Ch 8 Corrigé des activités

Act 1 :

① $1+2+2^2+2^3+2^4+\dots+2^{63}$

② cf 1re : Posons $n = 1+2+2^2+\dots+2^{63}$

idée ! $n(2-1) = (1+2+2^2+\dots+2^{63})(2-1)$

$$= (\cancel{2}-1)+(\cancel{2^2}-\cancel{2})+(\cancel{2^3}-\cancel{2^2})+\dots+(\cancel{2^{63}}-\cancel{2^{62}})+(2^{64}-\cancel{2^{63}})$$

$$= 2^{64}-1$$

Soit $n \cdot 1 = 2^{64}-1$

$n = 2^{64}-1 \cong 1{,}84 \cdot 10^{19}$

↑ calculatrice

③ $2^{64}-1 \cong 2^{64} = 2^{60} \cdot 2^4$

$$= (2^{10})^6 \cdot 16 = 1024^6 \cdot 16$$

$$\cong (10^3)^6 \cdot 16$$

$$= 16 \cdot 10^{18}$$

$$= 1{,}6 \cdot 10^{19}$$

④ $\dfrac{100 \text{ grains}}{1 \text{ gr}} = \dfrac{1{,}84 \cdot 10^{19}}{x \text{ gr}} \Leftrightarrow x\,[\text{gr}] = \dfrac{1{,}84 \cdot 10^{19}}{10^2} = 1{,}84 \cdot 10^{17}$

$= 1{,}84 \cdot 10^{11}$ tonnes

⑤ $A = 4\pi \cdot (6370)^2 \cong 509904363{,}8\ [\text{km}]^2$

29% de $A \cong 1{,}479 \cdot 10^8\,[\text{km}]^2 = 1{,}479 \cdot 10^{18}\,[\text{cm}^2]$

on aurait donc : $\dfrac{x \text{ grains}}{1 \text{ cm}^2} = \dfrac{1{,}84 \cdot 10^{19}}{1{,}479 \cdot 10^{18}} \cong 12{,}47 \text{ gr/cm}^2$

Act 2 :

$2^{\frac{1}{2}} = \sqrt{2}$; $4^{0{,}1} = 4^{\frac{1}{10}} = (4^{\frac{1}{2}})^{\frac{1}{5}} = (\sqrt{4})^{1/5} = 2^{1/5} = \sqrt[5]{2}$

$0^3 = 0$; $3^0 = 1$

0^0 non défini $[0^1 = 0^2 = 0^3 = 0^4 = \dots = 0$ mais $1^0 = 2^0 = 3^0 = 4^0 = \dots = 1 \Rightarrow 0^0 = ???]$

$\left(\frac{1}{3}\right)^{-2/3} = \dfrac{1}{\left(\frac{1}{3}\right)^{2/3}} = \dfrac{1}{\frac{1}{3^{2/3}}} = 3^{2/3} = (3^2)^{1/3} = 9^{1/3} = \sqrt[3]{9}$

$(-1)^n = \begin{cases} 1 & \text{si } n \text{ est pair} \\ -1 & \text{si } n \text{ est impair} \end{cases}$; $(-2)^{\frac{1}{2}}$ "=" $\sqrt{-2}$ n'existe pas car aucun nombre positif élevé au carré ne donne -2

Act 3

① 2^{222}

$2^{(22^2)} = 2^{484}$

$22^{22} = (11\cdot 2)^{22} > (8\cdot 2)^{22} = (2^3\cdot 2)^{22} = (2^4)^{22} = 2^{88}$

et $(11\cdot 2)^{22} < (16\cdot 2)^{22} = (2^4\cdot 2)^{22} = (2^5)^{22} = 2^{110}$

$\Big\}$ donc $2^{88} < 22^{22} < 2^{110}$

$(2^{22})^2 = 2^{44}$

donc $(2^{22})^2 < 22^{22} < 2^{222} < 2^{(22^2)}$

② (a) $\dfrac{(7\cdot 2\cdot 5)^{762}\cdot 3^{598}\cdot 1\cdot 2^{60}}{(3\cdot 7)^{599}\cdot 5^{634}\cdot 2^{692}} = \dfrac{7^{762}\cdot 2^{762}\cdot 5^{762}\cdot 3^{598}\cdot 2^{60}}{3^{599}\cdot 7^{599}\cdot 5^{634}\cdot 2^{692}}$

$= \dfrac{7^{762-599}\cdot 5^{762-634}\cdot 2^{762+60-692}}{3^{599-598}} = \dfrac{7^{163}\cdot 5^{128}\cdot 2^{130}}{3}$

(b) $\dfrac{x^{-18}y^3x^{12}y^{-3}}{x^{-30}\cdot x^6\cdot x^{18}} = \dfrac{x^{-6}y^0}{x^{-6}} = x^0\cdot y^0 = 1$ (pour $x\neq 0$, $y\neq 0$!)

Act 4

① (a) $\sqrt{\dfrac{50}{242}} = \sqrt{\dfrac{25}{121}} = \dfrac{5}{11}$ (b) $\sqrt{0{,}04} = 0{,}2$ (c) $\dfrac{\sqrt{12}}{\sqrt{75}} = \sqrt{\dfrac{12}{75}} = \sqrt{\dfrac{4}{25}} = \dfrac{2}{5}$

② (a) $\sqrt{24} = \sqrt{4\cdot 6} = \sqrt{4}\sqrt{6} = 2\sqrt{6}$

(b) $\sqrt{147} = \sqrt{49\cdot 3} = 7\sqrt{3}$

(c) $3\sqrt{5} - 4\sqrt{4\cdot 5} + 5\sqrt{9\cdot 5} - 3\sqrt{16\cdot 5} = 3\sqrt{5} - 4\cdot 2\sqrt{5} + 5\cdot 3\sqrt{5} - 3\cdot 4\sqrt{5}$

$= 3\sqrt{5} - 8\sqrt{5} + 15\sqrt{5} - 12\sqrt{5} = -2\sqrt{5}$

③ (a) $\dfrac{1}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$ (b) $\dfrac{15}{\sqrt{80}} = \dfrac{15}{\sqrt{16\cdot 5}} = \dfrac{15}{4\sqrt{5}}\cdot\dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{\overset{3}{\cancel{15}}\sqrt{5}}{4\cdot\cancel{5}} = \dfrac{3\sqrt{5}}{4}$

(c) $\dfrac{3\sqrt{2}-\sqrt{5}}{\sqrt{10}-1}\cdot\dfrac{\sqrt{10}+1}{\sqrt{10}+1} = \dfrac{(3\sqrt{2}-\sqrt{5})(\sqrt{10}+1)}{(\sqrt{10})^2 - 1^2} = \dfrac{3\sqrt{2}\sqrt{10}+3\sqrt{2}-\sqrt{5}\sqrt{10}-\sqrt{5}}{10-1}$

$= \dfrac{3\sqrt{20}+3\sqrt{2}-\sqrt{50}-\sqrt{5}}{9} = \dfrac{3\sqrt{4\cdot 5}+3\sqrt{2}-\sqrt{25\cdot 2}-\sqrt{5}}{9}$

$= \dfrac{3\cdot 2\sqrt{5}+3\sqrt{2}-5\sqrt{2}-\sqrt{5}}{9} = \dfrac{5\sqrt{5}-2\sqrt{2}}{9}$

(d) $\frac{\sqrt{75}+\sqrt{6}}{2\sqrt{3}+\sqrt{2}} \cdot \frac{2\sqrt{3}-\sqrt{2}}{2\sqrt{3}-\sqrt{2}} = \frac{2\sqrt{3}\sqrt{75}+2\sqrt{6}\sqrt{3}-\sqrt{75}\sqrt{2}-\sqrt{6}\sqrt{2}}{(2\sqrt{3})^2-(\sqrt{2})^2}$

$= \frac{2\cdot\sqrt{3\cdot 3\cdot 25}+2\sqrt{2\cdot 3\cdot 3}-\sqrt{3\cdot 25\cdot 2}-\sqrt{2\cdot 3\cdot 2}}{4\cdot 3-2}$

$= \frac{2\sqrt{9}\sqrt{25}+2\sqrt{2}\sqrt{9}-\sqrt{25}\sqrt{6}-\sqrt{4}\sqrt{3}}{10}$

$= \frac{2\cdot 3\cdot 5+2\cdot 3\sqrt{2}-5\sqrt{6}-2\sqrt{3}}{10} = \frac{30+6\sqrt{2}-5\sqrt{6}-2\sqrt{3}}{10}$

(4) $\sqrt{10+\sqrt{2}}+\sqrt{10-\sqrt{2}} = ?$ Appelons-le x

idée : on met au carré : $x^2 = \left(\sqrt{10+\sqrt{2}}+\sqrt{10-\sqrt{2}}\right)^2$

$= (10+\cancel{\sqrt{2}}) + 2\sqrt{10+\sqrt{2}}\sqrt{10-\sqrt{2}} + (10-\cancel{\sqrt{2}})$

$= 20 + 2\sqrt{(10+\sqrt{2})(10-\sqrt{2})}$

$= 20 + 2\sqrt{100-(\sqrt{2})^2}$

$= 20 + 2\sqrt{96}$

$= 20 + 2\sqrt{16\cdot 6}$

$= 20 + 2\cdot 4\sqrt{6}$

$= 20 + 8\sqrt{6}$

donc $x^2 = 20+8\sqrt{6}$

$x = \pm\sqrt{20+8\sqrt{6}}$

comme x est une somme de 2 racines, $x > 0$:

$x = \sqrt{20+8\sqrt{6}}$

(5) <u>Conjecture</u> : $\sqrt{6}-\sqrt{2} = 2\sqrt{2-\sqrt{3}}$

<u>dém</u> : $(\sqrt{6}-\sqrt{2})^2 = 6-2\sqrt{6}\sqrt{2}+2 = 8-2\sqrt{12} = 8-2\sqrt{4\cdot 3} = 8-4\sqrt{3}$

$(2\sqrt{2-\sqrt{3}})^2 = 4(2-\sqrt{3}) = 8-4\sqrt{3}$

les 2 nombres sont positifs et ont le même carré, donc ils sont égaux

Act 5

① (a) $\sqrt[5]{-32} = -2$ (b) $\sqrt[3]{0,027} = 0,3$ (c) $\sqrt[4]{81} = 3$

② (a) $\sqrt[3]{3}\,\sqrt[3]{9} = \sqrt[3]{27} = 3$ (c) $\sqrt{\sqrt{16}} = \sqrt{4} = 2$ (d) $(1024^{1/2})^{1/5} = 1024^{1/10} = 2$

(b) $\sqrt[5]{16}\,\sqrt[5]{2} = \sqrt[5]{32} = 2$ (e) $\sqrt[7]{\sqrt{7^7}} = \left((7^7)^{1/2}\right)^{1/7} = 7^{\cancel{7}\cdot\frac{1}{2}\cdot\frac{1}{\cancel{7}}} = \sqrt{7}$

③ $\sqrt{2^4} + 3\sqrt[3]{3^9} - 4\sqrt[4]{2^{16}} - \sqrt[5]{4^{10}} - 2\sqrt[6]{2^{12}}$

$= 2^2 + 3\cdot(3^9)^{1/3} - 4\cdot(2^{16})^{1/4} - (4^{10})^{1/5} + 2\cdot(2^{12})^{1/6}$

$= 4 + 3\cdot 3^3 - 4\cdot 2^4 - 4^2 + 2\cdot 2^2$

$= 4 + 3^4 - 2^6 - 16 + 2^3 = 4 + 81 - 64 - 16 + 8 = 13$

④ (a) $\sqrt[3]{a\sqrt{a^4}} = \sqrt[3]{a\cdot a^2} = \sqrt[3]{a^3} = a$ ($a \in \mathbb{R}$ quelconque)

(b) $\sqrt[3]{a\sqrt[3]{\frac{1}{\sqrt[3]{a}}}} = \sqrt[3]{a\sqrt[3]{\frac{1}{a^{1/3}}}} = \sqrt[3]{a\sqrt[3]{a^{-1/3}}}$

$= \sqrt[3]{a(a^{-1/3})^{1/3}} = \sqrt[3]{a\,a^{-1/9}}$

$= \sqrt[3]{a^{1-\frac{1}{9}}} = \sqrt[3]{a^{8/9}}$

$= (a^{8/9})^{1/3} = a^{8/27} = \sqrt[27]{a^8}$ $(a \in \mathbb{R}^*)$

Act 6

① (a) $5^{\frac{1}{2}} = \sqrt{5}$ (b) $1024^{\frac{1}{10}} = \sqrt[10]{1024} \left(= \sqrt[10]{2^{10}} = 2\right)$

(c) $36^{\frac{3}{2}} = \sqrt{36^3} = (\sqrt{36})^3 \left(= 6^3\right)$

(d) $\left(\frac{4}{9}\right)^{-\frac{1}{2}} = \dfrac{1}{\left(\frac{4}{9}\right)^{1/2}} = \dfrac{1}{\sqrt{\frac{4}{9}}} \left(= \dfrac{1}{\frac{2}{3}} = \dfrac{3}{2}\right)$

② (a) $\sqrt{7^3} = 7^{3/2}$ (b) $\sqrt[5]{3^2} = 3^{2/5}$

(c) $\sqrt[n]{a}\cdot\sqrt{a} = a^{\frac{1}{n}} \cdot a^{\frac{1}{2}} = a^{\frac{1}{n}+\frac{1}{2}} = a^{\frac{n+2}{2n}}$ $(a \geq 0)$

(d) $\dfrac{a}{\sqrt[3]{a^2}\,\sqrt[4]{a}} = \dfrac{a}{a^{2/3}a^{1/4}} = \dfrac{a}{a^{2/3+1/4}} = \dfrac{a}{a^{\frac{8+3}{12}}} = \dfrac{a}{a^{11/12}} = a^{1-\frac{11}{12}} = a^{\frac{1}{12}}$ $(a>0)$

③ (a) $a^{\frac{3}{4}} \cdot a^{\frac{4}{5}} = a^{\frac{15+16}{20}} = a^{\frac{31}{20}} = \sqrt[20]{a^{31}} = \sqrt[20]{a^{20}\cdot a^{11}} = a\sqrt[20]{a^{11}}$ (a

(b) $\left(\dfrac{6a^2}{b}\right)^{3/5} \cdot \left(\dfrac{b}{8a}\right)^{3/5} = \dfrac{6^{3/5} \cdot a^{6/5}}{\cancel{b^{3/5}}} \cdot \dfrac{\cancel{b^{3/5}}}{8^{3/5}a^{3/5}} = \dfrac{6^{3/5} \cdot a^{6/5-3/5}}{8^{3/5}}$

$= \left(\dfrac{6}{8}\right)^{3/5} \cdot a^{3/5} = \left(\dfrac{3}{4}\right)^{3/5} a^{3/5} = \left(\dfrac{3a}{4}\right)^{3/5} = \sqrt[5]{\left(\dfrac{3a}{4}\right)^3}$ $\left(\begin{matrix} b \neq 0 \\ a \neq 0 \end{matrix}\right.$

(c) $(a^3b^3)^{\frac{1}{4}} \cdot (a^{11} b)^{\frac{1}{12}} = a^{\frac{3}{4}} b^{\frac{3}{4}} \cdot a^{\frac{11}{12}} b^{\frac{1}{12}}$

$= a^{\frac{3}{4}+\frac{11}{12}} \cdot b^{\frac{3}{4}+\frac{1}{12}} = a^{\frac{9+11}{12}} \cdot b^{\frac{9+1}{12}}$

$= a^{\frac{20}{12}} \cdot b^{\frac{10}{12}} = a^{5/3}\, b^{5/6} = \sqrt[3]{a^5}\ \sqrt[6]{b^5}$ $(a, b \geq 0)$

Act 9

Soit x le contenu initial.

après 1 jour, il reste : $x - \frac{1}{10}x = x\left(1 - \frac{1}{10}\right) = x \cdot \frac{9}{10}$

" 2 jours, " " : $\left(x \cdot \frac{9}{10}\right) - \frac{1}{10}\left(x \cdot \frac{9}{10}\right) = \left(x \cdot \frac{9}{10}\right)\left[1 - \frac{1}{10}\right] = x \cdot \frac{9}{10} \cdot \frac{9}{10} = x\left(\frac{9}{10}\right.$

" 3 " , " " : $\ldots\ x\left(\frac{9}{10}\right)^3$

⋮

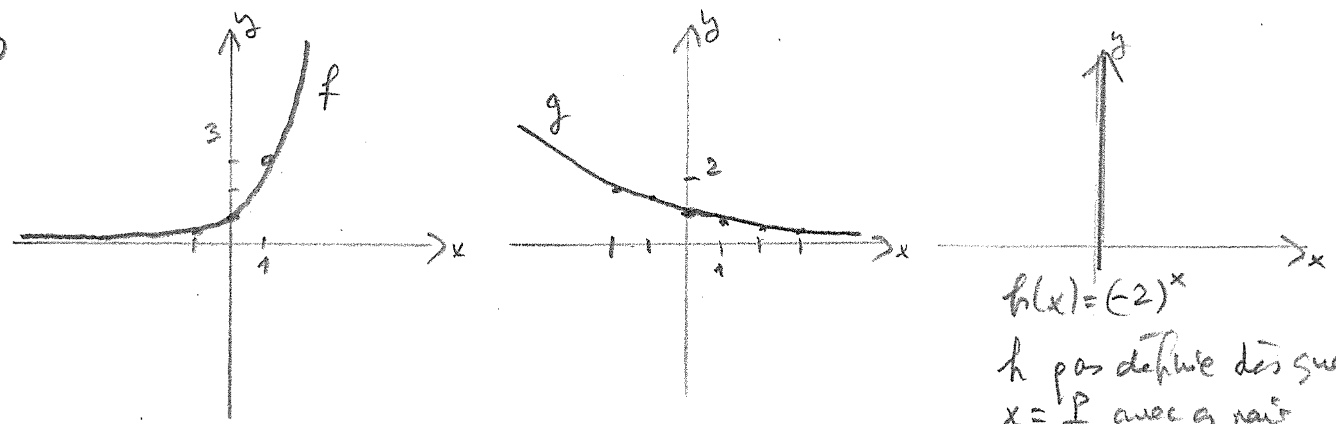
" n jours, " " : $x \cdot \left(\frac{9}{10}\right)^n$

On cherche n tel que $x \cdot \left(\frac{9}{10}\right)^n = \frac{x}{2} \Leftrightarrow \left(\frac{9}{10}\right)^n = \frac{1}{2}$

C'est une équation exponentielle … qu'on va apprendre à résoudre !

Act 10

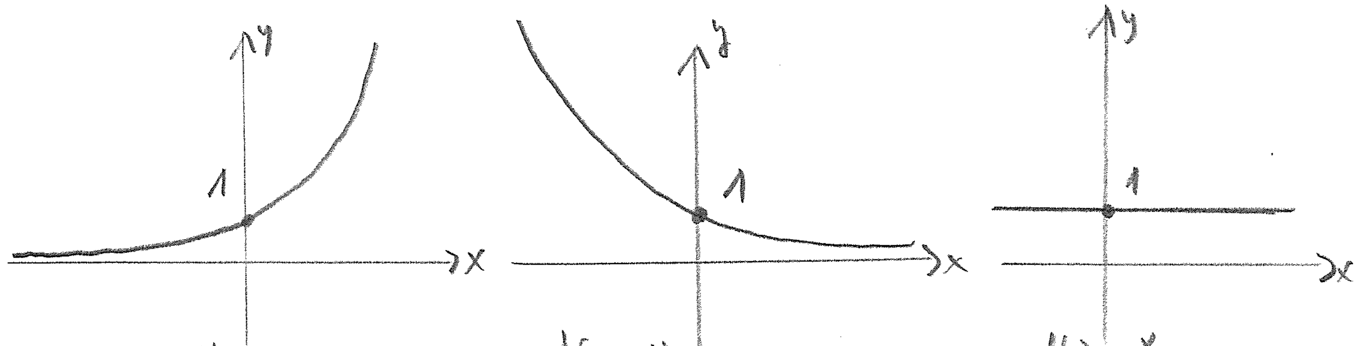
(1)



$h(x) = (-2)^x$

h pas définie dès que $x = \frac{p}{q}$ avec q pair

(2)



$f(x) = a^x$ avec $a > 1$ $\qquad$ $f(x) = a^x$ avec $0 < a < 1$ $\qquad$ $f(x) = a^x$ avec $a = 1$

si $a < 0$: pas défini pour trop de valeurs de x $\Rightarrow$ on ne les considère pas

Act 11

(a) $7^{3x} = 7^{2x+5} \Leftrightarrow 3x = 2x+5 \Leftrightarrow x = 5$

car $f : \mathbb{R} \to \mathbb{R}_*^+$, $x \mapsto 7^x$ est bijective $\qquad$ $S = \{5\}$

(b) $10^{-100x} = 0{,}01^{x-4} \Leftrightarrow 10^{-100x} = (10^{-2})^{x-4}$

$\Leftrightarrow 10^{-100x} = 10^{-2x+8}$

car $f : \mathbb{R} \to \mathbb{R}_*^+$, $x \mapsto 10^x$ bijective

$\Leftrightarrow -100x = -2x+8 \Leftrightarrow -98x = 8 \Leftrightarrow x = -\frac{8}{98} = -\frac{4}{49}$

$S = \left\{-\frac{4}{49}\right\}$

(c) $2^x = 4\sqrt{2} \Leftrightarrow 2^x = 2^2 \cdot 2^{1/2} \Leftrightarrow 2^x = 2^{5/2}$

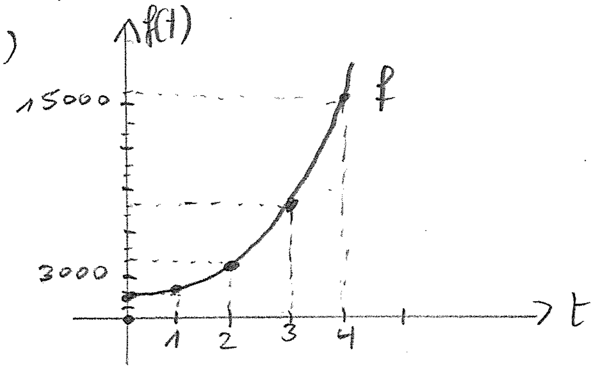
$\Leftrightarrow x = \frac{5}{2}$ $\qquad$ $S = \left\{\frac{5}{2}\right\}$

car $f : \mathbb{R} \to \mathbb{R}_*^+$, $x \mapsto 2^x$ bijective

Act 12

1 (a) $f(t) = 1000 \cdot 2^t$

(b)



(c) graphiquement : $f(1,5) \simeq 2800$
avec la calculatrice : $f(1,5) \simeq 2828,4$

(d) t tel que $f(t) = 10 \cdot f(0)$

$$\cancel{1000} \cdot 2^t = 10 \cdot \cancel{1000}$$

$$2^t = 10$$

graphiquement : $t \simeq 3,3$ (avec un graphe plus précis et complet !)
algébriquement : ? [à suivre]

2 (a) $t_0 = 20$ mg

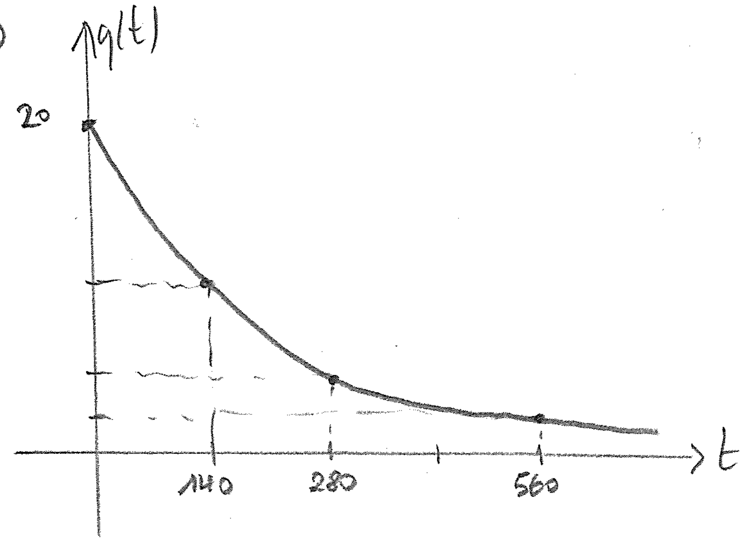
$t_{140} = \frac{20}{2} = 10$ mg

$t_{280} = t_{140}/2 = 5$ mg

$t_{560} = t_{280}/2 = 2,5$ mg

(b) $g(t) = 20 \cdot \left(\frac{1}{2}\right)^{\frac{t}{140}}$

(c)



d) $g(t) = 1$ [mg]

$\Leftrightarrow 20\left(\frac{1}{2}\right)^{\frac{t}{140}} = 1$

$\Leftrightarrow \left(\frac{1}{2}\right)^{\frac{t}{140}} = \frac{1}{20}$

$\Leftrightarrow 2^{\frac{t}{140}} = 20$

exploration avec la calc : $t \simeq 600$ [j]

(3) C_0 capital initial
i intérêt (annuel)

après 1 an : $C_0 + C_0 \cdot i = C_0(1+i)$

" 2 ans : $C_0(1+i) + [C_0(1+i)] \cdot i = C_0(1+i)[1+i] = C_0(1+i)^2$

" 3 ans : ... $C_0(1+i)^3$

" t ans : .... $C_0(1+i)^t$

donc, si on pose $C(n)$ le capital après n ans : $C(n) = C_0(1+i)^n$

(a) intérêt : $9\% \cdot 1000 = 90.-$
$C(1) = 1000(1+0{,}09)^1 = 1090.-$

(b) $C(2) = 1000(1+0{,}09)^2 = 1188.-10$

(c) $C(t) = C(1+0{,}09)^t = 1000(1{,}09)^t$ (d) $C(t) = C_0(1+i)^t$

(e) $\left.\begin{array}{l} C(10) = 1500.- \\ i = 3\% \text{ (annuel)} \\ C_0 \text{ inconnu} \\ t = 10 \text{ ans} \end{array}\right\} \Rightarrow 1500 = C_0 \cdot (1+0{,}03)^{10}$

$C_0 = \dfrac{1500}{1{,}03^{10}} \cong 1116.-15$

(f) $\left.\begin{array}{l} C_0 = 1000.- \\ i = 5\% \text{ (annuel)} \\ C(t) = 2000.- \end{array}\right\} \Rightarrow 2000 = 1000(1+0{,}05)^t$

$2 = 1{,}05^t$

exploration avec la calculatrice : $t \cong 14$ ans