Ma2 ; Ch 6 ; Exercice

a) $\boxed{f(x) = -2\cdot\sin(3x)}$

a) $D_f : \mathbb{R}$ ✓

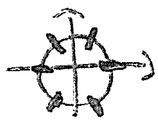
b) <u>images :</u>

$f\left(\frac{\pi}{6}\right) = -2$ $\quad f(0) = 0$ $\quad f\left(\frac{\pi}{2}\right) = 2$ ✓ $\quad f(\pi) = 0$ $\quad f\left(-\frac{\pi}{2}\right) = -2$

c) <u>ensemble des zéros :</u>

Pour que $f(x)=0$, il faut que $\sin(3x) = 0$

$\Rightarrow 3x = k\pi \rightarrow x = k\cdot\frac{\pi}{3}$ ✓



période :

$-2\sin(3(x+P)) = -2\sin(3x)$

$3(x+P) = 3x + 2k\pi$ (ou ...

$P = \frac{2k\pi}{3}$

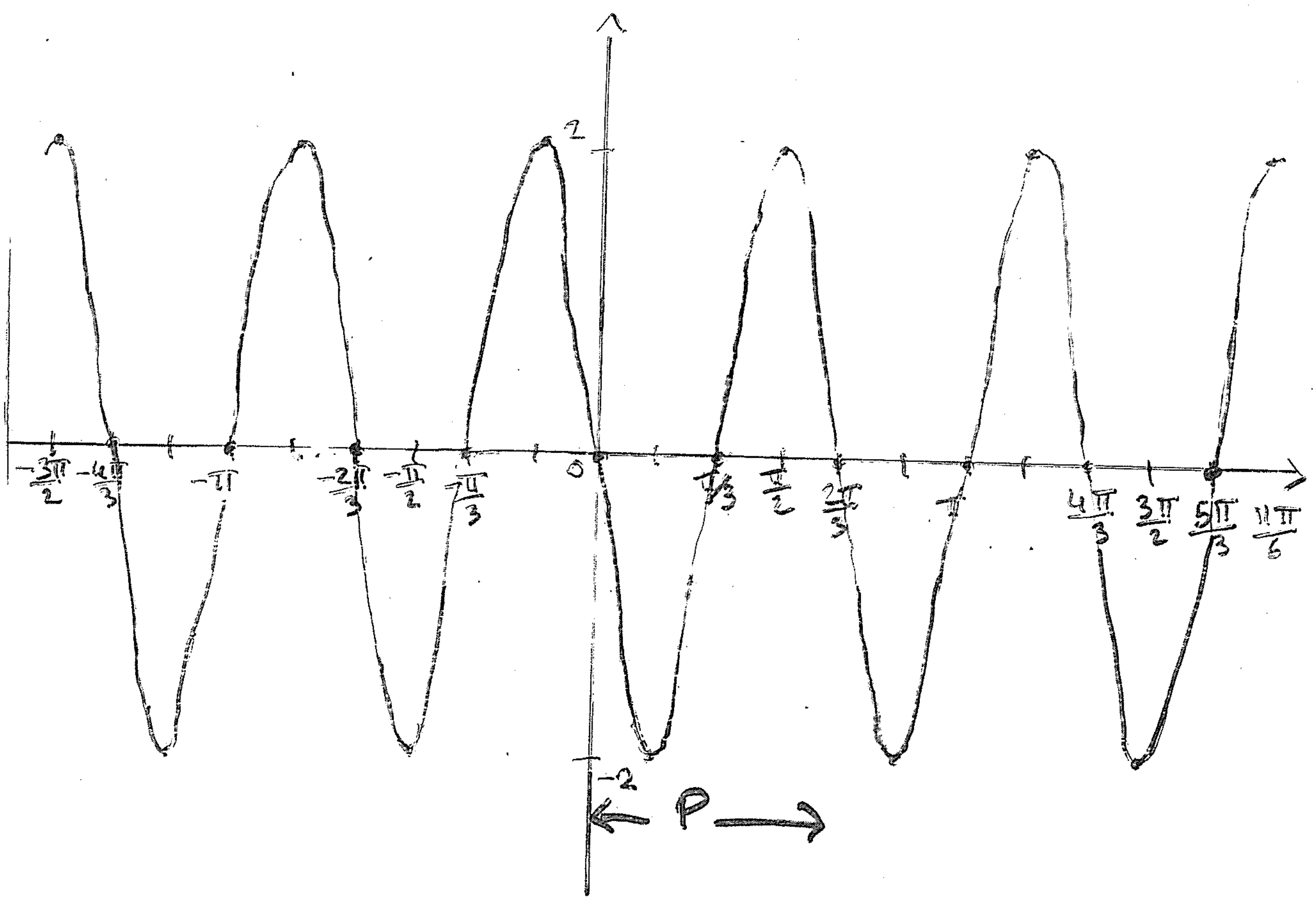
↳ $\boxed{P = \frac{2\pi}{3}}$

d) valeur maximale : 2 ✓

valeur minimale : −2

e) <u>tableau des signes :</u>

x			$-\frac{2\pi}{3}$		$-\frac{\pi}{3}$		0		$\pi/3$		$\frac{2\pi}{3}$	
$f(x)$	...	+	0	−	0	+	0	−	0	+	0	− ...



6)

$f(x) = 3\sin\left(\frac{x}{2}\right)$

a) $D_f = \mathbb{R}$ ✓

b) $f(0) = 0$; $f(\pi) = 3$;
$f(2\pi) = 0$; $f(3\pi) = -3$;
$f(4\pi) = 0$; $f(-\pi) = -3$;
$f(-2\pi) = 0$; $f(-3\pi) = 3$;
$f(-4\pi) = 0$ ✓

c) $S = \{2k\pi\}$
+ ... (détails) ✓

d) minimum : -3
maximum : 3 ✓

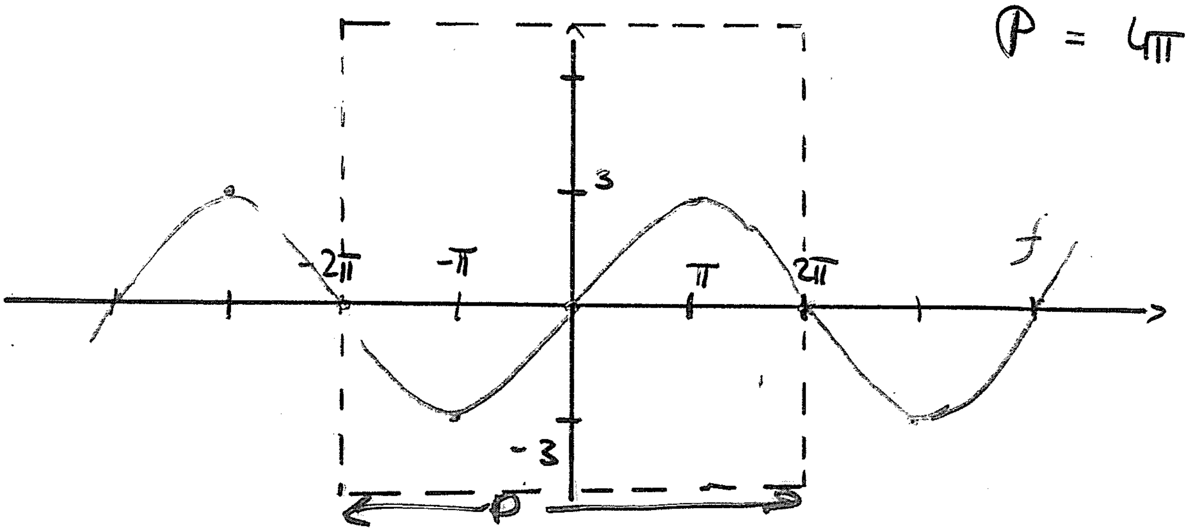
e)

x		-2π		0		2π		4π	
$f(x)$	+	0	−	0	+	0	−	0	+

période :
$\cancel{3}\sin\left(\frac{x+P}{2}\right) = \cancel{3}\sin\left(\frac{x}{2}\right)$
$\frac{x+P}{2} = \frac{x}{2} + 2k\pi$ (ou ...
$P = 4k\pi$
↳ $\boxed{P = 4\pi}$

f)

$P = 4\pi$



c) $f(x) = \sin\left(2x + \frac{\pi}{2}\right)$

a) $D_f = \mathbb{R}$ ✓

b) $f(0) = 1$ $\quad f\left(\frac{\pi}{4}\right) = 0$ $\quad f\left(\frac{\pi}{2}\right) = -1$ $\quad f(\pi) = 1$

$f\left(\frac{3\pi}{4}\right) = 0$ ✓

$f\left(-\frac{\pi}{4}\right) = 0$ $\quad f\left(-\frac{\pi}{2}\right) = -1$ $\quad f\left(-\frac{3\pi}{4}\right) = 0$ $\quad f(-\pi) = 1$

c) $2x + \frac{\pi}{2} = 0 + 2k\pi$ $\qquad 2x + \frac{\pi}{2} = \cancel{\pi} + 2k\pi$

$\Leftrightarrow \quad x = -\frac{\pi}{4} + k\pi$ ou $\Leftrightarrow \quad x = \frac{\pi}{4} + k\pi$

$\Rightarrow$ zéros ds $\mathbb{R}$: $\left\{ \frac{\pi}{4} + \frac{k\pi}{2} \right\}$

d) min $= -1$ $\qquad$ max $= 1$

e) $f(x+P) = f(x)$

$\sin\left(2(x+P) + \frac{\pi}{2}\right) = \sin\left(2x + \frac{\pi}{2}\right)$

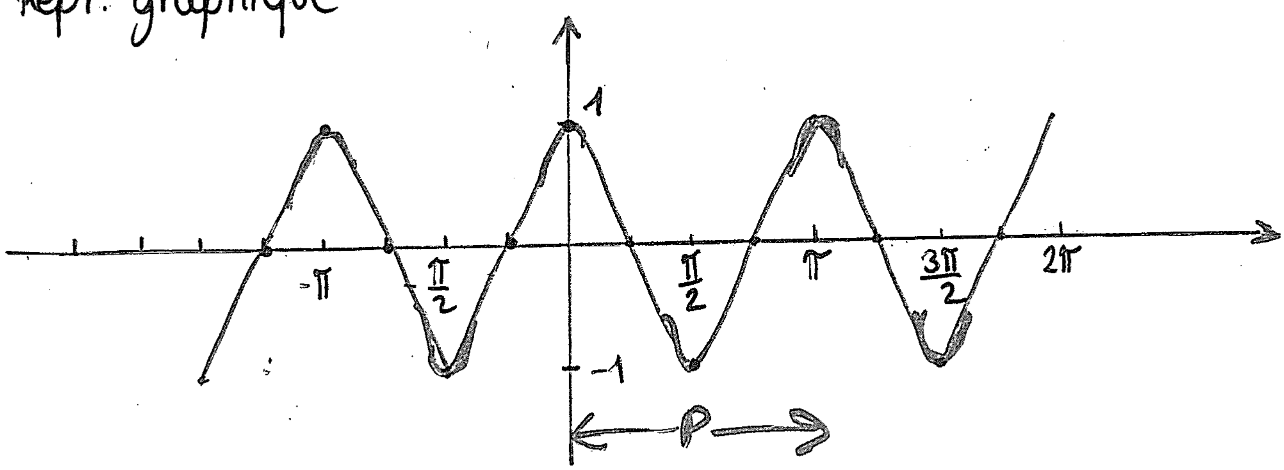
$2(x+P) + \frac{\pi}{2} = 2x + \frac{\pi}{2} + k2\pi$ $\qquad$ (ou ... $S = \emptyset$)

$\cancel{2x} + 2P + \cancel{\frac{\pi}{2}} = \cancel{2x} + \cancel{\frac{\pi}{2}} + k2\pi$

$\cancel{2}P = k\cancel{2}\pi$

$P = k\pi \rightarrow P = \pi$

f) Repr. graphique



1) $f(x) = 2\cos\left(\frac{3x}{4}\right)$

a. $D_f = \mathbb{R}$

b. $f(\pi) \simeq -1,41$ $\quad f\left(\frac{2\pi}{3}\right) = 0$ $\quad f\left(\frac{\pi}{3}\right) = \sqrt{2}$

$f\left(\frac{4\pi}{9}\right) = 1$ $\quad f\left(\frac{2\pi}{9}\right) = \sqrt{3}$

<u>période</u> :

$2\cos\left(\frac{3(x+P)}{4}\right) = 2\cos\left(\frac{3x}{4}\right.$

$\frac{3(x+P)}{4} = \frac{3x}{4} + k2\pi$ (ou ...

$P = \frac{8\pi}{3}$

c. $2\cos\left(\frac{3x}{4}\right) = 0$

$\frac{3}{4}x = \frac{\pi}{2} + k\pi$

$3x = 2\pi + 4k\pi$

$x = \frac{2\pi}{3} + \frac{4}{3}k\pi$

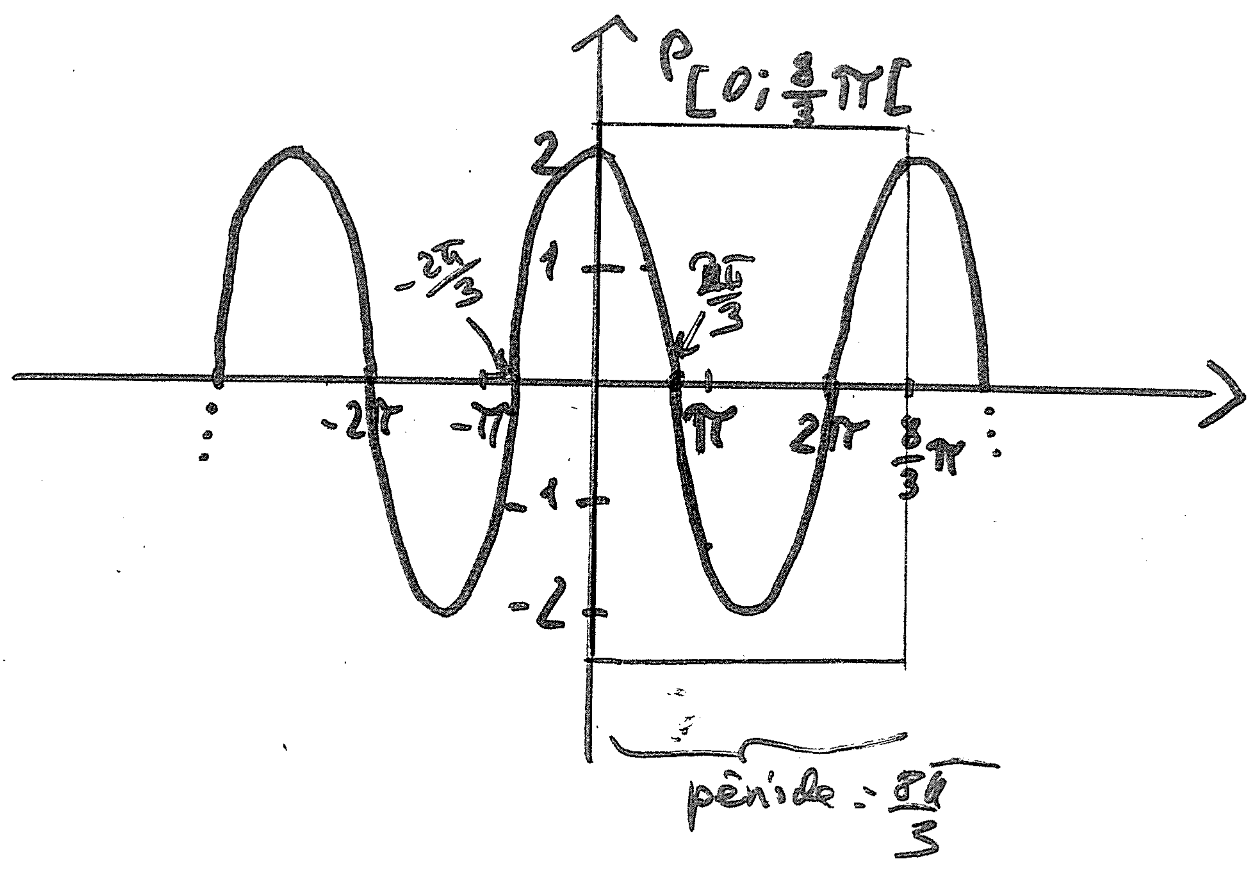
$S_{\mathbb{R}} = \left\{\frac{2\pi}{3} + \frac{4}{3}k\pi\right\}$ ✓

d. $E_f = [-2 ; 2]$ ✓

e.

x $[0; \frac{8}{3}\pi[$		$\frac{2}{3}\pi$		2π	
$2\cos\left(\frac{3}{4}x\right)$	+	0	−	0	+
$f(x)$	+	0	−	0	+

f.



e)

a) $D_f : \mathbb{R}$ ✓

$$\boxed{f(x) = -2\left(\cos\left(\frac{x}{3} + \frac{\pi}{6}\right)\right)}$$

c) "$-2\cos$" vaut zéro lorsque $\left(\frac{x}{3} + \frac{\pi}{6}\right)$ vaut $\frac{\pi}{2}$ ou $\frac{3\pi}{2}$ (ou pareil $+ k\pi 2$)

① $\longrightarrow \frac{x}{3} + \frac{\pi}{6} = \frac{\pi}{2} + k\pi 2$

$\frac{x}{3} = \frac{\pi}{2} - \frac{\pi}{6 + k2\pi} = \frac{2\pi}{6} + k2\pi$

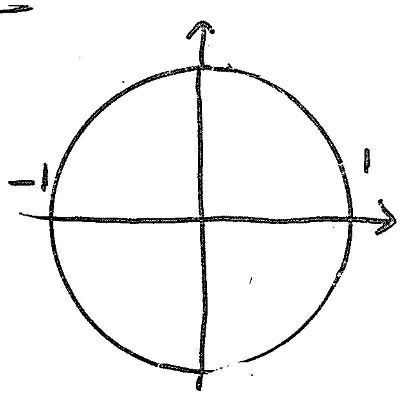
$\underline{x = \pi} + k6\pi$

② $\longrightarrow \frac{x}{3} + \frac{\pi}{6} = \frac{3\pi}{2} + k2\pi$

$\frac{x}{3} = \frac{3\pi}{2} - \frac{\pi}{6 + k2\pi} = \frac{8\pi}{6} + k2\pi$

$\underline{x = 4\pi} + k6\pi$

d)



Le cos max vaut +1 et le min vaut -1. Comme il y a un coefficient -2, cela correspond à -2 et +2 ✓

période: $-\cancel{2}\cos\left(\frac{x+P}{3} + \frac{\pi}{3}\right) = -\cancel{2}\cos\left(\frac{x}{3} + \frac{\pi}{6}\right)$

$\Leftrightarrow \frac{x+P}{3} + \frac{\pi}{3} = \frac{x}{3} + \frac{\pi}{6} + k2\pi$ ou ...
↳ pas de solution

$\frac{P}{3} = k2\pi$

$P = k6\pi \rightarrow \boxed{P = 6\pi}$

e)

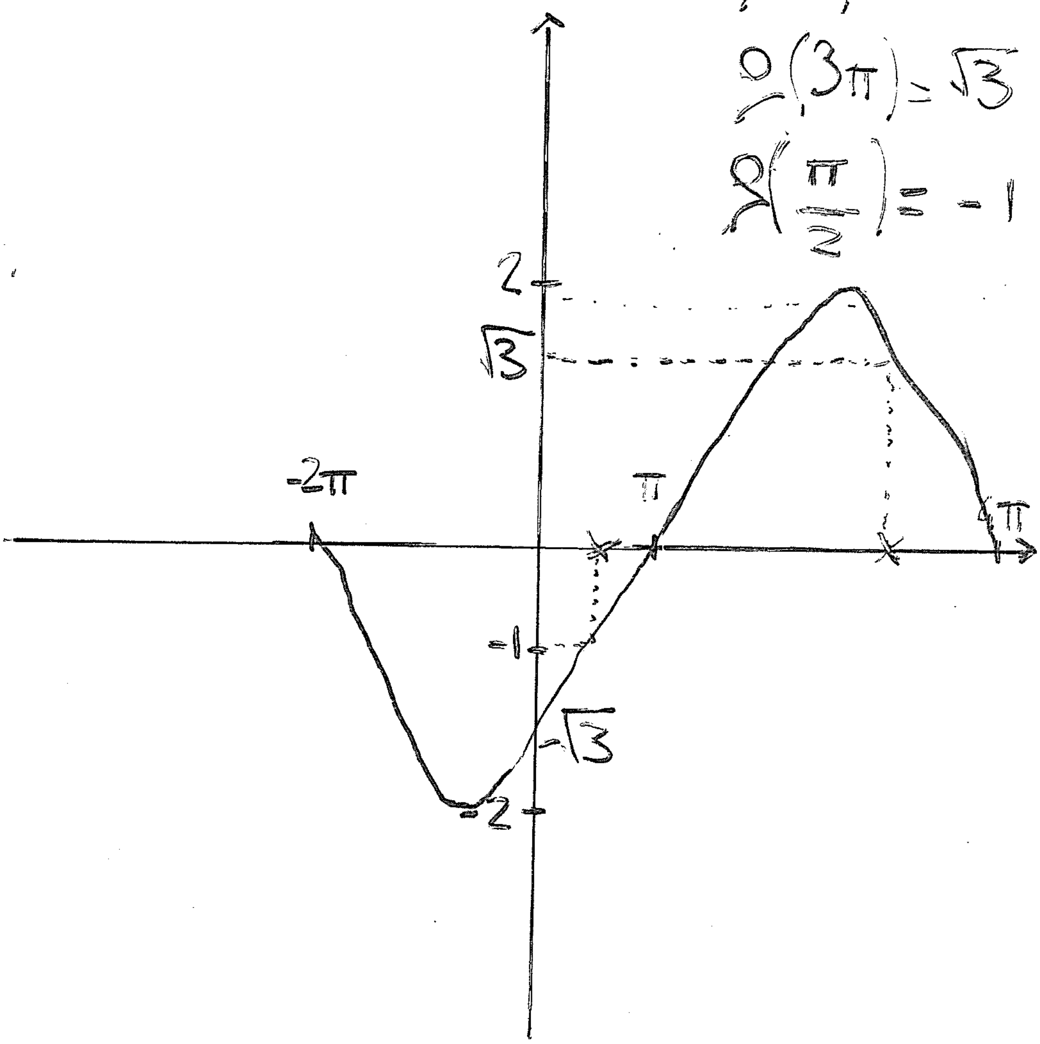
x		π		4π	
-2	$-$	$-$	$-$	$-$	$-$
$\cos\left(\frac{x}{3}+\frac{\pi}{6}\right)$	$+$	0	$-$	0	$+$
$f(x)$	$-$	0	$+$	0	$-$

Pour les autres on fait $+k2\pi$

$f(0) = -\sqrt{3}$

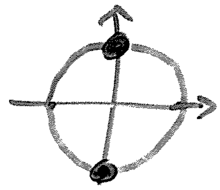
$f(3\pi) = \sqrt{3}$

$f\left(\frac{\pi}{2}\right) = -1$



4) $f(x) = -\tan\left(2x + \frac{\pi}{4}\right)$

a) D_f : problème si $2x + \frac{\pi}{4} = \frac{\pi}{2} + k\pi$



$\Leftrightarrow 2x = \frac{\pi}{4} + k\pi$

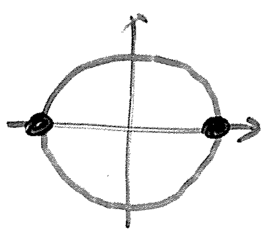
$\Leftrightarrow x = \frac{\pi}{8} + k\frac{\pi}{2}$

$D_f = \mathbb{R} \setminus \left\{ \frac{\pi}{8} + k\frac{\pi}{2} \mid k \in \mathbb{Z} \right\}$

b) $f(0) = -\tan\left(\frac{\pi}{4}\right) = -1$ $\quad f\left(\frac{\pi}{4}\right) = -\tan\left(\frac{3\pi}{4}\right) = 1$ $\quad f\left(-\frac{\pi}{4}\right) = -\tan\left(-\frac{\pi}{4}\right) =$

$f\left(\frac{\pi}{2}\right) = -1$

c) zéros : $-\tan\left(2x + \frac{\pi}{4}\right) = 0$



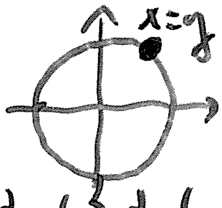
$\Leftrightarrow 2x + \frac{\pi}{4} = k\pi$

$\Leftrightarrow 2x = -\frac{\pi}{4} + k\pi$

$\Leftrightarrow x = -\frac{\pi}{8} + k\frac{\pi}{2}$

[+] période : $f(x+P) = f(x)$

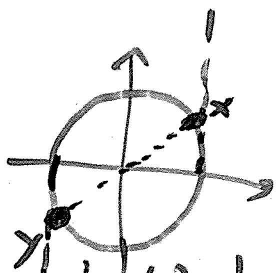
$\Leftrightarrow \cancel{-}\tan\left(2(x+P) + \frac{\pi}{4}\right) = \cancel{-}\tan\left(2x + \frac{\pi}{4}\right)$



(tan(x) = tan(y))

$$\underbrace{\left[2(x+P) + \frac{\pi}{4}\right] = \left[2x + \frac{\pi}{4}\right] + 2k\pi}_{\text{cas 1}}$$

ou



tan(x) = tan(y)

$$\underbrace{\left[2(x+P) + \frac{\pi}{4}\right] = \left[2x + \frac{\pi}{4} + \pi\right] + 2k\pi}_{\text{cas 2}}$$

cas 1 : $\cancel{2x} + 2P + \cancel{\frac{\pi}{4}} = \cancel{2x} + \cancel{\frac{\pi}{4}} + 2k\pi$

$\Leftrightarrow P = k\pi$

candidat minimal : $T = \pi$

cas 2 : $\cancel{2x} + 2P + \cancel{\frac{\pi}{4}} = \cancel{2x} + \cancel{\frac{\pi}{4}} + \pi + 2k\pi$

$\Leftrightarrow P = \frac{\pi}{2} + k\pi$

candidat minimal : $T = \frac{\pi}{2}$

donc $T = \frac{\pi}{2}$

d) pas de valeurs min ou max puisque la tangente prend des valeurs infinies (l'infini n'est pas un nombre !)

e)

x	$-\frac{\pi}{4}$		$-\frac{\pi}{8}$		0		$\frac{\pi}{8}$		$\frac{\pi}{4}$		$\frac{3\pi}{8}$		$\frac{\pi}{2}$
$f(x)$	+	+	0	−	−	−	‖	+	1	+	0	−	−

Une période

f)