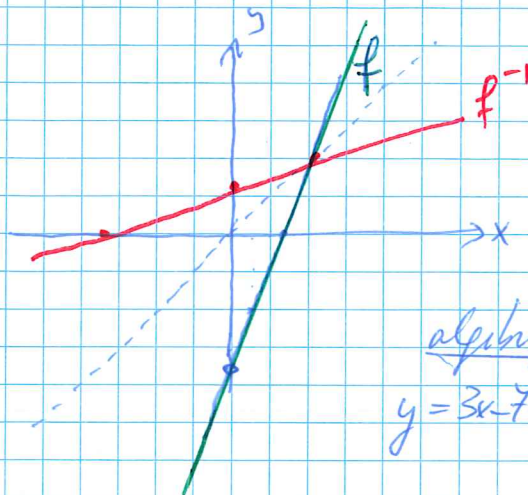


Act 9

(a) $f: \mathbb{R} \rightarrow \mathbb{R}$
 $x \mapsto 3x-7$

Géométriquement



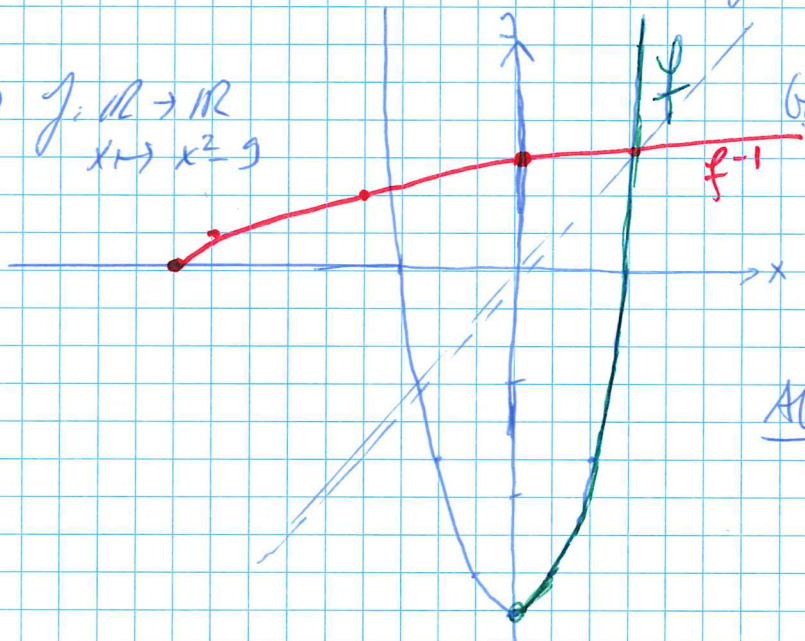
bij $\mathbb{R} \rightarrow \mathbb{R}$

algébriquement:

$$y = 3x - 7 \Leftrightarrow 3x = y + 7 \\ \Leftrightarrow x = \frac{y+7}{3}$$

fonction réciproque: à tout y correspond un unique x
 $f^{-1}: \mathbb{R} \rightarrow \mathbb{R}$
 $y \mapsto \frac{y+7}{3}$ ou $f^{-1}: \mathbb{R} \rightarrow \mathbb{R}$
 $x \mapsto \frac{x+7}{3}$

(b) $f: \mathbb{R} \rightarrow \mathbb{R}$
 $x \mapsto x^2 - 9$



Géom:

pas bij $\mathbb{R} \rightarrow \mathbb{R}$

bij $\mathbb{R}^+ \rightarrow [-9; +\infty[$

Alg: $y = x^2 - 9$
 $y + 9 = x^2$
 $x = \sqrt{y+9}$

pas bij de $\mathbb{R} \rightarrow \mathbb{R}$!

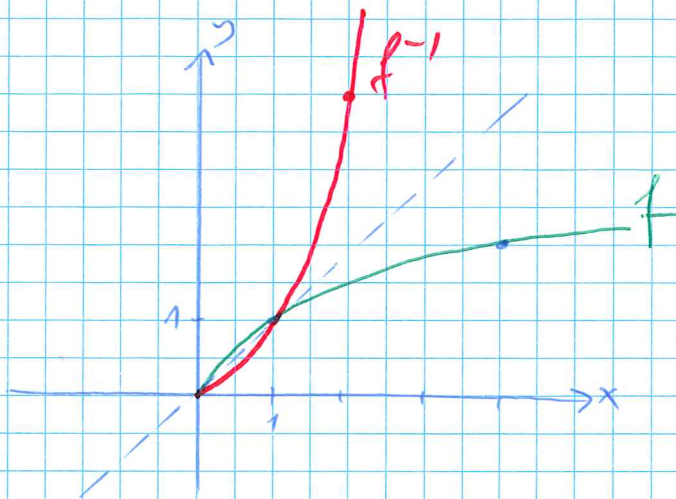
fonction réciproque: $f^{-1}: [-9; +\infty[\rightarrow \mathbb{R}^+$
 $y \mapsto \sqrt{y+9}$ ou $f^{-1}: [-9; +\infty[\rightarrow \mathbb{R}^+$
 $x \mapsto \sqrt{x+9}$

c) $f: \mathbb{R} \rightarrow \mathbb{R}$
 $x \mapsto \sqrt{x}$

pas bij $\mathbb{R} \rightarrow \mathbb{R}$
 bij $\mathbb{R}^+ \rightarrow \mathbb{R}^+$

$$y = \sqrt{x} \Leftrightarrow x = y^2$$

$$f^{-1}: \mathbb{R}^+ \rightarrow \mathbb{R}^+ \quad \text{ou} \quad f^{-1}: \mathbb{R}^+ \rightarrow \mathbb{R}^+ \\ y \mapsto y^2 \quad x \mapsto x^2$$

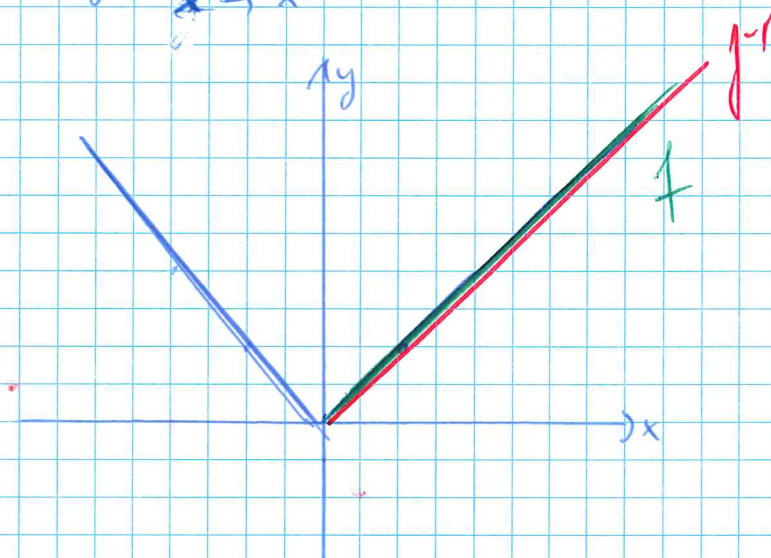


d) $f: \mathbb{R} \rightarrow \mathbb{R}$
 $x \mapsto |x|$

pas bij $\mathbb{R} \rightarrow \mathbb{R}$
 bij $\mathbb{R}^+ \rightarrow \mathbb{R}^+$

$$y = |x| \Leftrightarrow y = x \quad \text{si } x \geq 0$$

$$f^{-1}: \mathbb{R}^+ \rightarrow \mathbb{R}^+ \quad \text{ou} \quad f^{-1}: \mathbb{R}^+ \rightarrow \mathbb{R}^+ \\ y \mapsto y \quad x \mapsto x \quad (\text{ou } |x|)$$



e) $f: \mathbb{R} \rightarrow \mathbb{R}$
 $x \mapsto \sqrt{4-x^2}$

pas bij $\mathbb{R} \rightarrow \mathbb{R}$
 bij $\mathbb{R}[0,2] \rightarrow [0,2]$

$$y = \sqrt{4-x^2} \Leftrightarrow y^2 = 4-x^2$$

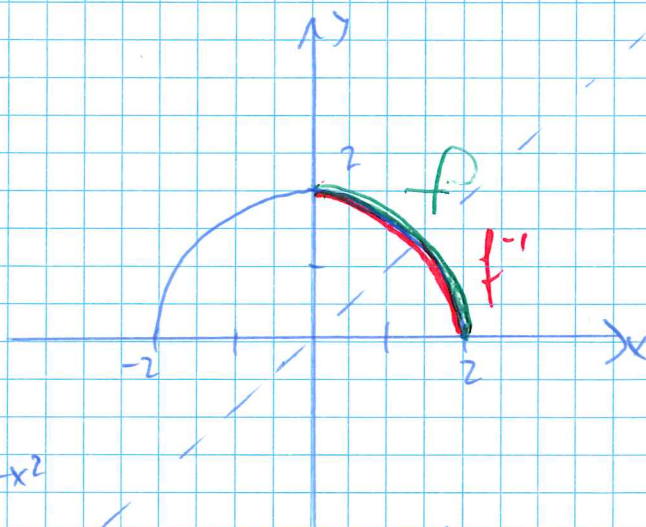
$$\Leftrightarrow x^2 = 4-y^2$$

$$\Leftrightarrow x = (\pm)\sqrt{4-y^2}$$

représente les 2 $\frac{1}{2}$ cercles!

$$f^{-1}: [0,2] \rightarrow [0,2] \\ y \mapsto \sqrt{4-y^2}$$

$$\text{ou} \\ f^{-1}: [0,2] \rightarrow [0,2] \\ x \mapsto \sqrt{4-x^2}$$



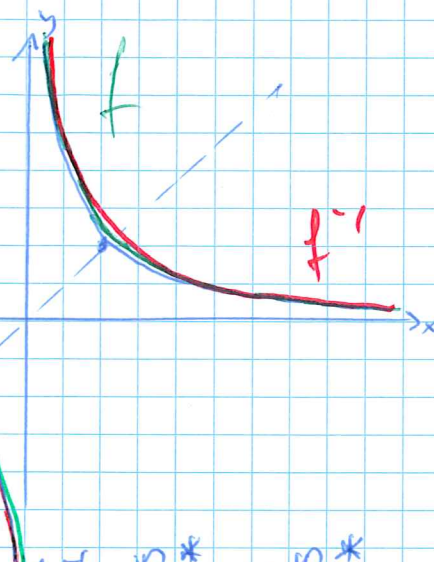
$$f) : f: \mathbb{R} \rightarrow \mathbb{R} \\ x \mapsto \frac{1}{x}$$

per bij $\mathbb{R} \rightarrow \mathbb{R}$
bij $\mathbb{R}^* \rightarrow \mathbb{R}^*$

$$y = \frac{1}{x} \Leftrightarrow x = \frac{1}{y}$$

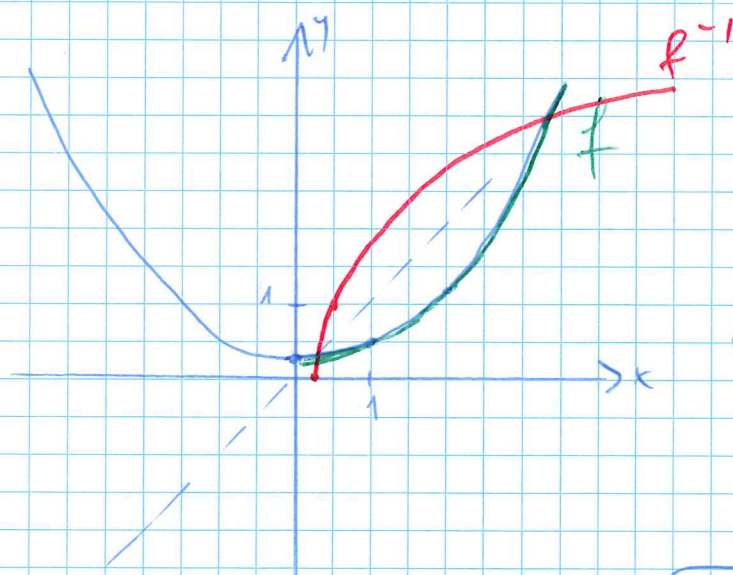
$$f^{-1}: \mathbb{R}^* \rightarrow \mathbb{R}^* \\ y \mapsto \frac{1}{y}$$

$$f^{-1}: \mathbb{R}^* \rightarrow \mathbb{R}^* \\ x \mapsto \frac{1}{x}$$



Akt 10

$$\boxed{1} \boxed{3} y = \frac{x^2+1}{4} \Leftrightarrow x^2+1 = 4y \Leftrightarrow x^2 = 4y-1 \\ \Leftrightarrow x = \pm \sqrt{4y-1}$$



$$f \text{ per bij } \mathbb{R} \rightarrow \mathbb{R} \\ f \text{ bij } \mathbb{R}^+ \rightarrow \left[\frac{1}{4}; +\infty\right[\\ f^{-1}: \left[\frac{1}{4}; +\infty\right[\rightarrow \mathbb{R}^+ \\ x \mapsto \sqrt{4x-1}$$

$$\boxed{2} f^{-1}(f(x)) = f^{-1}\left(\frac{x^2+1}{4}\right) = \sqrt{4\left(\frac{x^2+1}{4}\right)-1} = \sqrt{x^2} = x \\ \begin{matrix} \uparrow \\ x \in \mathbb{R}^+ \end{matrix} \quad \begin{matrix} \uparrow \\ x \geq 0 \end{matrix}$$

$$f(f^{-1}(x)) = f(\sqrt{4x-1}) = \frac{(\sqrt{4x-1})^2+1}{4} = \frac{4x-1+1}{4} = x$$

14 (a) $f: \mathbb{R} \rightarrow \mathbb{R}$ bij
 $x \mapsto \frac{3}{4}x + 1$

$$y = \frac{3}{4}x + 1 \Leftrightarrow 4y = 3x + 4 \Leftrightarrow x = \frac{4y - 4}{3}$$

$$f^{-1}: \mathbb{R} \rightarrow \mathbb{R} \quad \approx \quad f^{-1}: \mathbb{R} \rightarrow \mathbb{R}$$

$$y \mapsto \frac{4y - 4}{3} \quad x \mapsto \frac{4x - 4}{3}$$

(b) $f: \mathbb{R} \rightarrow \mathbb{R}$ pos bij
 $x \mapsto 2\sqrt{x-3}$

$$f: [3; +\infty[\rightarrow \mathbb{R}^+ \text{ bij}$$

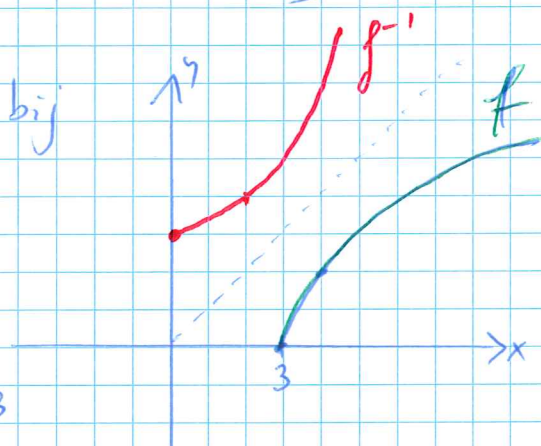
$$x \mapsto 2\sqrt{x-3}$$

$$y = 2\sqrt{x-3} \Leftrightarrow \left(\frac{y}{2}\right)^2 = x-3$$

$$\Leftrightarrow x = \frac{y^2}{4} + 3$$

$$f^{-1}: \mathbb{R}^+ \rightarrow [3; +\infty[$$

$$x \mapsto \frac{x^2}{4} + 3$$



(c) $f: \mathbb{R}^+ \rightarrow \mathbb{R}^+$ bij $f \text{ and } g$
 $x \mapsto \frac{1}{x}$

(d) $f: \mathbb{R} \rightarrow \mathbb{R}$ pos bij
 $x \mapsto 4x^2 - 9$

$$f: \mathbb{R}^+ \rightarrow [-9; +\infty[\text{ bij}$$

$$y = 4x^2 - 9$$

$$\Leftrightarrow 4x^2 = y + 9$$

$$\Leftrightarrow x = \pm \sqrt{\frac{y+9}{4}} = \pm \frac{1}{2} \sqrt{y+9}$$

$$f^{-1}: [-9; +\infty[\rightarrow \mathbb{R}^+$$

$$x \mapsto \frac{1}{2} \sqrt{y+9}$$

