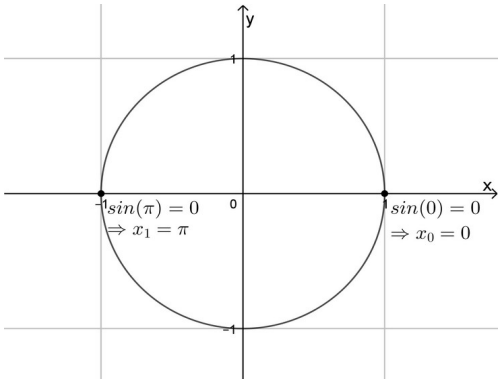
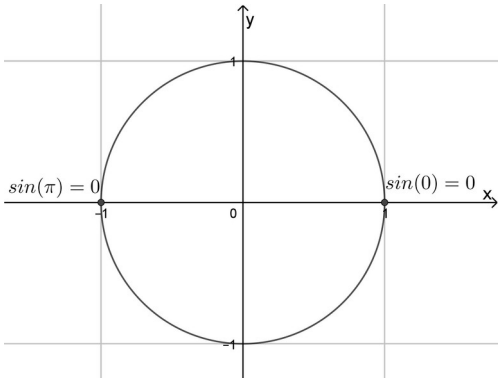
Exercice 13

a) $\sin(x)=0$



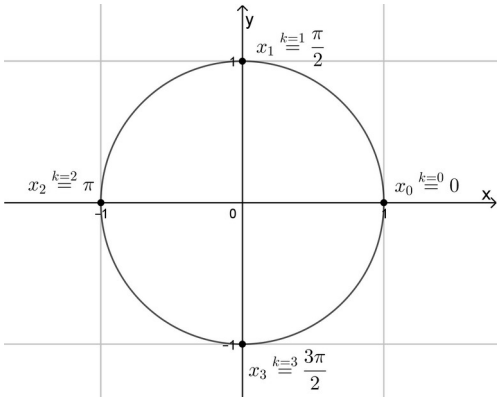
$$S=\{\pi+2k\pi|k\in\mathbb{Z}\}\cup\{0+2k\pi|k\in\mathbb{Z}\}=\{k\pi|k\in\mathbb{Z}\}$$

b) $\sin(2x)=0$

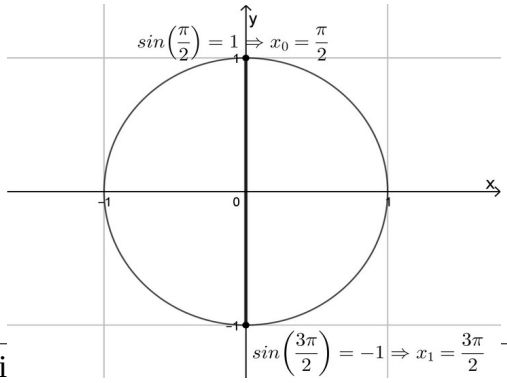


$$\Leftrightarrow \quad 2x=0+2k\pi \quad \text{ou} \quad 2x=\pi+2k\pi$$
$$\Leftrightarrow \quad x=k\pi \quad \text{ou} \quad x=\frac{\pi}{2}+k\pi$$
$$\Leftrightarrow \quad S=\{k\pi|k\in\mathbb{Z}\} \quad \cup \quad \left\{\frac{\pi}{2}+k\pi|k\in\mathbb{Z}\right\}$$
$$\Leftrightarrow \quad S=\left\{\frac{k\pi}{2}|k\in\mathbb{Z}\right\}$$

$\Leftrightarrow$

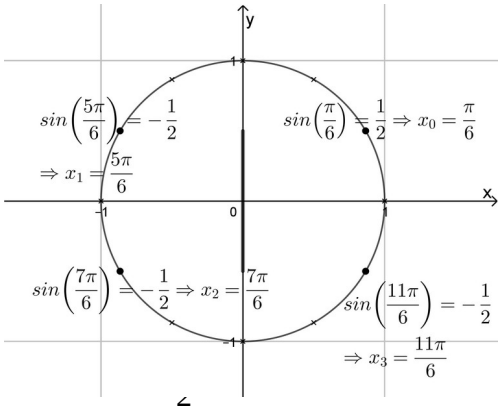


c) $\sin^2(x)=1 \Leftrightarrow \sin(x)=\pm 1$



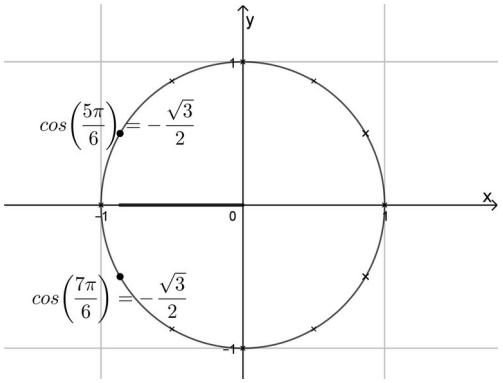
$$S=\left\{\frac{\pi}{2}+2k\pi|k\in\mathbb{Z}\right\}\cup\left\{\frac{3\pi}{2}+2k\pi|k\in\mathbb{Z}\right\}=$$
$$\left\{\frac{\pi}{2}+k\pi|k\in\mathbb{Z}\right\}$$

d) $\sin^2(x)=\frac{1}{4}\Leftrightarrow \sin(x)=\pm\frac{1}{2}$



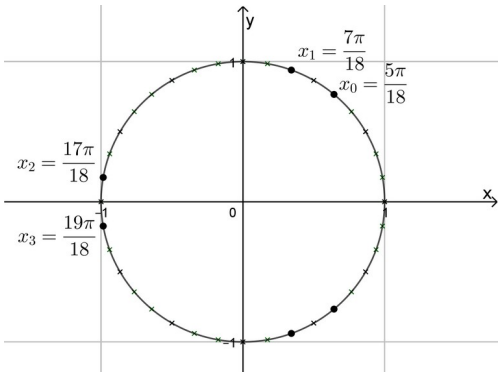
$$S=\left\{\frac{\pi}{6}+2k\pi|k\in\mathbb{Z}\right\}\cup\left\{\frac{5\pi}{6}+2k\pi|k\in\mathbb{Z}\right\}\cup\left\{\frac{7\pi}{6}+2k\pi|k\in\mathbb{Z}\right\}\cup\left\{\frac{11\pi}{6}+2k\pi|k\in\mathbb{Z}\right\}$$
$$=\left\{\frac{(4k+1)\pi}{6}|k\in\mathbb{Z}\right\}$$

e)

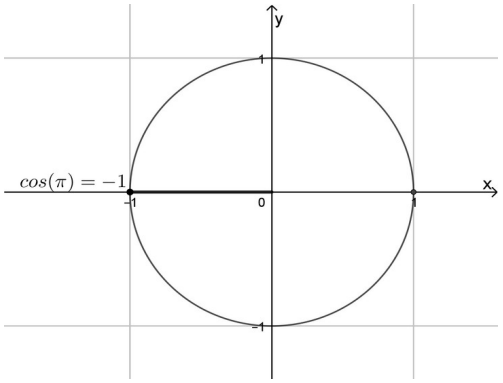


$$\Leftrightarrow \quad 3x=\frac{5\pi}{6}+2k\pi \quad \text{ou} \quad 3x=\frac{7\pi}{6}+2k\pi$$
$$\Leftrightarrow \quad x=\frac{5\pi}{18}+\frac{2k\pi}{3} \quad \text{ou} \quad x=\frac{7\pi}{18}+\frac{2k\pi}{3}$$
$$\Leftrightarrow \quad S=\left\{\frac{\pi}{18}(5+12k)|k\in\mathbb{Z}\right\}\cup\left\{\frac{\pi}{18}(7+12k)|k\in\mathbb{Z}\right\}$$

$\Leftrightarrow$



f) $\cos\left(x-\frac{\pi}{3}\right)=-1$

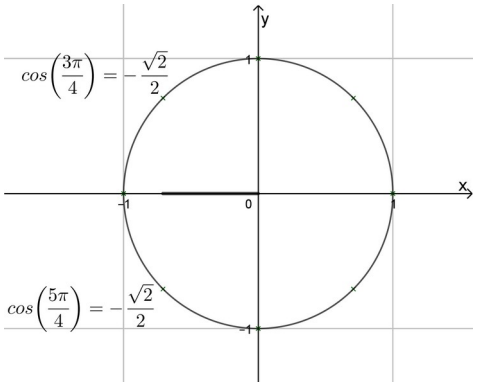


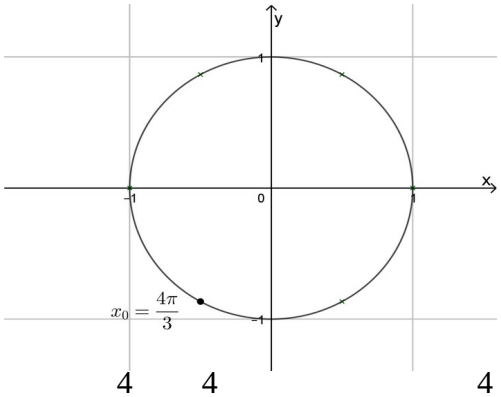
$\Leftrightarrow \quad x-\frac{\pi}{3}=\pi+2k\pi$

$\Leftrightarrow \quad x=\frac{4\pi}{3}+\frac{6k\pi}{3}$

$\Leftrightarrow \quad S=\left\{\frac{2\pi}{3}(2+3k)\,|\,k\in\mathbb{Z}\right\}$

g) $2\cos\left(\frac{1}{4}x\right)=-\sqrt{2}\Leftrightarrow\cos\left(\frac{1}{4}x\right)=-\frac{\sqrt{2}}{2}$





$\Leftrightarrow$

$4 \qquad 4 \qquad\qquad 4 \quad x=\frac{5\pi}{4}+2k\pi$

$\Leftrightarrow \quad x=3\pi+8k\pi \qquad \text{ou} \qquad x=5\pi+8k\pi$

$\Leftrightarrow \quad S=\{\pm 3\pi+8k\pi\,|\,k\in\mathbb{Z}\}$

aucune solution dans $[0;2\pi[$

h) $\cos^2(x)-\sin^2(x)=1 \underset{\cos^2(x)+\sin^2(x)=1}{\Leftrightarrow} \cos^2(x)-\sin^2(x)=\cos^2(x)+\sin^2(x)\Leftrightarrow 2\sin^2(x)=0\Leftrightarrow$
$\sin^2(x)=0\Leftrightarrow\sin(x)=0 \underset{\text{voir a)}}{\Leftrightarrow} S=\{k\pi\,|\,k\in\mathbb{Z}\}$

Exercice 14

a) $\cos(x)=-0.51812\Leftrightarrow x_1=\arccos(-0.51812)\simeq 2.11545$

Attention, la calculatrice ne tient pas compte de $\cos(x)=\cos(-x)$ donc

$\cos(-x)=-0.51812\Leftrightarrow -x\simeq 2.11545\Leftrightarrow x\simeq -2.11545$ qu'on considère dans l'intervalle

$[0;2\pi[$ pour finalement obtenir $x_2\simeq -2.11545+2\pi\simeq 4.16773$.

b) $\sin(x)=-0.49712\Leftrightarrow x=\arcsin(-0.49712)\simeq -0.52027\notin[0;2\pi[$

La calculatrice ne donne aucune solution ici. Attention, on a toutefois :

$\sin(x)=\sin(x+2\pi)$ donc $x_1=\arcsin(-0.49712)\simeq -0.52027+2\pi\simeq 5.76291\in[0;2\pi[$

Et $\sin(x)=\sin(\pi-x)$ donc $x_2=\arcsin(-0.49712)\simeq \pi-(-0.52027)\simeq 3.66187\in[0;2\pi[$

c) $\cos(x)=-1.24\Leftrightarrow x=\arccos(-1.24)$ *Domain Error*

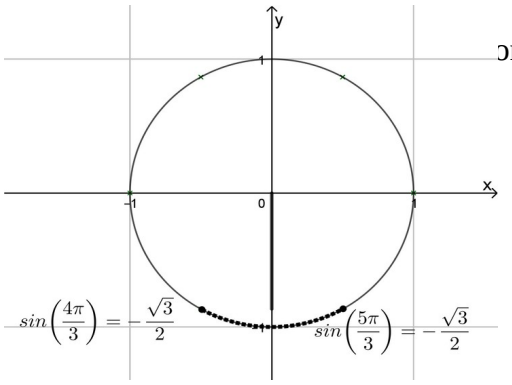
Il y a en effet pas de solution à cette équation car $-1\leq\cos(x)\leq 1$, $\forall x\in\mathbb{R}$.

d) $\tan(x)=138.245\Leftrightarrow x_1=\arctan(138.245)\simeq 1.56356$

Attention, la calculatrice ne donne pas la solution $x_2=x_1+\pi\simeq 4.70516$ issue de $\tan(x)=-\tan(\pi-x)$.

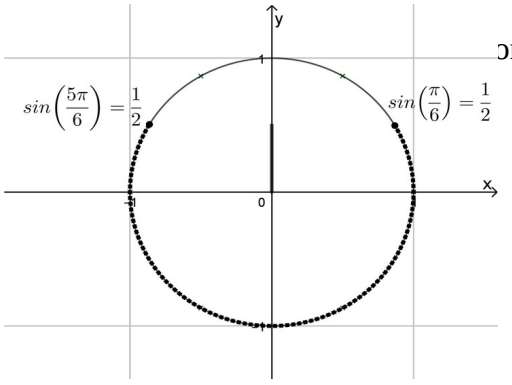
Exercice 15

a) $\sin(x)<-\frac{\sqrt{3}}{2}$



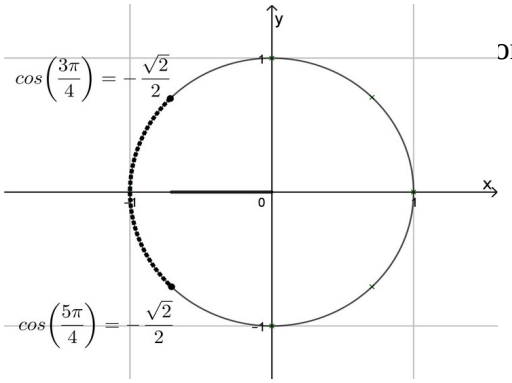
donc $S=\left]\frac{4\pi}{3};\frac{5\pi}{3}\right[$

b) $\sin(x)<\frac{1}{2}$



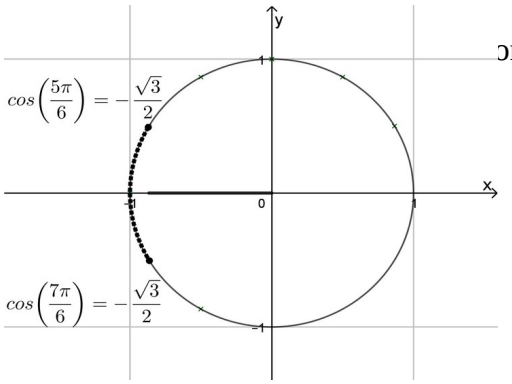
donc $S=[0;2\pi]\setminus\left[\frac{\pi}{6};\frac{5\pi}{6}\right]$

c) $\cos(x)<-\frac{\sqrt{2}}{2}$



donc $S=\left]\frac{3\pi}{4};\frac{5\pi}{4}\right[$

d) $-2\cos(x)>\sqrt{3}\Leftrightarrow\cos(x)<-\frac{\sqrt{3}}{2}$



donc $S=\left]\frac{5\pi}{6};\frac{7\pi}{6}\right[$

Exercice 16

a) $\sin^2(x)+\cos^2(x)=1\Leftrightarrow\left(\frac{1}{3}\right)^2+\cos^2(x)=1\Leftrightarrow\cos^2(x)=\frac{8}{9}\Leftrightarrow\cos(x)=\pm\sqrt{\frac{8}{9}}\Leftrightarrow\cos(x)=\pm\frac{2\sqrt{2}}{3}$

$\Leftrightarrow\cos(x)=-\frac{2\sqrt{2}}{3}$ car $\cos(x)=\frac{2\sqrt{2}}{3}\Rightarrow x\notin\left]\frac{\pi}{2};\pi\right[$

b) $\tan(x)=10\underset{\tan(x)=\frac{\sin(x)}{\cos(x)}}{\Leftrightarrow}\frac{\sin(x)}{\cos(x)}=10\Leftrightarrow\frac{\sin^2(x)}{\cos^2(x)}=100\Leftrightarrow$

$\sin^2(x)=100\cos^2(x)\underset{\cos(x)^2=1-\sin^2(x)}{\Leftrightarrow}\sin^2(x)=100(1-\sin^2(x))\Leftrightarrow\sin^2(x)=100-100\sin^2(x)$

$101\sin^2(x)=100\Leftrightarrow\sin(x)=\pm\sqrt{\frac{100}{101}}\Leftrightarrow\sin(x)=-\sqrt{\frac{100}{101}}$ car $\sin(x)=\sqrt{\frac{100}{101}}\Rightarrow x\notin\left]\pi;\frac{3\pi}{2}\right[$