 1}$ et $D_{\frac{f}{g}} = D_f \cap D_g \setminus \{x \mid g(x) = 0\} = \mathbb{R} \setminus \left\{ \frac{\pm\sqrt{3}}{3} \right\}$

b) $(f+g)(x) = f(x) + g(x) = \sqrt{1+x} + \sqrt{1-x}$ et $D_{f+g} = D_f \cap D_g = [-1; \infty[\cap]-\infty; 1] = [-1; 1]$
 $(f-g)(x) = f(x) - g(x) = \sqrt{1+x} - \sqrt{1-x}$ et $D_{f-g} = D_f \cap D_g = [-1; 1]$
 $(f \cdot g)(x) = f(x) \cdot g(x) = \sqrt{1+x} \cdot \sqrt{1-x} = \sqrt{1-x^2}$ et $D_{f \cdot g} = D_{f+g} = D_{f-g} = [-1; 1]$
 $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{1+x}}{\sqrt{1-x}} = \sqrt{\frac{1+x}{1-x}}$ et $D_{\frac{f}{g}} = D_f \cap D_g \setminus \{x \mid g(x) = 0\} = [-1; 1] \setminus \{1\} = [-1; 1[$

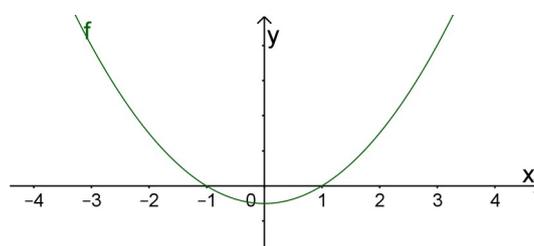
c) $(f+g)(x) = f(x) + g(x) = \sqrt{9-x^2} + \sqrt{x^2-1}$ voir ci-dessous pour le domaine.
 $(f-g)(x) = f(x) - g(x) = \sqrt{9-x^2} - \sqrt{x^2-1}$ voir ci-dessous pour le domaine.
 $(f \cdot g)(x) = f(x) \cdot g(x) = \sqrt{9-x^2} \cdot \sqrt{x^2-1} = \sqrt{-x^4 + 10x^2 - 9}$ voir ci-dessous pour le domaine.
 $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{9-x^2}}{\sqrt{x^2-1}} = \sqrt{\frac{9-x^2}{x^2-1}}$ voir ci-dessous pour le domaine.

$D_f: 9-x^2 \geq 0 \Leftrightarrow (3-x)(3+x) \geq 0$ et comme f est concave avec $Z_f = \{\pm 3\}$ on a $D_f = [-3; 3]$



$D_g: x^2-1 \geq 0 \Leftrightarrow (x-1)(x+1) \geq 0$ et comme g est convexe avec $Z_g = \{\pm 1\}$ on a

$D_g =]-\infty; -1] \cup [1; +\infty[$



$D_{f+g} = D_{f-g} = D_{f \cdot g} = D_f \cap D_g = [-3; -1] \cup [1; 3]$

$D_{\frac{f}{g}} = D_f \cap D_g \setminus \{x \mid g(x) = 0\} = [-3; -1] \cup [1; 3] \setminus \{\pm 1\} = [-3; -1[\cup]1; 3]$

Exercice 2

a) $(f \circ g)(x) = f(g(x)) = f(x^2 - 3x + 5) = 2(x^2 - 3x + 5) - 5 = 2x^2 - 5x + 5$

$(g \circ f)(x) = g(f(x)) = g(2x - 5) = (2x - 5)^2 - 3(2x - 5) + 5 = 4x^2 - 20x + 25 - 6x + 15 + 5 = 4x^2 - 26x + 45$

$(g \circ g)(x) = g(g(x)) = g(x^2 - 3x + 5) = (x^2 - 3x + 5)^2 - 3(x^2 - 3x + 5) + 5 = x^4 - 6x^3 + 19x^2 - 30x + 25 - 3x^2 + 9x - 15 + 5 = x^4 - 6x^3 - 3x^2 - 21x + 15$

b) $(f \circ g)(x) = f(g(x)) = f(x^2 - 2) = \frac{2(x^2 - 2) - 1}{(x^2 - 2) + 4} = \frac{2x^2 - 5}{x^2 + 2}$

$(g \circ f)(x) = g(f(x)) = g\left(\frac{2x-1}{x+4}\right) = \left(\frac{2x-1}{x+4}\right)^2 - 2 = \frac{4x^2 - 4x + 1 - 2(x^2 + 8x + 16)}{x^2 + 8x + 16} =$

$\frac{2x^2 - 20x - 31}{x^2 + 8x + 16}$

$(g \circ g)(x) = g(g(x)) = g(x^2 - 2) = (x^2 - 2)^2 - 2 = x^4 - 4x^2 + 2$

c) $(f \circ g)(x) = f(g(x)) = f\left(\frac{2x-1}{x-3}\right) = \frac{3\left(\frac{2x-1}{x-3}\right) - 4}{\left(\frac{2x-1}{x-3}\right) + 1} = \frac{\frac{6x-3-4(x-3)}{x-3}}{\frac{2x-1+x-3}{x-3}} = \frac{\frac{2x+9}{x-3}}{\frac{3x-4}{x-3}} = \frac{2x+9}{3x-4}$

$(g \circ f)(x) = g(f(x)) = g\left(\frac{3x-4}{x+1}\right) = \frac{2\left(\frac{3x-4}{x+1}\right) - 1}{\left(\frac{3x-4}{x+1}\right) - 3} = \frac{\frac{6x-8-(x+1)}{x+1}}{\frac{3x-4-3(x+1)}{x+1}} = \frac{\frac{5x-9}{x+1}}{\frac{-7}{x+1}} = -\frac{5x-9}{7}$

$(g \circ g)(x) = g(g(x)) = g\left(\frac{2x-1}{x-3}\right) = \frac{2\left(\frac{2x-1}{x-3}\right) - 1}{\left(\frac{2x-1}{x-3}\right) - 3} = \frac{\frac{4x-2-(x-3)}{x-3}}{\frac{2x-1-3(x-3)}{x-3}} = \frac{\frac{3x+1}{x-3}}{\frac{-x+8}{x-3}} = \frac{3x+1}{8-x}$

d) $(f \circ g)(x) = f(g(x)) = f(\sqrt{1-x}) = \sqrt{(\sqrt{1-x})^2 - 1} = \sqrt{1-x-1} = \sqrt{-x}$

$(g \circ f)(x) = g(f(x)) = g(\sqrt{x^2-1}) = \sqrt{1-\sqrt{x^2-1}}$

$(g \circ g)(x) = g(g(x)) = g(\sqrt{1-x}) = \sqrt{1-\sqrt{1-x}}$

Exercice 3

a) $f_1(x) = \frac{x-1}{x+1}$

$$f_2(x) = (f_1 \circ f_1)(x) = f_1(f_1(x)) = f_1\left(\frac{x-1}{x+1}\right) = \frac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1} = \frac{\frac{x-1-x-1}{x+1}}{\frac{x-1+x+1}{x+1}} = \frac{\frac{-2}{x+1}}{\frac{2x}{x+1}} = \frac{-2}{2x} = -\frac{1}{x}$$

$$f_3(x) = (f_1 \circ f_2)(x) = f_1(f_2(x)) = f_1\left(-\frac{1}{x}\right) = \frac{-\frac{1}{x} - 1}{-\frac{1}{x} + 1} = \frac{\frac{-1-x}{x}}{\frac{-1+x}{x}} = \frac{-1-x}{-1+x} = -\frac{x+1}{x-1}$$

$$f_4(x) = (f_1 \circ f_3)(x) = f_1(f_3(x)) = f_1\left(-\frac{x+1}{x-1}\right) = \frac{-\frac{x+1}{x-1} - 1}{-\frac{x+1}{x-1} + 1} = \frac{\frac{-x-1-x+1}{x-1}}{\frac{-x-1+x-1}{x-1}} = \frac{-2x}{-2} = x$$

$$f_5(x) = (f_1 \circ f_4)(x) = f_1(f_4(x)) = f_1(x)$$

- b) On constate que $f_5(x) = f_1(x)$ et donc on a aussi $f_6 = f_2$, $f_7 = f_3$, $f_8 = f_4$, $f_9 = f_1$, etc.
 Finalement, on a $f_{1000} = f_4$ ainsi $f_{1000}(3) = f_4(3) = 3$.

c) $(f \circ g)(x) = f(g(x)) = f\left(\frac{x+1}{1-x}\right) = \frac{\frac{x+1}{1-x} - 1}{\frac{x+1}{1-x} + 1} = \frac{\frac{x+1-1+x}{1-x}}{\frac{x+1+1-x}{1-x}} = \frac{\frac{2x}{1-x}}{\frac{2}{1-x}} = \frac{2x}{2} = x$

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{x-1}{x+1}\right) = \frac{\frac{x-1}{x+1} - 1}{1 - \frac{x-1}{x+1}} = \frac{\frac{x-1-x+1}{x+1}}{\frac{x+1-x+1}{x+1}} = \frac{\frac{2x}{x+1}}{\frac{2}{x+1}} = \frac{2x}{2} = x$$

On constate que g est la réciproque de f .

Exercice 4

$$f(x) = \frac{1}{1-x} \quad x \xrightarrow{v} 1-x \xrightarrow{u} \frac{1}{1-x} \quad \text{donc } f = u \circ v$$

$$g(x) = 1-2x \quad x \xrightarrow{w} 2x \xrightarrow{v} 1-2x \quad \text{donc } g = v \circ w$$

$$h(x) = \frac{x-1}{x} \quad x \xrightarrow{u} \frac{1}{x} \xrightarrow{v} 1 - \frac{1}{x} = \frac{x-1}{x} \quad \text{donc } h = v \circ u$$

$$i(x)=x \quad x \xrightarrow[u]{\quad} \frac{1}{x} \xrightarrow[u]{\quad} \frac{1}{\frac{1}{x}}=x \quad \text{donc } i=u \circ u$$

$$k(x)=\frac{x}{x-2} \quad x \xrightarrow[u]{\quad} \frac{1}{x} \xrightarrow[w]{\quad} \frac{2}{x} \xrightarrow[v]{\quad} 1-\frac{2}{x}=\frac{x-2}{x} \xrightarrow[u]{\quad} \frac{1}{\frac{x-2}{x}}=\frac{x}{x-2}$$

$$\text{donc } k=u \circ v \circ w \circ u$$

Exercice 5

a) $(h \circ f)(x) = h(f(x)) = h(x+2) = (x+2)^2 = x^2 + 4x + 4$

$$x \in D_{h \circ f} \Leftrightarrow x \in D_f \text{ et } f(x) \in D_h$$

$$\Leftrightarrow x \in \mathbb{R} \text{ et } f(x) \in \mathbb{R}$$

$$\Leftrightarrow x \in \mathbb{R} \text{ et } x+2 \in \mathbb{R}$$

$$\Leftrightarrow x \in \mathbb{R}$$

$$\text{Donc } D_{h \circ f} = \mathbb{R}$$

b) $(f \circ g)(x) = f(g(x)) = f\left(\frac{2x+5}{x+2}\right) = \left(\frac{2x+5}{x+2}\right) + 2 = \frac{2x+5}{x+2} + \frac{2(x+2)}{x+2} = \frac{4x+9}{x+2}$

$$x \in D_{f \circ g} \Leftrightarrow x \in D_g \text{ et } g(x) \in D_f$$

$$\Leftrightarrow x \in \mathbb{R} \setminus \{-2\} \text{ et } g(x) \in \mathbb{R}$$

$$\Leftrightarrow x \in \mathbb{R} \setminus \{-2\} \text{ et } \frac{2x+5}{x+2} \in \mathbb{R}$$

$$\Leftrightarrow x \in \mathbb{R} \setminus \{-2\} \text{ et } x \in \mathbb{R} \setminus \{-2\}$$

$$\text{Donc } D_{f \circ g} = \mathbb{R} \setminus \{-2\}$$

c) $(g \circ g)(x) = g(g(x)) = g\left(\frac{2x+5}{x+2}\right) = \frac{2\left(\frac{2x+5}{x+2}\right)+5}{\left(\frac{2x+5}{x+2}\right)+2} = \frac{\frac{4x+10+5x+10}{x+2}}{\frac{2x+5+2x+4}{x+2}} = \frac{\frac{9x+20}{x+2}}{\frac{4x+9}{x+2}} = \frac{9x+20}{4x+9}$

$$\begin{aligned}
 x \in D_{g \circ g} &\Leftrightarrow x \in D_g \text{ et } g(x) \in D_g \\
 &\Leftrightarrow x \in \mathbb{R} \setminus \{-2\} \text{ et } g(x) \in \mathbb{R} \setminus \{-2\} \\
 &\Leftrightarrow x \in \mathbb{R} \setminus \{-2\} \text{ et } \frac{2x+5}{x+2} \in \mathbb{R} \setminus \{-2\} \\
 &\Leftrightarrow x \in \mathbb{R} \setminus \{-2\} \text{ et } \frac{2x+5}{x+2} \neq -2 \quad \text{On résout } \frac{2x+5}{x+2} = -2 \Leftrightarrow 2x+5 = -2(x+2) \Leftrightarrow x = -\frac{9}{4} \\
 &\Leftrightarrow x \in \mathbb{R} \setminus \{-2\} \text{ et } x \in \mathbb{R} \setminus \left\{-\frac{9}{4}\right\}
 \end{aligned}$$

$$\text{Donc } D_{g \circ g} = \mathbb{R} \setminus \left\{-\frac{9}{4}; -2\right\}$$

d) $(f \circ j)(x) = f(j(x)) = f(\sqrt{x+1}) = \sqrt{x+1} + 2$

$$\begin{aligned}
 x \in D_{f \circ j} &\Leftrightarrow x \in D_j \text{ et } j(x) \in D_f \\
 &\Leftrightarrow x \in [-1; +\infty[\text{ et } j(x) \in \mathbb{R} \\
 &\Leftrightarrow x \in [-1; +\infty[\text{ et } \sqrt{x+1} \in \mathbb{R} \\
 &\Leftrightarrow x \in [-1; +\infty[\text{ et } x \in [-1; +\infty[
 \end{aligned}$$

$$\text{Donc } D_{f \circ j} = [-1; +\infty[$$

e) $(g \circ f)(x) = g(f(x)) = g(x+2) = \frac{2(x+2)+5}{(x+2)+2} = \frac{4x+9}{x+4}$

$$\begin{aligned}
 x \in D_{g \circ f} &\Leftrightarrow x \in D_f \text{ et } f(x) \in D_g \\
 &\Leftrightarrow x \in \mathbb{R} \text{ et } f(x) \in \mathbb{R} \setminus \{-2\} \\
 &\Leftrightarrow x \in \mathbb{R} \text{ et } x+2 \in \mathbb{R} \setminus \{-2\} \quad \text{On résout } x+2 = -2 \Leftrightarrow x = -4
 \end{aligned}$$

$$\text{Donc } D_{g \circ f} = \mathbb{R} \setminus \left\{-\frac{9}{4}; -2\right\}$$

f) $(g \circ h)(x) = g(h(x)) = g(x^2) = \frac{2x^2+5}{x^2+2}$

$$\begin{aligned}
 x \in D_{g \circ h} &\Leftrightarrow x \in D_h \text{ et } h(x) \in D_g \\
 &\Leftrightarrow x \in \mathbb{R} \text{ et } h(x) \in \mathbb{R} \setminus \{-2\} \\
 &\Leftrightarrow x \in \mathbb{R} \text{ et } x^2 \in \mathbb{R} \setminus \{-2\} \quad \text{On résout } x^2 = -2 \Leftrightarrow S = \emptyset \\
 &\Leftrightarrow x \in \mathbb{R} \text{ et } x \in \mathbb{R}
 \end{aligned}$$

$$\text{Donc } D_{g \circ h} = \mathbb{R}$$

Exercice 6

a) $(f \circ g)(x) = f(g(x)) = f(\sqrt{3x}) = (\sqrt{3x})^2 - 4 = 3x - 4$

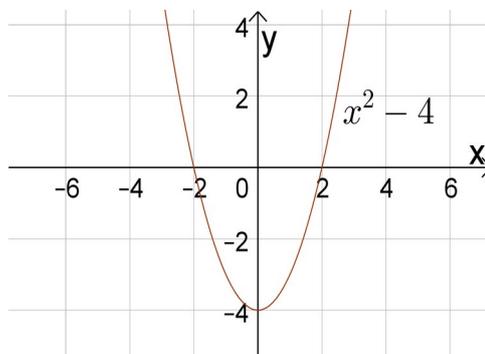
$$\begin{aligned} x \in D_{f \circ g} &\Leftrightarrow x \in D_g \text{ et } g(x) \in D_f \\ &\Leftrightarrow x \in \mathbb{R} \text{ et } g(x) \in \mathbb{R} \\ &\Leftrightarrow x \in \mathbb{R} \text{ et } \sqrt{3x} \in \mathbb{R} \\ &\Leftrightarrow x \in \mathbb{R} \text{ et } x \in [0; +\infty[\end{aligned}$$

Donc $D_{f \circ g} = x \in [0; +\infty[$

$$(g \circ f)(x) = g(f(x)) = g(x^2 - 4) = \sqrt{3(x^2 - 4)}$$

$$\begin{aligned} x \in D_{g \circ f} &\Leftrightarrow x \in D_f \text{ et } f(x) \in D_g \\ &\Leftrightarrow x \in \mathbb{R} \text{ et } f(x) \in [0; +\infty[\\ &\Leftrightarrow x \in \mathbb{R} \text{ et } x^2 - 4 \in [0; +\infty[\quad \text{On résout } x^2 - 4 \geq 0 \Leftrightarrow (x+2)(x-2) \geq 0 \\ &\Leftrightarrow x \in \mathbb{R} \text{ et } x \in]-\infty; -2] \cup [2; +\infty[\end{aligned}$$

Donc $D_{g \circ f} =]-\infty; -2] \cup [2; +\infty[$



b) $(f \circ g)(x) = f(g(x)) = f(\sqrt{x+5}) = \sqrt{\sqrt{x+5}} - 2$

$$\begin{aligned} x \in D_{f \circ g} &\Leftrightarrow x \in D_g \text{ et } g(x) \in D_f \\ &\Leftrightarrow x \in [-5; +\infty[\text{ et } g(x) \in [2; +\infty[\\ &\Leftrightarrow x \in [-5; +\infty[\text{ et } \sqrt{x+5} \in [2; +\infty[\quad \text{On résout } \sqrt{x+5} \geq 2 \Leftrightarrow \underbrace{x+5}_{\geq 4} \geq 4 \Leftrightarrow x \geq -1 \end{aligned}$$

(l'inégalité ne change pas de sens car $2 \geq 0$)

Donc $D_{f \circ g} = x \in [-1; +\infty[$

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x-2}) = \sqrt{\sqrt{x-2}} + 5$$

$$\begin{aligned} x \in D_{g \circ f} &\Leftrightarrow x \in D_f \text{ et } f(x) \in D_g \\ &\Leftrightarrow x \in [2; +\infty[\text{ et } f(x) \in [-5; +\infty[\\ &\Leftrightarrow x \in [2; +\infty[\text{ et } \sqrt{x-2} \in [-5; +\infty[\quad \text{On résout } \sqrt{x-2} \geq -5 \Leftrightarrow x \geq 2 \\ &\Leftrightarrow x \in [2; +\infty[\text{ et } x \in [2; +\infty[\end{aligned}$$

Donc $D_{g \circ f} = [2; +\infty[$